Chapter 114

Common Mistakes Due to Mathematical Sophistry

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ABSTRACT
Mathematics is present in people's daily lives, whether formally in the school or academic environment, or in places where formalities are not necessarily observed. Using mathematical concepts carelessly can generate results that do not satisfy or do not represent reality. Thus, the objective of this article was to present some sophistry in mathematics, expose some justifications and highlight the errors made in the arguments. From basic expressions, known to a large part of the students, it was demonstrated how to obtain mathematical absurdities. Some resolutions presented were used, generally by students who do not pay attention to the definitions and hypotheses necessary for their correct applications. Also a theorem has been created that affirms the obvious. Thus, it can be concluded that when using invalid mathematical arguments in a demonstration or application, such as those that violate axioms, logical arguments, definitions, or hypotheses, it is possible to deduce contradictions or frightening results, which can bring surprises.

Keywords: Mathematical expressions; Formal education; Fallacies.

1 INTRODUCTION

Mathematical operations and expressions are present in everyday everyday situations, being used from fairs to scientific research. However, its applications are, on several occasions, unnoticed by its users because they are outsidethe educational scope (Castro & Pereira, 2020).

The use of mathematical software for problem solving has been increasing over the years. Being used and analyzed more than a decade ago (Romeiro, et al., 2021; Ribeiro, et al., 2020; Silva, et al., 2012), having an exceptional increase with the establishment of the pandemic caused by SARS-CoV-2, better known as COVID-19 (Holanda & Soares, 2021). However, the use of these softwares in the resolution of mathematical expressions without due care with their necessary hypotheses, can cause an even greater disorder, because students come to believe in the response provided by the machine, for granted, regardless of any speculation. Even with the use of different teaching techniques, the importance in the hypotheses of each mathematical concept should be perceived by students (Sobral & Soares, 2012; Castro, 2020; Santo & Santos, 2020).

However, often the use of some mathematical techniques without taking due care with the necessary hypotheses for such application can lead to quite controversial results. By using arguments that are not valid in a resolution or demonstration, such as those that injure axiomas, logical arguments, or definitions, it is possible to deduce contradictions or frightening results, which can bring surprises.

Several times, these arguments are purposely elaborated, so that, from true premises, it is possible to deduce false conclusions (it is common for some politicians to use this type of argument). In this case, we say that, this type of reasoning is a sophism (or fallacy). Sophism or fallacy is a sequence of seemingly valid arguments that can be used to deduce false results. The term sophism comes from the philosophers of ancient Greece called Sophists (4th century BC), against which the philosopher Socrates (470-399BC) opposed through the School of Plato and Aristotle. Freitas (2012) conceptualizes sophistry as logical errors, whether conscious or not, used by deceivers, in order to deceive and form misconceptions, leading to illegitimate prejudices, stereotypes and, thus provoking, bad decisions.

Sophism are little worked in basic education classrooms, and they could be used in appropriate situations, and, as long as they are used correctly, they become a tool to arouse students' curiosity. Thus, such sophism could become a fun way to reinforce the learning of certain mathematical contents, contributing to a greater and more motivated learning (Machado, et al., 2013, Souza, et al., 2019).

Although it may be thought that such inadequate premises are present only in expressions present in elementary school, it would be a mistake to accept such a statement. In this sense of emphasizing logical errors that promulgate absurdities and that are present in some specific topics of mathematics, Klymchuk and Staples (2013) presented a large amount of sophistries and mathematical paradoxes, aimed at a first year of a differential and integral calculus course. They addressedsome of thepossibilities of sophistry in functions, boundaries, derivatives and integrals.

However, it is worth mentioning that a sophist is not a paradox! A paradox is a self-contradictory sentence to which one arrives by valid arguments. Lages (2014), presents five very interesting mathematical paradoxes, that is: barber, liar, unexpected hanging, the problem of Monty Hall and the Achilles race and the turtle. The paradoxes in mathematics consist of an interesting area of mathematical logic, which amuse and can be used in the introduction of various themes, whether in elementary school, high school and higher education (Pinto, 2010; Brasileiro Filho, 2010; Dorta, 2013, Monteiro & Mondini, 2019).

Thus, the aim of this study is to highlight some errors, usually made, in the resolution of mathematical equations, resulting from the lack of observance of the necessary hypotheses.

2 METHODOLOGY

The methodology of this work consists in the presentation of some examples, the demonstration of these properties and the application of these mathematical concepts, without the necessary attention to their hypotheses, leading to mathematical absurdities.

2.1 Distributivity in the multiplication of radicals

In elementary school we learn that we can separate a radica that is composed of a product in product of two radicals, that is:

$$\sqrt{a \cdot b} = \sqrt{a} \cdot \sqrt{b} \quad (1)$$

This property can be demonstrated as follows (BARTLE 2019): be and then we have:$x_0 = \sqrt{a} y_0 = \sqrt{b}$

$$x_0 y_0 = \sqrt{a} \cdot \sqrt{b} \quad (2)$$

Like and. As soon as$x_0^2 = a y_0^2 = b$

$$(x_0 y_0)^2 = x_0^2 y_0^2 = ab \Rightarrow x_0 y_0 = \sqrt{ab} \quad (3)$$

When comparing equations (2) and (3), we obtain equation (1).

$$\sqrt{ab} = \sqrt{a}\sqrt{b}$$

Using this property and the imaginary unit i, where $i^2 = -1$, we can be led to "prove" that, through the $1 = -1$following sophistry:

$$1 = \sqrt{1}$$
$$\sqrt{1} = \sqrt{(-1).(-1)}$$
$$1 = \sqrt{(-1)} \cdot \sqrt{(-1)}$$
$$1 = i \cdot i$$
$$1 = i^2$$
$$1 = -1$$

2.2 PRODUCT EQUAL TO ZERO

When we have a product of two (or more) numbers, where the result of this product equals to zero, necessarily one of them is the null element, i.e.: $\forall a, b \in \mathbb{R}$

$$ab = 0 \Leftrightarrow a = 0 \; ou \; b = 0 \quad (4)$$

Proof of this statement can be easily obtained (LIMA 2020). Suppose, then, it comes:$a \neq 0$

$$ab = 0 \Rightarrow (a^{-1}a)b = a^{-1}0 = 0 \Rightarrow b = 0 \quad (5)$$

Proceeding analogously, if then$b \neq 0$ $a = 0$, and therefore or or . $a = 0 b = 0$Like this:

Step 1 - be two numbers any equal

$$x = y$$

Step 2 – we can multiply both sides by any number, including one of those already present in the equation, i.e.:

$$x^2 = xy$$

Step 3 - we can add the opposite number to both sides, getting:

$$x^2 - xy = 0$$

Step 4 – as we have elements common to the monomials of the equation, we can put it into evidence, that is:

$$x(x - y) = 0$$

step5 – and thus, using (erroneously) the principle of multiplying two terms to be equal to zero, conclude that necessarily

$$x = 0$$

2.3 DIFFERENCE SQUARE

Considering that , we $(a-b)^2 = a^2 - 2a.b + b^2$can demonstrate that the numbers 2 and 3 are the same, following the following steps:

Step 1 - any number is equal to itself, let's say:

$$-6 \;\; = -6$$

Step 2 - we can write this number as a result of different expressions

$$4 - 10 = 9 - 15$$

Step 3 - we can also add the same number to both sides of an equation, without modifying its result:

$$4 - 10 + \frac{25}{4} = 9 - 15 + \frac{25}{4}$$

Step 4 – we can write this new expression as remarkable square product developments of difference:

$$2^2 - 2 \cdot 2 \cdot \left(\frac{5}{2}\right) + \left(\frac{5}{2}\right)^2 = 3^2 - 2 \cdot 3 \cdot \left(\frac{5}{2}\right) + \left(\frac{5}{2}\right)^2$$

Step 5 – now we contract this remarkable product, finding:

$$\left(2-\frac{5}{2}\right)^2=\left(3-\frac{5}{2}\right)^2$$

Step 6 – as both sides of the equation are squared, we can simplify it by finding that:

$$2-\left(\frac{5}{2}\right)=3-\left(\frac{5}{2}\right)$$

Step 7 – finally we can simplify the expression by eliminating the common terms and concluding that:

$$2=3$$

2.4 LOGICAL IMPLICATIONS

Last but not least, we leave a theorem that is the dream of many students:

The Fundamental Theorem: All students will be approved.

Demonstration:

(i) In class X students are intelligent and unstudious, so, withdrawing the students little scholars, there will be only scholarly students left, so they must be approved.

(ii) In class X students are very studious and few intelligent, so, removing the unintelligent students from the class, there will be only scholarly students left, so they must be approved.

(iii) In class X the students are very intelligent and studious, so that we will only have scholarly students, so they must be approved.

It follows from items (i),(ii) and (iii) that the only thing missing for all is approved: is to study.

Conclusion: Soon, all students will be approved.

3 RESULTS AND DISCUSSION(CAN BE SEPARATED OR TOGETHER) (SOURCE TNR 12 – LEFT ALIGNED)

As results, we will present considerations of how mathematical absurdities were generated.

3.1 DISTRIBUTIVITY IN THE MULTIPLICATION OF RADICALS

This absurdity stems from the lack of attention in not verifying the initial hypothesis of proof of property, which $\forall a,b\in R, a,b\geq 0\sqrt{a\cdot b}=\sqrt{a}\cdot\sqrt{b}$

3.2 PRODUCT EQUAL TO ZERO

It is quite common to find between exercises and evaluations to find this type of situation, where the student uses only part of the conclusion of a theorem. However, without realizing that the possibility that any one term of the equation can be zero, one can come to the absurdity that if two numbers are equal, necessarily it is zero.

3.3 DIFFERENCE SQUARE

The dismemberment of a remarkable product can also be of great for us to do so if we don't get into the details of the properties we use. In this way, due attention was not paid to the elimination of the powers existing in the expression (step 6), since the possibility of equality with the negative element of the expression was not considered. For a modular function, we have to:

$$\forall a, b \in R, a^2 = b^2 \Rightarrow |a| = |b| \tag{5}$$

The statement of this statement is presented in two cases, and are presented as follows (MORAIS FILHO, 2012):

1st Case: if then$x \geq 0$

$$|x|^2 = |x||x| = x \cdot x = x^2$$

2nd Case: if then$x < 0$

$$|x|^2 = |x||x| = (-x).(-x) = x^2$$

Consequently, of the two cases, I fear that:

$$\sqrt{(x^2)} = |x|$$

In the example above, to find the correct shape in the resolution, we have to realize that:

$$\sqrt{\left(2-\frac{5}{2}\right)^2} = \left|2-\frac{5}{2}\right| = \left|-\frac{1}{2}\right| = \frac{1}{2} \quad \text{and} \quad \sqrt{\left(3-\frac{5}{2}\right)^2} = \left|3-\frac{5}{2}\right| = \left|\frac{1}{2}\right| = \frac{1}{2}$$

3.4 LOGICAL IMPLICATIONS

In fact, the theorem says nothing, only uses logical implications levam to a single possibility, which students should study.

4 CONCLUSION

The article sought to show in an exemplified way how the use of mathematical concepts, without due knowledge of their hypotheses can promote mathematical absurdities. Four mathematical sophistry were presented, which used: zero equality product, modular equation, remarkable product, complex numbers, among others.

It was evidenced that sophistry are extremely important factors with regard to the application of mathematics, and that one cannot allow itself to be deceived with algebraic manipulations that induce error, since they can generate great damage, both in research and in everyday situations.

It is concluded that it is of fundamental importance for the correct learning and subsequent applications of mathematics, so that the manipulations of certain premises are not accepted in the demonstration and determinations of some results.

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