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Optimization algorithm and the inverse problems applied to the structural damage identification in steel beams with the use of fem and experimental data

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ABSTRACT

Optimization problems are recurrent both in academia and in industry, in addition, the conditions of structural elements throughout their useful life

when in use tend to acquire different damages arising from their natural deterioration or even due to the exceptional causes that may lead to these magnitudes. of different damages. In this sense, combining the identification of these damages through optimization processes in the continuous search for more appropriate means and consequently the improvement of the available information that characterizes the problems under study is essential, and constitutes one of the needs of current engineering, because of the evolutionary state structural elements when in use. In this work, it is proposed the adjustment of experimentally tested metallic beam structures, from a static analysis by the Finite Element Method (FEM) using ANSYS© to obtain displacements, as well as the use of inverse problems and an optimization method. From the adjusted models, the damage is simulated (reduction of the stiffness properties of the elements) in the structures, and, then, an optimization and damage identification technique is applied through the Differential Evolution Method (DE). The modeled and experimentally tested metallic beams presented mostly consistent results and the ED technique showed to have good potential for solving damage identification problems using Inverse Problems, managing to converge practically in all cases.

Keywords: Optimization, Finite Element Method, Inverse Problems, Damage Identification, Differential Evolution.

1 INTRODUCTION

As there is a multitude of engineering works that are at an advanced age or even that was poorly designed and executed, starting to manifest various deteriorations arising from various situations, facts that demonstrate the importance of the areas of damage prediction.

This research seeks to continue the advances achieved in the areas of Damage Prognosis (DP) and Structural Health Monitoring (SHM) in the field of engineering, with optimal processes aimed at increasingly better solutions for the problems that arise. present.

As stated by Yang et al. (2016) [1], optimization analyzes are crucially important in the design process to find a good balance between economy and safety in all areas of engineering.

It is often clear that maintenance, when it occurs, is only corrective and is only carried out when the work is on the threshold of its limited state of use or collapse. The field of engineering that involves the conservation of existing structures through preventive interventions programmed throughout their service life only becomes evident when a structural accident occurs with relevant work, hence the importance of related themes that often depend on effective action management with adequate periodic technical follow-up.

Optimization processes may have within their scope, for example, the intention of bringing the behavior of the numerical model closer to the experimental data, to make it more accurate, as is the case of this research. Damage models, on the other hand, can indicate variations in the stiffness and mass parameters of a structure. Destructive and non-destructive testing methods are used for identification/detection that helps detect failures or even changes in the properties of materials that make up certain structures. Numerical methods are also used, via the Finite Element Method (FEM) through calculations of natural frequency variations and vibration modes, before and after the onset of damage. Modal methods, derived from dynamic analysis, require a more refined study.

Another determining point depends on the specification of the cost / objective function, defined as the relationship between the experimental and numerical results, they must efficiently drive the optimization process, having as main properties, for example, the experimental data points on the curve and all experimental curves should have equal opportunities to be optimized and different units and/or the number of curves in each sub-goal should not affect the overall performance of the fixture. These two criteria must be met without manually choosing the weighting factors. However, for some specific non-analytical problems this is very difficult in practice. Null values of experimental or numerical models also make the task difficult.

The use of the Differential Evolution Method to detect damage in steel beams based on numerical and experimental results is proposed in this paper in the context of Structural Health Monitoring Methods.

2 THEORETICAL FRAMEWORK

2.1 OPTIMIZATION ALGORITHMS

The idea of the optimizer is to make successive changes to the damage variables of the damaged model to find the damage to the tested elements. The process that involves this procedure is composed of the Inverse Problem that will plot the state that the structure maintains.

Its efficiency is achieved through the choice of an adequate objective function and the design variables. A general scheme of an optimization problem demonstrated by Gomes et. al. (2016) [2], can be seen in Figure 1 below.

Figure 1 - Two-dimensional elastic structure where Ω -domain; Γ_u -boundary with fixed displacements $(\bar{u} = 0)$; Γ_t boundary with applied tractions; Γ_d -de - signboundary $(t=0)$; u -displacements; t -tractions; x_i - Cartesian coordinates

The optimization techniques used for the damage detection procedure essentially follow the measurement of the displacement from the damaged structure, the displacement calculation from the undamaged model, the verification of the convergence, and if this criterion is reached, the change to a new point, the calculation of the displacement, in the new point and the iteration from the convergence analysis. The advantages are the low sensitivity to noise compared to other techniques. It uses the static displacements and stiffness of each element since damage to structures is generally defined as a reduction in the stiffness of the element (Choi, 2002) [3].

In this context, from the structures modeled by the Finite Element Method (FEM) based on static experimental data (displacements), through inverse problems and optimization methods, the adjusted models will be simulated damage to the structures, testing their efficiency for this purpose and seeking the development of an ideal optimizer.

2.2 DIFFERENTIAL EVOLUTION METHOD

According to Storn and Price (1995) [4], proponents of the Differential Evolution method (DE – Differential Evolution), the classic version of this algorithm is very simple and presents some advantages, such as it has only three control parameters; it works with real domains, that is, it does not require that the design variables are encoded in binary numbers; it has good convergence properties and can be easily adapted for use in parallel computing.

According to Sobrinho et. al (2020) [5], the DE method uses algorithms that are based on the population of individuals. Each individual represents a search point in the space of potential solutions to a given problem and imitates nature principles to create optimization procedures.

The behavioral schemes of a Differential Evolution algorithm can be seen below in Figure 2.

A Tabela 1 apresenta o esquema de ED para o comportamento das variáveis envolvidas.

 Being a population-based optimizer, it starts by solving the problem by sampling the objective function at several starting points. Starting points can be randomly chosen or not, depending on available information about the search space. Classical Differential Evolution has four main phases: initialization, mutation based on vector difference, crossover/recombination, and selection. The algorithm is controlled by three parameters:

> ❖ I_NP is the size of i population and the number of competing solutions in a given generation G (I_itermax = maximum number of iterations or generations). It can also be called the number of vectors in the population. This population size is directly proportional to I_D, which is the number of parameters of the objective function, or even the variables involved, or even the dimensionality of the problem. Indicating to obtain I_NP the value of 10 times I_D;

> \bullet F is the scale factor or weighting constant, typically between 0 and 2, that controls the differential mutation of the process (also called the step size of the differential evolution). It is the pass rate, which defines the probability of a test vector surviving;

> \div F CR is the crossover rate, specified in the interval between 0 and 1 (or also called the crossover probability constant). The higher this rate is, the greater the probability that the components of the candidate vector will be the same as the components of the mutant vector.

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The general characteristics of the Differential Evolution method are:

❖ Proposed by Rainer Storn and Kenneth Price in 1995 [4];

❖ Very popular in nonlinear optimization with continuous variables;

❖ Basic search mechanism: differential mutation operator;

❖ Considered a Stochastic Algorithm, although it is not inspired by a natural process;

❖ Interesting computational qualities, such as: simplicity of implementation, robustness and efficiency, self-adaptation and versatility.

Consider the nonlinear optimization problem with continuous real variables, according to Equation (1):

$$
x^* = arg_x^{min} f(x) \quad \text{Subject to: } \begin{cases} g(x) \le 0 \\ h(x) = 0 \end{cases}
$$
 (1)

Initially, the problem is considered unrestricted, that is, without the constraint functions $g(x)$ and $h(x)$ and arg related to the function arguments. The notation is as follows:

U ([a,b]): sampling with uniform distribution between a and b; N ([μ,σ]): sampling with normal distribution with mean μ and standard deviation σ;

Let X t={x (t,i);i=1,…,I_NP} be a population of candidate solutions. Each individual is represented by a column vector, according to Equation (2):

$$
x_{t,i} = \begin{bmatrix} x_{t,i,1} \\ x_{t,i,2} \\ x_{t,i,n} \end{bmatrix}
$$
 (2)

Where the third index indicates one among the "n" variables of the problem. The search mechanism of the Differential Evolution method that uses difference vectors, determined by the following points (see diagrams in Table 1):

❖ Two individuals are randomly selected to create a difference vector;

❖ This difference vector is added to a third individual, also randomly selected, producing a mutant solution;

❖ The mutant solution is therefore the result of a disturbance in some individuals

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of the population;

❖ This perturbation is a randomly constructed difference vector.

3 USE OF OPTIMIZATION TECHNIQUES AND STRUCTURAL DAMAGE DETECTION

The optimization techniques used for the damage detection procedure essentially follow the measurement of the displacement from the damaged structure, the displacement calculation from the undamaged model, the verification of the convergence, and if this criterion is reached, the change to a new point, the calculation of the displacement, in the new point and the iteration from the convergence analysis.

The advantages are the low sensitivity to noise compared to other techniques. It uses the static displacements and stiffness of each element since damage to structures is generally defined as a reduction in the stiffness of the element (Choi, 2002) [3]. Following Equation (3) we observe:

$$
X = (\beta_1, \beta_2, \dots \dots \beta_{nl})
$$
\n⁽³⁾

Where:

β: ratio between the effective stiffness of the ith element.

Using both the static displacements acquired from the finite element analysis (FEM) of the intact structure and the displacements corresponding to the damaged structure, the optimization function is obtained, according to Equation (4) below.

$$
F = \sum_{i=1}^{nn} \left| \frac{D_i^M}{D_i^C} - 1 \right| \tag{4}
$$

On what:

 D_i^M : displacement measured at the ith node;

 D_i^C : calculated displacement at the ith node;

: number of nodes in the system.

In Sobrinho *et. al* (2020) [5], for a beam element, through the following Equation (4), the stiffness matrix establishes how the physical and material properties are stored and also how each beam is modified to incorporate the variable damage.

$$
\begin{bmatrix} K_j \end{bmatrix} = \frac{E\left(1 - \begin{bmatrix} d_i \end{bmatrix}\right)}{i^3} \begin{bmatrix} A^{i^2} & 0 & 0 & -AI^2 & 0 & 0 \\ 0 & 12I & 6II & 0 & -12I & 6II \\ 0 & 6II & 4II^2 & 0 & -6II & 2II^2 \\ -AI^2 & 0 & 0 & AI^2 & 0 & 0 \\ 0 & -12I & -6II & 0 & 12I & -6II \\ 0 & 6II & 2II^2 & 0 & -6II & 4II^2 \end{bmatrix}_{6x6}
$$
 (4)

where [di] is the variable design vector and, the variable design vectors "di" shown in Equation (4), could assume values between 0 (intact element) and 1 (damaged element).

3.1 THE GENERAL BEHAVIOR OF THE DIFFERENTIAL EVOLUTION METHOD

Santos (2009) [7] presents Equation (5) in the analytical form of a function of 2 variables with some minimum points, where a 3-D graphical analysis of the same can be obtained, through the Peak Functions (Peaks Functions – MATLAB© R2014a [8]). Figure 3 (a) below shows the panorama of the Peak Functions, as well as the same one, through the Peak Functions (MATLAB© R2014a [8]). Figure 3 (b) shows the detail of the perception of the 3 minima of the function.

$$
z = 3(1-x)^{2}e^{-(x^{2})} - (y+1)^{2} - 10\left(\frac{x}{5} - x^{3} - y^{5}\right)e^{(-x^{2}-y^{2})}
$$
\n
$$
-\frac{1}{3}e^{(-(x+1)^{2}-y^{2})}
$$
\n(5)

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3.2 BEHAVIOR OF THE DIFFERENTIAL EVOLUTION METHOD IN THE SEARCH FOR A GLOBAL MINIMUM OF A FUNCTION

First, to understand the behavior of the Differential Evolution Method in the search for a global minimum of a function, one must observe the initial generation that gives rise to this process, as well as the updates made to each new generation.

In this sense, Figure 4 below, shows the verification of a random distribution of the points by the defined design space, where the simulation of the same is identified by reducing the sample space of the Peak Function until the second generation (maximum number of iterations or generations: I_itermax=2). In the graphical analysis of the contour and profile of the Peak Function, it is observed that the distribution of red dots represents the locations of individuals in the population, in which the area in red observed in the graphs is the region of the maximum peak of the function.

Figure $4 - 2nd$ Generation: Peaks Function graphical representation and the random distribution verification of points by defined design space and sample space reduction

Figure 5, where the graphic analysis is performed, represents the minimum value of the objective function, through its update at each new generation, as well as demonstrating the value of the best individual of the Peak Function.

Figure 6 already represents the distribution of the vector difference through the vector difference between individuals, implying the following relationship: the greater the distance between individuals, the greater the vector difference, and consequently the greater the distance between individuals in the design space.

Figure $6 - 2nd$ Generation: Difference vector distribution graphical representation between individuals in each generation project space

With the decrease of this vector difference in the distance between the individuals, it is stated that they are converging to a region with a minimum point in common. The minimization of the vector difference, that is, this tending to zero, shows that the method reached a minimum point of the function.

There is also the random distribution of individuals in the search space to facilitate the sweep of the design space to reach regions with minimum points, this is also one of the characteristics of the Differential Evolution Method.

In Figure 7, in the 5th and 7th generation, it is already observed that in the vicinity of the dark blue region, there is a small convergence of individuals, as well as some other individuals trying to search other regions looking for another better point where the global minimum is located off the Peak Function.

Figure 7 – Generation evolution graphical representation where the region points tend to converge and the global minimum function is found

From there, in Figure 8 (a), the path to the region where the global minimum is located is found, initiating a local search for a new position that generates the minimum value of the objective function, that is, the other individuals start a migratory process to the region that presents the lowest value of the objective function up to the 10th Generation shown, as shown in Figure 8 (b). There is a

conversion to the region close to zero, confirming the convergence process, where the vectors are increasingly close to each other so that the minimum point approaches its path and the beginning of stabilization.

Figure 8 – Graphical representation of individuals when they start a migration process to the region that has the lowest value of the objective function

Figure 9 illustrates the positioning of individuals in the design space, where there is a single red dot in the center of the dark blue region, confirming the path for convergence of the optimizer population, through the minimum value of the Objective Function.

Figure 9 – 15th Generation: Graphical representation of the Peaks Function and its consequent convergence to the region that has a minimum point

Finally, Figure 10 illustrates the final positioning of the individuals in the design space, where there is a single point of clusters of red individuals in the center of the dark blue region, confirming the path for the convergence of the optimizer population, through the value minimum of the Objective Function.

Figure 10 – 25th Generation: Graphical representation of the Peaks Function and its final consequent convergence to the region that has a minimum point

4 ANALYZED MODELS AND RESULTS

Studies were sought with verifications essentially arising from experiments in metallic beams (Iprofile), made of MR-250 steel with total lengths of 6.00 meters, in bias-supported condition, subjected to different load stages applied in mid-span and also in other points, as well as the damages and their different positions throughout the studies carried out. These cases are listed in Silva (2015) [9] in his doctoral thesis.

The damages were induced in the beams through transverse notches to the longitudinal axis of the beams (Figure 11) and numerically they were simulated by eliminating mesh elements in finite elements (Figure 12).

Figure 11 – Real induced damage (Silva, 2015) [9] Figure 12 – Numerically simulated damage (Silva, 2015) [9]

The geometric and material properties of the beams in the experimental analyses, as well as the damage conditions to be verified, can be seen below in Table 2.

The load stages used for evaluation with the optimization methods refer to those applied only in the load stage immediately before the maximum load (according to Silva, 2015 [9]: 4373 N). The damage was induced through transverse notches to the longitudinal axis of the beams, and the adoption of these open vertical cracks is caused by different requests, such as behaviors found in buildings with metallic structural elements and special works of art. In general terms, experimental models were sought with the maximum similarity with current construction standards for future practical contributions in the various systems built and formed by different structural elements.

The beams of the experimental tests were divided into 16 elements of 37.5 cm in length each, with 17 nodes equally spaced along the beam.

The objective functions produced by the accumulation of the quadratic differences of the intact and damaged responses used can be observed in Equation (6) below, which includes:

$$
F = \sum_{i=1}^{16} \left(Y_m^{ij} - Y_a^{ij} \right)^2 \tag{6}
$$

where $Y^{\hat{y}}_{m}$ are the static displacements measured (intact structure), $Y^{\hat{y}}_{a}$ are the static displacements obtained analytically (damaged structure), "i" is the degree of freedom, and "j" is the static shipment condition in a particular case.

The damage will be simulated to be identified by the DE, through intact and damaged responses of structures. The optimizer will make successive changes to the damage variables of the damaged model so that damage is found in all tested elements. The process that involves this procedure is composed of the Inverse Problem that will plot the state that the structure maintains.

The results of applying the Differential Evolution Method to different situations of beams with constant numbers of elements will be presented (called within the method the number of parameters of the objective function, even the variables involved, or even dimensionality). The static displacements were used as numerical and experimental results.

The measured data were simulated synthetically using the finite element analytical model of the structure with certain damaged elements (reduced stiffness) as well as the experimental models.

In general terms, the results to be obtained for each type of beam model and their respective load steps will take into account the displacements of the intact and damaged structure, with the insertion of damage in any element of the structure and the consequent analysis of an objective function involving these obtained results and finally being worked on in the optimization method.

The information from the synthetic (numerical) experimental data is used in terminology related to inverse problems since the structure response was derived from the intact and damaged responses. The damage assessment through experimental data brings a closer approximation of the real behavior of the structures considering that they can be produced by a measuring device, despite the practical limitations to obtain a lot of information.

5.1 STATIC NUMERICAL AND EXPERIMENTAL ANALYSIS: INTACT SUPPORTED METALLIC BEAM

In this case, now an intact slat-supported metallic beam (VR), with punctual loading in the middle of the span, as represented in Figure 13, from the work by Silva (2015).

The intact condition will be used as a parameter for checking the static damaged conditions of the beams in case 1 (VD1-2), case 2 (VD1-4) and case 3 (VD2-2), all with a loading order of around 3 kN.

5.1.1 Case 1 - Static Numerical and Experimental Analysis: Supported Metallic Beam (VD1-2)

The damaged simply supported metallic beam (Case 1: VD1-2), with punctual loading in the middle of the span, can be seen in Figure 14 below.

The intact and damaged graphic analyzes corresponding to the displacements for the beam in case 1, shown in Figure 15, where: the x-axis (abscissas) corresponds to the length of the beam (6m) and the yaxis (ordinates) corresponds to the displacements generated by the application of the loads, in this case, loads of approximately 3 kN and little are used.

Figure 15 – Intact and damaged graphical analysis corresponding to the displacements for the beam of case 1 (load approximately 3 kN)

The simulations proposed in this approach concern the results of the values obtained in the essentially experimental analysis used to identify the damaged elements in the structure. Due to the reasonable computational processing time, uses of 100 iterations were defined for this experimental analysis. In this analysis, only the intact and damaged static displacement values of the beam elements were considered. Figure 16 shows the result of the problem solution.

Figure 16 – Damaged identification VR \leftrightarrow VD1-2 (100 iterations): Beam load Case 1 (case of damage from left support – 1,5m=L/4)

5.1.2 Case 2 - Static Numerical and Experimental Analysis: Supported Metallic Beam (VD1-4)

On the other hand, the damaged slatted steel beam (Case 2: VD1-4), with punctual loading in the middle of the span, and damaged configuration as represented in Figure 17, from the work by Silva (2015) [9].

The intact and damaged graphic analyzes corresponding to the displacements for the beam in case 2, shown in Figure 18, where: the x-axis (abscissas) corresponds to the length of the beam (6m) and the yaxis (ordinates) corresponds to the displacements generated by the application of the loads, in this case, loads of approximately 3 kN and little are used.

Figure 18 – Intact and damaged graphical analysis corresponding to the displacements for the beam of case 2 (load approximately 3 kN)

The simulations proposed in this approach concern the results of the values obtained in the essentially experimental analysis used to identify the damaged elements in the structure. Due to the reasonable computational processing time, uses of 100 iterations were defined for this experimental analysis. In this analysis, only the intact and damaged static displacement values of the beam elements were considered. Figure 19 shows the result of the problem solution.

Figure 19 – Damaged identification VR \leftrightarrow VD1-4 (100 iterations): Beam load approximately 3 kN Case 2 (case of damage from left support $-1, 5m=L/4$) Objective Function (Minimum Value): 11.9614

5.1.3 Case 3 - Static Numerical and Experimental Analysis: Supported Metallic Beam (VD2-2)

The damaged slung-supported metallic beam (Case 3: VD2-2), from the work by Silva (2015) [9], with damaged configuration and punctual loading in the middle of the span, can be seen in Figure 20 below.

The intact and damaged graphic analyzes corresponding to the displacements for the beam in case 3, shown in Figure 21, where: the x-axis (abscissas) corresponds to the length of the beam (6 m) and the yaxis (ordinates) corresponds to the displacements generated by the application of loads, in this case, loads of approximately.

The simulations proposed in this approach concern the results of the values obtained in the essentially experimental analysis used to identify the damaged elements in the structure. Due to the reasonable computational processing time, uses of 100 iterations were defined for this experimental analysis. In this analysis, only the intact and damaged static displacement values of the beam elements were considered. Figure 22 shows the result of the problem solution.

Figure 22 – Damaged identification VR \leftrightarrow VD2-2 (100 iterations): Beam load approximately 3 kN Case 3 (case of damage from left support -1.8 m and 4.2 m)

6 FINAL CONSIDERATIONS

The structural responses of beams, under different loading conditions, through their static displacements, were used to identify damage. Since the damage is considered through the alteration (decreasing) of the stiffness properties of these elements. These constitute studies originating from Inverse Problem Methods or Systems Identification Methods.

The summary of the results obtained can be seen in Table 3 below.

With these minimum values of the objective functions found in the hundredth iteration and with the damage values of the elements following the proposed problem, finding damage of approximately 2% in element 4 for the analysis with the beam VD1-2 and 4% in element 4 for the analyzes with beam VD1- 4, and some disturbances for the other elements.

As for the minimum values of the objective functions found in the hundredth iteration, several damages of 2% in elements 5 and 12 for beam VD2-2, with the damage values of the elements following the proposed problem, in addition to some small disturbances for the other elements, mainly near the supports.

In the vicinity of the damage points, there were some distortions, probably because of the disturbance caused, where, for a more realistic adaptation to the model, a greater number of iterations could be used.

The damage identification analyses in this example was restricted to the displacements obtained in the intact and damaged experimental analyses, but as previously reported, a greater number of iterations were used again, which allowed a decrease in the generated residues, even where there were large differences in displacements, presence of punctual loads, proximities of the supports or even in the proximities of the damaged regions, despite this the damage values of the elements follow under the proposed problem. It is emphasized that increasing the number of iterations, in some cases, helps to solve the local minimum approximation problem.

With the analysis of these beams, it can also be stated that a greater number of displacement information would also help in the work of the optimizer. Even so, the tool met the ability to locate and quantify damage in any element of the structures under study.

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