O CAMPO CONCEITUAL MULTIPLICATIVO: UMA PROPOSTA DE APRENDIZAGEM À LUZ DA TEORIA DAS SITUAÇÕES DIDÁTICAS NO 8º ANO DO ENSINO FUNDAMENTAL

THE MULTIPLICATIVE CONCEPTUAL FIELD: A PROPOSAL FOR LEARNING IN THE LIGHT OF THE THEORY OF DIDACTIC SITUATIONS IN THE 8TH GRADE OF ELEMENTARY SCHOOL

EL CAMPO CONCEPTUAL MULTIPLICATIVO: UNA PROPUESTA PARA EL APRENDIZAJE A LA LUZ DE LA TEORÍA DE LAS SITUACIONES DIDÁCTICAS EN 8º CURSO DE EDUCACIÓN PRIMARIA

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RESUMO

O presente artigo é um recorte de uma dissertação de mestrado que investiga os efeitos da abordagem de ensino baseada na Teoria das Situações Didáticas sobre o desempenho dos estudantes dos Anos Finais do Ensino Fundamental na resolução de situações-problema relacionadas ao Campo Conceitual Multiplicativo. Como fundamentação teórica, utilizou-se a Teoria dos Campos Conceituais de Vergnaud (1990, 2009) e a Teoria das Situações Didáticas de Brousseau (1996, 2008). O principal objetivo é analisar os efeitos dessa abordagem de ensino sobre o desempenho dos estudantes dos Anos Finais do Ensino Fundamental na resolução de situações-problema do Campo Conceitual Multiplicativo. Este estudo adota uma abordagem qualitativa, exploratória e descritiva, com o delineamento de um estudo de caso. A investigação foi realizada na Universidade Federal do Amazonas, dentro do projeto denominado Escola de Matemática Básica. Participaram da pesquisa 12 estudantes do 8º ano do Ensino Fundamental. Os resultados indicam uma melhoria significativa na habilidade dos alunos de compreender e aplicar conceitos multiplicativos em situações-problema envolvendo o Campo Conceitual Multiplicativo, sugerindo que a integração da Teoria das Situações Didáticas e da Teoria dos Campos Conceituais facilita um aprendizado matemático mais profundo e engajado. Conclui-se que a abordagem pedagógica que incorpora essas teorias não apenas contribui para superar dificuldades conceituais em matemática, mas também promove um aprendizado mais potencialmente significativo e duradouro.

Palavras-chave: Ensino de matemática. Teoria dos Campos Conceituais. Campo Conceitual Multiplicativo. Situações problemas.

ABSTRACT

This article is an excerpt from a master's thesis that investigates the effects of a teaching approach based on the Theory of Didactic Situations on the performance of students in the final years of elementary school when solving problem situations related to the Multiplicative

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Conceptual Field. Vergnaud's Theory of Conceptual Fields (1990, 2009) and Brousseau's Theory of Didactic Situations (1996, 2008) were used as theoretical foundations. The main objective is to analyze the effects of this teaching approach on the performance of students in the final years of elementary school in solving problem situations in the multiplicative conceptual field. This study adopts a qualitative, exploratory and descriptive approach, with the design of a case study. The research was carried out at the Federal University of Amazonas, as part of a project called the School of Basic Mathematics. Twelve 8th grade elementary school students took part. The results indicate a significant improvement in the students' ability to understand and apply multiplicative concepts in problem situations involving the Multiplicative Conceptual Field, suggesting that the integration of the Theory of Didactic Situations and the Theory of Conceptual Fields facilitates deeper and more engaged mathematical learning. It is concluded that a pedagogical approach that incorporates these theories not only helps to overcome conceptual difficulties in mathematics, but also promotes more potentially meaningful and lasting learning.

Keywords: Teaching mathematics. Conceptual Fields Theory. Multiplicative Conceptual Field. Problem situations.

RESUMEN

Este artículo es un extracto de una tesis de maestría que investiga los efectos de un enfoque didáctico basado en la Teoría de las Situaciones Didácticas sobre el rendimiento de los alumnos de los últimos cursos de primaria al resolver situaciones problemáticas relacionadas con el Campo Conceptual Multiplicativo. Se utilizaron como fundamentos teóricos la Teoría de los Campos Conceptuales de Vergnaud (1990, 2009) y la Teoría de las Situaciones Didácticas de Brousseau (1996, 2008). El objetivo principal es analizar los efectos de este enfoque didáctico en el rendimiento de los alumnos de los últimos cursos de primaria al resolver situaciones problemáticas en el campo conceptual multiplicativo. Este estudio adopta un abordaje cualitativo, exploratorio y descriptivo, con el diseño de un estudio de caso. La investigación se realizó en la Universidad Federal de Amazonas, como parte del proyecto Escuela de Matemática Básica. Participaron 12 alumnos del 8º año de la enseñanza primaria. Los resultados indican una mejora significativa en la capacidad de los alumnos para comprender y aplicar conceptos multiplicativos en situaciones problema que involucran el Campo Conceptual Multiplicativo, sugiriendo que la integración de la Teoría de las Situaciones Didácticas y la Teoría de los Campos Conceptuales facilita un aprendizaje matemático más profundo y comprometido. Se concluye que el enfoque pedagógico que incorpora estas teorías no sólo ayuda a superar las dificultades conceptuales en matemáticas, sino que también promueve un aprendizaje potencialmente más significativo y duradero.

Palabras clave: Enseñanza de las matemáticas. Teoría de los campos conceptuales. Campo conceptual multiplicativo. Situaciones problemáticas.

INTRODUCTION

Arithmetic is a branch of mathematics that studies the properties and relationships of numbers, especially the fundamental operations of addition, subtraction, multiplication, and division. This mathematical field deals with the basic concepts of number manipulation and its algebraic properties and is essential for understanding and solving problems related to numerical quantities and operations, being the basis for many other branches of mathematics.

From a technical and practical perspective, the four fundamental operations of arithmetic play important roles in our daily lives. Addition, subtraction, multiplication, and division are essential tools that permeate various situations, providing efficiency in the manipulation and interpretation of numerical information.

Thus, regardless of the specific application, understanding and mastering these operations are essential for accurate and efficient decision-making. They constitute the mathematical basis that permeates several areas of life, contributing to solving a variety of problems in a practical and accessible way.

Corroborating the above, Mendes (2021) points out that arithmetic operations play a fundamental role in the teaching of Mathematics, since they represent the primordial basis for understanding the human being. This is due to the imperative to clearly incorporate the teaching of Arithmetic at all levels of education, highlighting its relevance in the everyday context, especially in the educational approach of the twenty-first century.

However, although some mathematical fields have practical and relevant applications in the daily lives of students, it is worth remembering that mathematics education is often still characterized by the traditional approach in which the teacher introduces mathematical concepts and methods, followed by the students' practice in solving exercises (Oliveira, 2019). In this approach, the main focus is on the transmission of knowledge by the teacher and the repetitive application of the contents by the students.

This practice, although it has been used over the years and has its advantages, can also have some limitations. By focusing exclusively on memorization and the mechanical application of formulas and techniques, one can fail to stimulate critical thinking and deep understanding of mathematical concepts. Students may become more adept at following a path pre-established by the teacher rather than developing their problem-solving skills creatively and autonomously.

Thus, understanding that the arithmetic field of basic operations has salient implications in several areas of knowledge, to promote more relevant mathematical learning, it is necessary to adopt methodologies that actively involve students in the construction of knowledge, through real and challenging problems, allowing them to perceive mathematics as a powerful tool to face real-world issues and develop analytical and problem-solving skills (Silva, 2019).

In this scenario, the Theory of Didactic Situations (DST), proposed by Guy Brousseau, is a theory that establishes an educational model in which learning occurs through the adaptation of the student to the milieu (a material medium such as pieces of a game, a challenge or a problem, together with the associated rules of interaction), emphasizing the importance of assimilation and adaptation to the environment created during the process. She highlights the fundamental change that allows students to engage in autonomous activities (Brousseau, 1996).

Also noteworthy is the Theory of Conceptual Fields – CBT, proposed by Gerard Vergnaud and his collaborators, and this cognitivist theory has as its main objective to provide a theoretical basis to understand how knowledge is connected and how conceptual changes occur over time (Moreira, 2002).

It is emphasized that CBT is important for learning because it allows the identification and analysis of the connections between knowledge based on conceptual content. This enables the investigation of the students' difficulties, especially in the context of learning concepts, and enables the teacher to stimulate and value the students' activity. In addition, CBT focuses on investigating the conditions in which students can understand, assimilate and internalize specific concepts from school knowledge (Vergnaud, 2009).

Considering all of the above, we present a proposal for the teaching and learning of multiplication and division in Elementary School with the use of CBT-based tasks carried out through the DST approach as pedagogical tools.

In view of this, the present research is guided by the following question: How are the reasoning of students in the 8th grade of Elementary School in solving problems in the Multiplicative Field influenced by the teaching approach based on the Theory of Didactic Situations?

Based on all that has been exposed, the objective of this research was delimited into: analyze the effects of the teaching approach guided by the Theory of Didactic Situations on the performance of students in the Final Years of Elementary School in the resolution of problem situations in the Multiplicative Conceptual Field.

THEORETICAL FOUNDATION

This topic aims to present the theoretical references that underlie this research, addressing essential concepts for the understanding of the investigated phenomenon.

Initially, the Multiplicative Conceptual Field is discussed, which is a subfield of the Theory of Conceptual Fields, proposed by Vergnaud, in order to detail the elements that make up this specific domain and its relevance to the teaching and learning of mathematics. And then, the fundamental assumptions of Brousseau's Theory of Didactic Situations are presented, which support the analysis of the interactions between teacher, student and mathematical knowledge in the school context.

THE MULTIPLICATIVE CONCEPTUAL FIELD

From the perspective of Vergnaud's (1990) Theory of Conceptual Fields, the concepts of the division operation are inserted in the Multiplicative Conceptual Field (MCC). Thus, the MCC can be represented by the triad C = (S, I, R). This triad is composed of:

S (the reference): the set of situations that give meaning to the concepts of multiplication and division;

I (the meaning): the set of invariants that underlie the operability of the schemes used to solve tasks related to multiplication and division;

R (the signifier): the set of forms that enable the representation of multiplicative structures, encompassing their properties, situations and procedures to deal with tasks involving these concepts.

Next, the theoretical elements necessary for the construction of concepts concerning the multiplicative conceptual field will be presented.

It is relevant to first consider a conceptual field as a set of situations, and multiplicative structures, fundamental in mathematics, comprise operations of multiplication, division or combinations of these. Multiplication, representing the combination of equal groups, and division, distributing quantities equitably, are essential components of these structures.

In the context addressed, the term "situation" does not refer to a didactic situation, but to a task. The essence lies in the idea that any complexity can be broken down into tasks, whose specific nature and difficulty are crucial for understanding (Vergnaud, 1990).

For Vergnaud (1990), the difficulty of a task is not limited to the sum or product of the difficulties of the individual subtasks; However, it is evident that failure in a subtask implies the overall failure of the situation. Thus, the analysis and understanding of the distinct characteristics of each task are essential to effectively face complex situations.

In addition, Vergnaud (1990, p. 147-148) also emphasizes that

> The conceptual field of multiplicative structures is both the set of situations whose treatment involves one or more multiplications or divisions, as well as the set of

> concepts and theorems that allow the analysis of these situations: simple proportion and multiple proportion, linear and n- linear function, directly and indirectly, quotient and product of dimensions, linear combination and linear application, fraction, ratio, rational number, multiple and divisor, etc.

According to Santana et al. (2016) to understand multiplicative structures, it is essential to initially consider that their relationships can take on ternary and quaternary shapes. In the ternary context, there is a connection between two quantities, which may be of a similar or different nature, and the operation between them will result in a third quantity. On the other hand, the quaternary relation involves handling four quantities, combining them two by two, and addressing two distinct quantities.

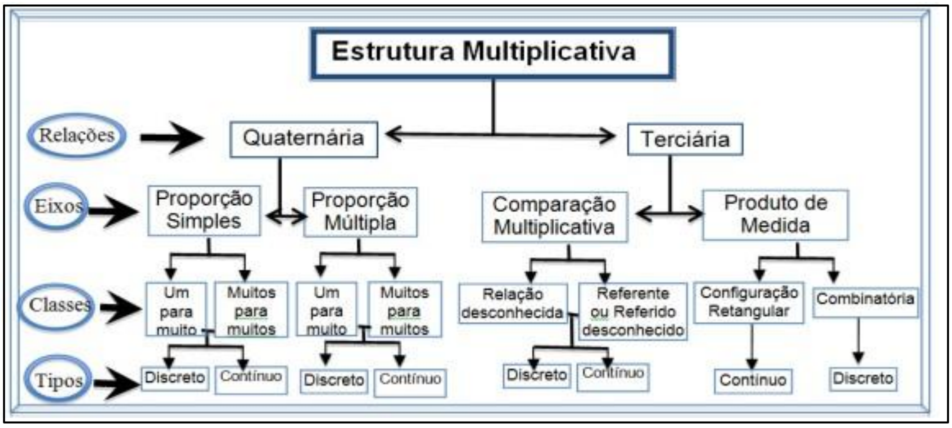
Also, according to the author, each of the relationships groups different axes. The axes related to quaternary relations are: single proportion, double proportion and multiple proportion. Those related to ternary relations are: multiplicative comparison and product of measurements.

Thus, problems of a multiplicative nature are systematically categorized into two major classes of relations.

> **Isomorphism of measurements**: the first great form of multiplicative relation is a quaternary relation between four quantities, two measures being of a certain type and the other two of another type; and **product of measurements**: this form of relation consists of a ternary relation between three quantities, of which one is the product of the other two, both on the numerical plane and on the dimensional plane (Vergnaud, 2009, p. 239).

For a more in-depth understanding of this structure, Merlini and Santos (2016) developed a scheme of the Multiplicative Conceptual Field (Figure 1), covering all the elements of the two relationships mentioned above.

Figure 1: Multiplicative Conceptual Field Schematic



Source: Merlini and Santos (2016).

Thus, to understand the dimension of Multiplicative Structures, a conceptual understanding is necessary, as well as the possibility of thinking about the formation of the concept in order to break with the additive field and, thus, expand the repertoire of mathematical knowledge for both the teacher and the students (Lopes, 2023).

It is evident that understanding and solving problems involving these relationships in the Multiplicative Field are fundamental skills for the development of students' critical and analytical thinking. According to Lopes (2023), by applying these concepts in everyday situations, students can improve not only their mathematical skills, but also their logical reasoning and problem-solving skills.

Next, the Theory of Didactic Situations is discussed, which is relevant to the training of Mathematics teachers. This theory aims to guide pedagogical practices during the planning and execution of classes, outlining the roles of the teacher and students.

THE THEORY OF DIDACTIC SITUATIONS

During the 1970s, the situation was predominantly a tool manipulated by the teacher. In this scenario, the educator used various resources, such as texts and materials, to create a learning environment. Under this paradigm, the teacher not only outlined the environment but also actively controlled the students' interactions with the content.

However, as the theory evolves, Brousseau (2008) highlights a fundamental change, that is, mathematical situations now allow the student to engage in autonomous activities, without direct intervention by the teacher. In this phase, the didactic situation becomes a model that comprehensively describes the complex interactions between the teacher, the student and the educational system.

Brousseau (2008), in his Theory of Didactic Situations, establishes an educational model in which learning occurs through the adaptation of the student to the *milieu*. The author emphasizes that learning only occurs when the subject assimilates and adapts to the environment created during the process.

According to Reges (2020, p. 90) when it comes to mathematics,

> [...] the *milieu* can be a problem-situation, a riddle, a game or a domino of fractions, for example. It is something that mobilizes the subject's cognitive function, that is, the student will need to make use of knowledge that he already has, but that is not enough for the immediate resolution of the activity, making it challenging.

Interaction with *milieu* occurs through the choice of device made by the teacher when planning his lesson. When faced with the device and its rules of interaction, the student interacts with it, usually collaboratively, experimenting, making decisions, formulating and testing hypotheses, seeking arguments to support their resolution strategies and sharing them with colleagues (Reges, 2020).

> The *milieu* must be prepared in the sense of causing imbalance in the student, so that he, through adaptive processes (assimilation and accommodation) can rebalance himself, thus occurring learning. In this case, the *milieu is considered to* be antagonistic, that is, a factor of difficulties, contradictions, and imbalance (Reges, 2020, p. 91).

Therefore, the *milieu* is conceived as an element that creates an environment conducive to the construction of knowledge, even if this implies facing difficulties and contradictions, because it is in this context that the true learning process occurs.

Three types of didactic situations are established by Brousseau (2008): Action Situation, Formulation Situation and Validation Situation, whose schemes will be presented below.

Status of action

In this context of action, an individual acts by choosing states of the environment according to his motivations. If the environment responds consistently, the subject learns from the feedback, anticipates responses, and incorporates them into future decisions. For Brousseau (2008, p. 28) "learning is the process in which knowledge is modified". Thus, learning modifies this knowledge, represented by tactics or procedures of what the individual considers, although they are projections. Knowledge allows you to adjust these anticipations, influencing the individual's choices.

In this scenario, the student performs a reflective analysis and simulates several approaches, opting for a resolution method in the context of an adaptive scheme. This choice is guided by the dynamic interaction with the '*milieu*', allowing the student to make the necessary decisions to effectively structure the resolution of the problem (Brousseau, 1996).

Formulation status

In this action, Brousseau (2008) discusses the complexity of implicit models of action and how they change as they are formulated. According to him, the formulation of implicit knowledge not only defines it, but also affects the way it can be treated, learned and acquired. This process involves the subject being able to recognize, identify, decompose, and reconstruct this knowledge into a linguistic system. In addition, the author suggests that the formulation of knowledge implies interaction with another subject, real or fictitious, to whom the information must be communicated.

In the formulation stage, according to Brousseau's (1996) approach, there is a dynamic exchange of information between the student and the '*milieu*'. This exchange takes place through a more adaptable language, without the obligation to explicitly resort to formal mathematical language. During this process, elements such as ambiguity, redundancy, metaphors, the creation of innovative semiological terms may arise, and there is the possibility of a lack of relevance and effectiveness in the transmission of the message, in the context of continuous feedback. Thus, in the formulation phases, students seek to change the common language, adapting it to the information they need to communicate more effectively.

Validation Status

In such a situation, the sender is not just an informant but an active proponent, while the receiver is a critical opponent. Both collaborate to connect knowledge to the situation, confronting and challenging their ideas. Together, they formulate relationships between context and mathematical knowledge. Each one can disagree and ask for justifications, promoting a collective construction of understanding and facilitating the resolution of complex problems.

According to Brousseau (1996), at this stage, students try to persuade the interlocutors (the other groups) of the validity of their statements through the use of appropriate mathematical language, often employing demonstrations. The phases of return, action, formulation and validation are distinctive of the didactic situation, in which the teacher grants the student the autonomy to follow the paths of discovery, refraining from revealing his didactic intentions. In this dynamic, the teacher assumes only the role of mediator.

In this context, it is clear that learning is not just a passive process of absorbing information, but rather a dynamic interaction between the subject and his environment. As Brousseau (2008) emphasizes, learning implies the modification of knowledge, where the individual's experiences and choices are fundamental. In addition, the interaction between individuals is crucial in the formulation of knowledge, revealing the need for effective communication and mutual understanding.

Institutionalization of situations

Nunes (2019) points out that in the institutionalization phase, the definition of the mathematical structure to be studied occurs. During the previous stages, a singular narrative develops, now elevated to a universal status, integrating itself into the culture that originates the knowledge in question. It is at this point that knowledge is formally validated and made official.

The process of institutionalization of knowledge begins, aimed at establishing social norms and reflecting the teacher's intentions. During this moment, the teacher resumes part of the responsibility previously granted to the students, evaluating and validating the students' productions or discarding those that do not meet the established criteria. This is achieved through formalization and generalization, where the objects of study are defined. It is in the institutionalization phase that the explicit role of the teacher is manifested, the object of study is officially assimilated by the student, and the teacher formally recognizes this learning process (Brousseau, 2008).

Thus, institutionalization is configured as the process by which knowledge transitions from its role as a means of immediate resolution of situations of action, formulation or validation, to the role of reference for future uses, both collective and personal.

It is a way for teachers to work in an interdisciplinary way, involving mathematical content, digital technologies and students' day-to-day situations, seeking to enrich the teaching-learning process inside and outside the classroom. In this way, the term "didactic situation" is related to the mathematical content that the student knows according to his or her surrounding environment, so this process will happen if the student shows interest in expanding and restructuring his or her knowledge.

In view of the above, it is perceived that the Theory of Conceptual Fields, the Multiplicative Conceptual Field and the Theory of Didactic Situations complement each other by providing theoretical subsidies for the understanding of mathematical learning processes. While Vergnaud's Theory of Conceptual Fields allows us to analyze how mathematical knowledge is organized into cognitive schemes based on different situations, the conceptual field of multiplicative structure deepens the specific understanding of the concepts and relationships involved in multiplication and division.

Brousseau's Theory of Didactic Situations, on the other hand, contributes by highlighting the importance of didactic interactions in the process of knowledge construction, outlining how teaching situations can favor the mobilization and appropriation of mathematical concepts by students. Thus, we start in this study from the assumption that these approaches dialogue with each other by articulating the cognitive, conceptual and didactic aspects of learning, providing a solid basis for the teaching of multiplicative structure in basic education.

METHODOLOGICAL PATH

This research adopts a qualitative approach, as defined by (D´Ambrosio, 2012, p. 93), who conceives it as a research "[...] focused on the individual, with all its complexity, and on its insertion and interaction with the sociocultural and natural environment".

The choice for this approach is justified by the fact that the research that is intended to be carried out has the following characteristics:

> 1) Interpretation as the focus. In this sense, there is an interest in interpreting the situation under study from the perspective of the participants themselves; 2) Subjectivity is emphasized. Thus, the focus of interest is the informants' perspective; 3) Flexibility in the conduct of the study. There is no a priori definition of the situations; 4) The interest is in the process and not in the result. This is followed by an orientation that aims to understand the situation under analysis; 5) The context as closely linked to people's behavior in the formation of experience; and 6) The recognition that there is an influence of the research on the situation, admitting that the researcher is also influenced by the research situation (Oliveira, 2008, p. 14).

The qualitative methodology is suitable for this study due to its emphasis on interpretation and subjectivity, allowing us to explore how students in the 8th grade of Elementary School understand and apply the concepts of multiplication and division. In addition, this approach allows capturing the complexity of students' interactions with the learning environment and with the proposed methodologies, such as the Theory of Conceptual Fields and the Theory of Didactic Situations, offering a detailed view of the development of students' mathematical thinking.

Research assumes both an exploratory and descriptive nature, which according to Gil (2002) exploratory and descriptive research are often considered essential preliminary steps to obtain scientific explanations.

The exploratory approach is characterized by probing the theme with the participants, aiming at the identification of their previous knowledge. This type of research aims mainly to develop, clarify and adjust concepts and ideas, with the aim of formulating more precise problems or hypotheses that can be investigated in subsequent studies (Gil, 2002).

The exploratory nature is justified by the search to probe students' previous knowledge, develop new insights into the application of the theories involved, and the methodological flexibility that allows adjustments as new data emerge. Furthermore, the research emphasizes the investigation of learning processes, rather than just measuring outcomes, thus contributing to the formulation of more accurate problems and hypotheses for future studies. This approach is suitable for exploring a poorly understood phenomenon, developing new teaching perspectives, and generating a clearer and more contextualized understanding of students' experiences in learning the Multiplicative Conceptual Field.

This is a descriptive research, as its focus is on the description of the phenomenon observed through CBT and DST. According to Gil (2002), descriptive research has as its main objective the detailed presentation of the characteristics of a given population or phenomenon, or even the analysis of the relationships between different variables. These studies cover a wide range of investigations and stand out for the adoption of standardized methods of data collection, such as questionnaires and systematic observations.

The design of this research uses the case study, considered by Yin (2005) as an ideal methodological strategy to investigate complex phenomena within their real contexts. Yin (2005) points out that case studies are particularly useful for exploring situations in which interventions and outcomes are not clearly distinguishable from the contexts in which they occur. He states: "The case study is an empirical inquiry that investigates a contemporary phenomenon (the 'case') in depth and within its real context, especially when the boundaries between phenomenon and context are not clearly evident" (Yin, 2005, p. 18).

Following this orientation, the present case study was designed to capture the nuances of pedagogical practices in mathematics, allowing a detailed analysis of how specific teaching strategies impact student learning. This method is particularly valuable for understanding the interplay between educational theory and pedagogical practice, offering deep insights into classroom dynamics and the effectiveness of didactic interventions.

Thus, in addition to the analysis of the students' performance, we sought to promote adjustments and reflections on their resolution strategies. In the context of the case study, this approach allowed for an in-depth investigation of the interactions between students and the teaching methodologies adopted, providing a more detailed understanding of the challenges and advances in mathematical learning. According to Yin (2005), the case study makes it possible not only to observe phenomena within their real contexts, but also to analyze how specific factors influence educational outcomes.

Thus, the continuous involvement of the participants in this study enabled a dynamic cycle of observation, analysis and adaptation, ensuring that the difficulties identified could be addressed through strategies adjusted to the learning context. This process reinforces the nature of the case study as a method that integrates theory and practice, making it possible to formulate recommendations based on concrete evidence obtained throughout the research.

Regarding the locus of the research for data collection, it was conducted at the Federal University of Amazonas – UFAM in the city of Manaus. The participants of the research were 12 students of Basic Education, more precisely, students of the 8th year of Elementary School who participated in the project called School of Basic Mathematics within the Institution, where classes were taught by undergraduates of the Mathematics Degree course. This specific selection aimed to address the research question in a targeted manner, aligning with the objectives outlined for the study.

The inclusion/exclusion criteria of the participants in this research are linked to the condition of being a student of the 8th grade of Elementary School who is part of the Basic Mathematics School project, at UFAM, whose parents or legal guardians have formalized the signature of the Informed Consent Form (ICF), authorizing participation in the study. Individuals who missed one of the meetings during the research were excluded from the study.

In the execution of the research, three main data collection techniques stand out: Participant observation and the detailed observation of mathematical situations in their natural context, the use of filming and photographs to document interactions, and the application of a structured protocol (framework to structure the answers) during mathematical activities. In addition, to improve the approach, instruments such as a logbook were used to record reflections and insights, the cell phone camera to capture key moments, and a printed evaluation to analyze the performance and understanding of the participants. This strategic combination of techniques and instruments provided a comprehensive and reasoned analysis of the mathematical situations under study.

In this context, the analysis of the research data was conducted through Content Analysis, which, according to Bardin (2016), comprises three distinct phases. The first, called pre-analysis, encompasses the selection of documents, the formulation of hypotheses and the preparation of material for analysis. In the second phase, the exploration of the material takes place, which includes the identification of the units, their enumeration and classification. Finally, the third stage consists of the treatment, inference, and interpretation of the data. In addition

> [...] it is a set of techniques for analyzing communications, aiming to obtain, through systematic and objective procedures for describing the content of messages, indicators (quantitative or not) that allow the inference of knowledge related to the conditions of production/reception (inferred variables) of these messages (Bardin, 2004, p.37).

Thus, Content Analysis seeks to discover the elements or units of meaning present in a set of messages. It involves the application of systematic and objective procedures to categorize these elements and draw inferences about the content, the conditions of production and reception of the messages, as well as the contexts in which they were generated. Bardin (2004) emphasizes the importance of a rigorous and structured approach to content analysis, which allows for a reliable and valid interpretation of communicative data.

Thus, the research used Content Analysis according to Bardin (2016) to examine the data collected, ensuring a systematic and in-depth interpretation of the participants' responses. Categorization was fundamental to organize and understand the emerging contents, allowing a structured analysis based on the established levels.

The data collection stage, called a posteriori evaluation, was structured to analyze the students' understanding of mathematical situations related to the Multiplicative Conceptual Field. This evaluation includes questions that explore knowledge about quaternary relations.

Chart 1 presents the distribution of the 4 questions, categorized by axis and class of quaternary relations, allowing a detailed analysis of the students' performance in different types of mathematical reasoning.

Table 1: Mathematical situations of the ex-post evaluation

QUATERNARY RELATIONSHIP		
Problem situation	**Axis**	**Class**
1) A horse wears 4 shoes. How many shoes does it take for 7 horses?	Simple Ratio	One for many
2) To paint a building, 5 painters take 40 days. How long does it take for 10 painters to do the same job?	Simple Ratio	Many to many
3) A light bulb consumes 60 watts in two hours. How many watts are consumed by 4 bulbs in 10 hours?	Multiple Ratio	One for many
4) A school has 4 teachers who, together, manage to correct 32 tests in two hours. If the school hires 3 more teachers, how many tests will they be able to correct in a period of 6 hours?	Multiple Ratio	Many to many

Source: Prepared by the authors

The results obtained from the situations in the subsequent evaluation were used to establish a dialogue with the findings of the literature review. At this stage, the focus was on analyzing the evolution of students' understanding in relation to the proposed mathematical situations, considering the ternary and quaternary relationships addressed in the evaluation. In addition, it was sought to verify how the interactions in the different phases of the activity (action, formulation and validation) contributed to the construction of knowledge and the overcoming of previously identified conceptual difficulties. This analysis allowed us to understand how the students mobilized and adjusted their thoughts throughout the process, evidencing learning patterns and possible conceptual gaps.

RESULTS AND DISCUSSION

The test was administered to 12 students, organized into four 8th grade groups, each composed of 3 students. The test, with the four questions, was administered by the researcher, using the evaluation instrument and the previously developed analysis categories.

In addition, the identification of the groups was carried out by means of a unique code for each one, which includes a prefix corresponding to the group to which they belonged during the application of the evaluation. Thus, the 8th grade group belonged to groups G8-1, G8-2, G8-3 and G8-4. This coding method allowed a structured analysis of the responses of each group, without compromising the privacy of the students.

In the context of this research, the *milieu* was represented by the mathematical questions of the test, which challenged the students to mobilize their previous knowledge and develop new solving strategies. According to Brousseau's Theory of Didactic Situations (2008), the *milieu* acts as a fundamental element for learning, as it imposes challenges and contradictions that drive the student to reflect, test hypotheses and adapt their strategies to overcome difficulties. Thus, the proposed questions played the role of a structured problem environment, in which students were encouraged to actively interact with mathematical knowledge.

In the Action Stage, the students were organized into four small groups of three participants. This initial organization lasted 50 minutes and allowed each group to engage independently in solving mathematical problems, without direct interference from the teacher, characterizing the action situation, in which students face the proposed challenges and build their own resolution strategies. As indicated by Brousseau (2008), this phase is essential, as it allows students to experiment with different approaches, interact with the *milieu* and develop their own hypotheses to solve the questions. After completing this step, the groups delivered their initial answers for further analysis.

Then, the Formulation Stage was carried out, in which the groups were reorganized to allow an exchange of ideas among the students. In this phase, which lasted 20 minutes, the four groups of the 8th grade were subdivided into two larger groups (G8-1 and G8-2; G8-3 and G8-4). This restructuring aimed to promote the exchange of strategies and reflections among the participants, allowing the students to analyze different approaches and refine their answers.

According to Brousseau (2008), the formulation situation involves a process of communication and reconstruction of knowledge, in which students share their ideas, adjust their strategies and reinterpret the mathematical concepts discussed. During this phase, students reviewed the questions, discussed alternatives, and elaborated a new version of their answers, thus consolidating a deeper understanding of mathematical problems.

Finally, in the Validation Stage, which lasted 40 minutes, the students returned to their original training of the initial small groups and solved the questions again, taking into account the knowledge acquired in the previous phase. This stage reflects the validation situation, in which students confront their answers, justify their mathematical choices, and present arguments to defend their solutions. According to Brousseau (2008), this phase is essential for mathematical learning, as it allows students to test the coherence of their answers and seek logical support for their strategies. After this final review, each group delivered a new version of the answers, reflecting a continuous process of knowledge construction.

This cycle of action, formulation and validation provided students with a dynamic and interactive experience of mathematical learning, based on the principles of the Theory of Didactic Situations. The process enabled students to face challenges, interact with their peers and develop a deeper understanding of the mathematical concepts worked on, highlighting the role of *the milieu* as a central element in the mediation of knowledge and in the promotion of the cognitive development of the participants.

It was decided to analyze only one group in the a posteriori evaluation stage, and group G8-4 was selected. This choice is directly based on the methodological design of the study, which takes a qualitative approach with the case study design. In this way, it was possible to carry out a more detailed and in-depth analysis of the reasoning used by the students, allowing a more accurate understanding of the particularities, difficulties and advances observed. Such a methodological decision is consistent with the assumptions of the case study, which seeks to deeply understand educational phenomena within real contexts, valuing the quality of the analysis to the detriment of the quantity of cases analyzed.

It is noteworthy that the qualitative analysis will be conducted adopting a holistic approach to the data collected, using the instrument and analysis categories developed by Magina, Santos and Merlini (2014). The strategies identified will subsequently be described, classified and quantified at four levels.

Level 1: Incomprehensible – This level includes strategies in which the student did not clearly record on paper the operation used to solve the problem, or, when recorded, the underlying reasoning was not identifiable. Answers in which the student produced a drawing irrelevant to the solution, simply repeated a piece of data from the problem, or chose a number at random without an understandable justification were also classified at this level.

Level 2: Additive Thinking – This level encompasses strategies that use addition operations, either through pictorial representations or numerical solutions. Students at this level use addition to solve problems, demonstrating a basic understanding of the additive process by combining elements or quantities visually or through direct calculations.

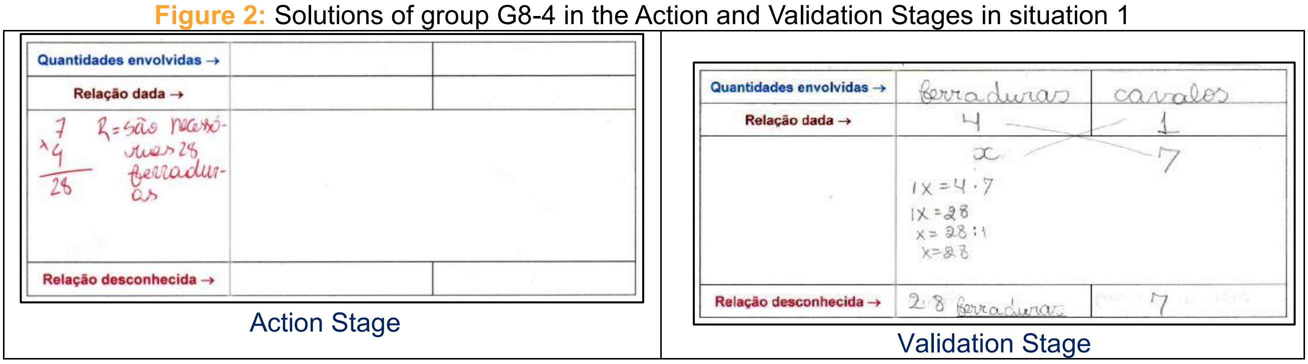
Level 3: Transition (from Additive to Multiplicative Thinking) – At this level, students' strategy involves forming groups with equal amounts. This approach consists of repeatedly adding the same amount, either through grouped icons (e.g., IIII IIII IIII = 12) or numerically (4 + 4 + 4 = 12). Although this strategy approaches multiplicative thinking, it is still based on additive reasoning, as it involves the formation of equal groups followed by the addition operation. The pictorial representation of this strategy is clearly demarcated by the groups drawn, while the numerical representation is manifested in the sum of equal portions. We call this phenomenon a transition strategy, because it connects the additive and multiplicative concepts.

Level 4: Multiplicative Thinking – At this level, the strategy adopted by the student is characteristically related to the multiplicative conceptual field, that is, demonstrating a direct understanding and application of the multiplicative structure in its resolutions.

Based on the categorization described, it becomes possible to identify and analyze with greater precision the different forms of mathematical reasoning adopted by students in the face of the proposed problem-situations. This classification into levels allows not only to observe the degree of complexity of the strategies used, but also to monitor possible progressions in the students' thinking, from incipient and poorly structured approaches to more advanced manifestations of multiplicative reasoning. Next, the analysis of the four Situations will be presented.

Situation 1 demonstrates a consistent resolution pattern, keeping the group at **Level 4: Multiplicative Thinking**. In the Action Step, the students correctly calculated 4×7=28 and specified "28 horseshoes are needed". In the Validation Stage, they maintained the same strategy and confirmed the result by registering "28 horseshoes". The stability in the approach and the clarity in the communication show a consolidated mastery of the multiplicative structure, see Figure 2.

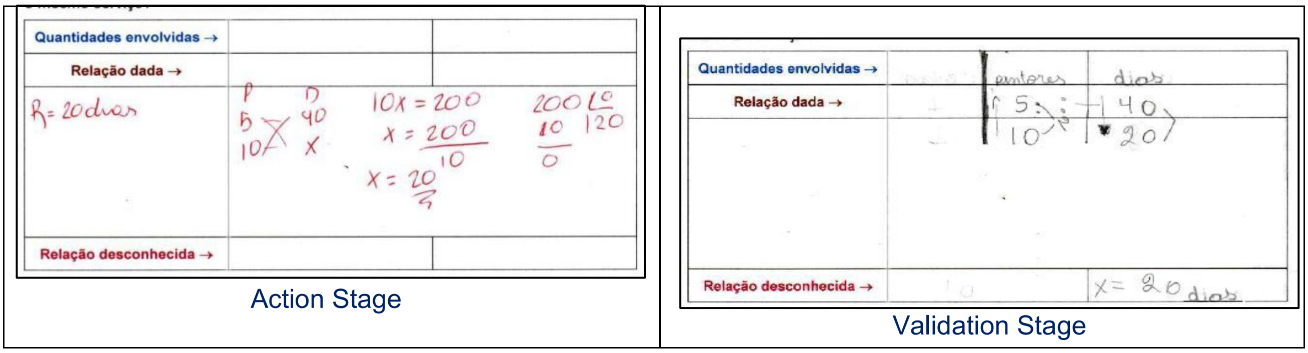
Figure 2: Solutions of group G8-4 in the Action and Validation Stages in situation 1



Source: The authors.

In **Situation 2**, the group demonstrated different strategies between the stages, showing flexibility in solving the problem and remaining at **Level 4: Multiplicative Thinking.** In the Action Stage, they used an algebraic procedure, structuring the equation 10x=200, solving x=200/10 and recording "20 days". In the Validation Stage, they reformulated the approach by applying proportional reasoning, identifying that the ratio between 10 and 5 painters reduced the time by half, correctly concluding with "20 days". This adjustment in the methodology reinforces the precision in the mathematical argumentation, as shown in Figure 3.

Figure 3: Group G8-4 Solutions in the Action and Validation Stages in Situation 2



Source: The authors.

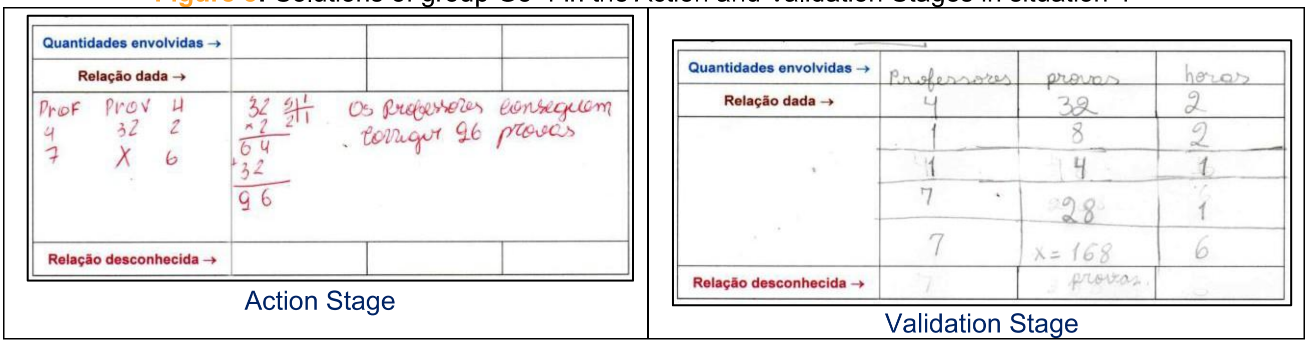
The resolutions in **Situation 3** show that group G8-4 started **at Level 1: Incomprehensible Thinking**, but advanced to **Level 4: Multiplicative Thinking** in the Validation Stage. In the Action Stage, they performed the operations 60×4=240 and 240×10=2400, but did not present any justification for the calculations, making the answer disconnected and without conceptual clarity. In the Validation Stage, they reorganized their reasoning by correctly identifying that a light bulb consumes 30 watts per hour, obtained by dividing 60 watts (consumption in 2 hours) by 2. Then, they calculated the consumption of a light bulb in 10 hours by multiplying 30 watts by 10, arriving at 300 watts. Finally, they determined the total consumption of 4 light bulbs over 10 hours by multiplying 300 watts by 4, correctly obtaining 1200 watts. This restructuring of the resolution strategy and the explicitness of the reasoning demonstrate a significant advance in the mathematical organization, as shown in Figure 4.

Figure 4: Solutions of group G8-4 in the Action and Validation Stages in situation 3

Source: The authors.

Finally, in **Situation 4**, the group showed a significant change in the understanding of the problem, starting from **Level 1: Incomprehensible Thinking** to **Level 4: Multiplicative Thinking**. In the Action Stage, they applied a set of operations without a clear structure, performing 32×2=64, adding 32 to obtain 96 and recording "teachers are able to correct tests", without indicating the correct total. This error compromises the accuracy of the answer. In the Validation Stage, they restructured their reasoning, starting with the identification that a teacher corrects 4 tests per hour, multiplying by 7 teachers to obtain 28 tests per hour, and then multiplying by 6 hours to correctly arrive at 168 tests. This review and correction of the strategy demonstrate a significant evolution in the organization of mathematical thinking, as shown in Figure 5.

Figure 5: Solutions of group G8-4 in the Action and Validation Stages in situation 4



Source: The authors.

Based on the above, the evaluation of the responses of group G8-4 demonstrates a consistent performance in **Level 4: Multiplicative Thinking**, with advances in organization and mathematical communication throughout the Validation Stage. In most situations, students demonstrated an understanding of the concepts of multiplication, division and proportionality, correctly applying the necessary operations. However, in the Action Stage, some answers presented a lack of clarity in the formulation or absence of explanation of the final result, in Situation 3 and Situation 4, the group initially did not present coherence in the calculations and justifications, being classified as **Level 1: Incomprehensible Thinking**.

In the Validation Stage, there was a relevant evolution in the way students recorded their answers, with improvements in the organization and mathematical justification. The group demonstrated an advance in multiplicative reasoning, because, in Situations 3 and 4, there was a complete restructuring of reasoning, allowing the correction of initial errors and the proper application of proportionality, ensuring the transition to **Level 4: Multiplicative Thinking**. This progress shows that the review and reflection on the calculations were fundamental to improve the clarity and precision of the answers, consolidating the mastery of the mathematical concepts involved.

Regarding quaternary relations, Magina, Santos and Merlini (2014) do not corroborate the results of the present research, as the authors found that the pictorial representation was of great value to the students, contributing to the success in solving the questions. However, in the present work, there was almost no use of pictorial representations, evidencing a different approach and indicating that this resource was not used as a strategy to solve the problems presented.

In the past, the group, when classified at Level 1 and evolved to Level 4, in questions 3 and 4, seems to present strategies that are aligned with the characteristics described by Almeida (2017) in relation to multiplicative structure, particularly in the direct application of multiplication to solve proportional problems. This group can demonstrate signs of understanding multiplicative relations, identifying proportions and scales in an apparently autonomous way, without resorting to explicit clues in the statement. In addition, there are signs of familiarity with the concepts underlying proportionality, using multiplication efficiently, after the formulation stage, which may indicate a progress in the internalization of the multiplicative structures described by the author.

In conclusion, the research indicated an improvement in the students' performance throughout the study, especially in relation to the resolution of problem situations in the Multiplicative Conceptual Field. Initially, the students presented difficulties in organizing the answers and applying multiplicative concepts in an autonomous and structured way. However, with the progressive implementation of the approach based on the Theory of Didactic Situations, it was possible to observe an evolution in the way students reflected on their strategies, correcting errors more efficiently, which suggests a deeper understanding of the content and the use of more effective approaches to solve problems.

Finally, the research was able to advance to the stage of institutionalization of the teaching-learning process by promoting, through the progressive implementation of the approach based on Brousseau's Theory of Didactic Situations, a transformation in the students' problem-solving strategies. Because, initially, the students faced relevant difficulties in the autonomous and structured application of multiplicative concepts. However, when challenged by carefully crafted problem situations, they began to adapt their strategies, correct mistakes, and gradually integrate the concepts more efficiently. This process of adaptation to the "milieu", as described by Brousseau (2008), resulted in the internalization of knowledge, characterizing the institutionalization of learning. Students began to apply the concepts more independently, with greater confidence and clarity, reflecting a significant evolution in the development of their mathematical skills and an advance towards the construction of mathematical knowledge autonomously.

FINAL CONSIDERATIONS

The main objective of this research was to investigate the effects of a teaching approach based on the Theory of Didactic Situations and the Theory of Conceptual Fields on the learning of mathematics by students in the Final Years of Elementary School, with a specific focus on solving problem-situations in the Multiplicative Conceptual Field. The results obtained throughout this research confirmed the initial hypothesis that the integration of these theoretical approaches contributes significantly to the improvement of students' performance in the understanding and application of fundamental mathematical concepts.

The DST proved to be effective in promoting didactic learning situations, in which the student assumes an active role in the construction of knowledge. The *milieu*, as a problematizing medium, favored the emergence of cognitive conflicts and conceptual adaptation, promoting the active and collaborative participation of students. CBT, in turn, enabled a more refined analysis of the reasoning mobilized, allowing the identification of significant advances in the internalization of concepts related to multiplication, division and proportionality. The four evaluative situations demonstrated that the use of contextualized and challenging tasks, combined with intentional pedagogical interventions, was decisive for the transition from elementary arithmetic thinking to more robust conceptual structures.

However, this research has limitations that should be considered. The case study, with a small sample of 12 students, restricts the generalization of the results to other educational contexts. The methodological choice of deepening in a single group limits the scope of possible inferences. In addition, the reduced intervention time may not have been sufficient for the full consolidation of the concepts addressed, considering the complexity of multiplicative structures. Such limitations, however, do not compromise the relevance of the findings, but reinforce the need for future studies with larger samples, longer duration, and diversity of school contexts to validate and expand the results obtained here.

It is concluded that the articulation between theory and pedagogical practice favored the development of students' mathematical reasoning, contributing to the overcoming of conceptual difficulties and promoting more meaningful and lasting learning. The experience evidenced in this study reinforces the importance of adopting didactic strategies that value student protagonism, the use of problem situations and qualified teacher mediation as central elements in the teaching-learning process in mathematics.

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