COMPUTATIONAL IMPLEMENTATION OF A THREE-DIMENSIONAL FRAME ELEMENT SUBJECTED TO UNIFORM TEMPERATURE VARIATION

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Christopher Rabelo Centofante[1], Michell Macedo Alves[2], Agda Rabelo Centofante[3] and Elias Centofante[4].

ABSTRACT
The development of computational tools for structural analysis has solved structural problems. However, due to existing limitations for users who do not have the full license or simply the tool does not have a certain function, the analyses may be restricted to more limited theoretical models that do not consider phenomena of temperature action on structures. Thus, this work proposes the development of a free and educational computational code capable of calculating displacements and requesting forces in three-dimensional frames subjected to uniform temperature variation. This code has been implemented in the Python language on the Visual Codes platform. The code has four processes for its creation, the first being the bibliographic review, the second the construction of the code, the third performing the test to verify the functionality and finally comparing the results with examples from the literature. The code that fulfilled the objectives established in this work was developed, a free educational tool for the calculation of three-dimensional frames with uniform temperature influence, having four validations for it, the first being simpler, a member embedded in the X axis with displacement only in this axis, the second a group of materials with displacement only in the X axis, the third a two-dimensional frame, being X and Y with uniform temperature in one bar and the last more complex, a three-dimensional frame composed of 4 pillars and 4 beams with the addition of uniform temperature at the top of the structure. The results were compatible with those obtained in the literature.

Keywords: Matrix Analysis. Thermal Expansion. Rigidity Method. Three-dimensional gantry.

[1] Bachelor of Science in Civil Engineering
Federal Institute of Education, Science and Technology Goiano - Rio Verde Campus – Goiás, Brazil
[2] Dr. in Structural Engineering from USP, São Paulo, Brazil
Federal Institute of Education, Science and Technology Goiano - Rio Verde Campus – Goiás, Brazil
[3] Dr. in Agronomy
Federal Institute of Education, Science and Technology Goiano - Rio Verde Campus – Goiás, Brazil
[4] Master in Ecology and Conservation
State University of Mato Grosso, University Campus of Xavantina- MT, Brazil

INTRODUCTION

Technological development has brought advances in all areas of knowledge, helping human beings in their tasks, with Civil Engineering it was no different. Currently, it is practically impossible to execute a construction project without the help of some software, generating facilities ranging from an Excel spreadsheet to programs that are capable of performing drawings, calculations, representations and simulations, facilitating the work of engineers, such as: AutoCAD, which is one of the most well-known CAD-type software in the world, Revit is based on BIM technology, unlike AutoCAD, SketchUp, TQS is a CAD-type software, produced by Brazilians, aimed at the design of structures in reinforced concrete, structural masonry, among others.

To understand the behavior of the structures in each type of building, it is necessary to use calculation software by the finite element method and real tests in the most adverse conditions and to know the aspects that influence the structures at high temperatures, highlighting the reduction of mechanical resistance and the modulus of elasticity of the structural elements of the building, leading to additional deformations, which can lead to the structure collapsing. However, to have access to these tools, generally, you must buy their licenses of use, which in many cases have a high cost for beginners, with the need to resort to other more affordable software.

One tool option is Python, open source available on several platforms, created in 1991 (its first version) by the Dutchman Guido Van Rossum, it is an easy-to-interpret language that demands more from the machine and less from the programmer, being, therefore, an easily readable language, which does not prioritize the speed of execution, this language is widely used due to the vast number of free libraries available, which enable an infinity of achievements, where it is possible to use it to compile a matrix analysis of the desired structure, which will be in the case of a spatial portico. According to Kassimali (2015), matrix analysis is the prediction of the results of actions on a structure, these actions can be loads and/or external effects, such as movement of supports or temperature changes.

Structural analysis is an essential part of structural design, according to Martha (2017), "it aims to design a structure that meets all the needs for which it will be built". To this end, it is necessary to know both the actions acting on it, as well as its behavior, which, in general, can be expressed by its requesting efforts and displacements

Methodologies that use programming language have the ability to generate results that match reality with great fidelity. In addition, they are safe, as they are in a virtual environment, enabling diverse tests and new approaches, ensuring a wide variety of arrangements.

This work aims to understand the structural behavior of metallic spatial frames with uniform temperature variation, generating as a result displacement, forces and reactions due to the variation, restricted only to the uniform temperature variation, as well as the elaboration of an open source code in the Python language, providing greater knowledge in the use of computational languages, it can be an educational tool in the teaching and learning process for students of the Civil Engineering Course, where its approach must be contextualized and clear, also serving to evaluate the effects of temperature on three-dimensional frames generating deformations, changing internal structures, as well as requesting efforts and support reactions.

THEORETICAL FRAMEWORK

GANTRY

Frames are linear and coplanar planar structures formed by the association of straight bars articulated with each other and not competing with active and reactive loads. They are the result of the association between pillars and beams of massive or hollow structures. The frames, together with the secondary load-bearing elements, form the resistant skeleton of the construction system, in which the roofing and lateral closing elements are fixed. The union of these elements is usually considered by designers to be perfectly rigid connections or perfectly articulated connections. (Soares; Hanai, 2001).

Figure 1 - Fresh gantry cranes.

Source: Fachini Civil Engineering

Kimura (2018) defines a spatial portico as a three-dimensional model in which it comprises the structure in its entirety, without restrictions in the positioning of the nodes and direction of the bars, thus allowing the concomitant application of vertical and horizontal actions. Thus, this structural model translates the most common type of lattice structures, since they represent all the columns and beams present in a building, which allows a complete and efficient evaluation of the overall behavior of the structure.

Gantry cranes are subject to compression, tensile, bending, torsional and shear stresses. The loads and forces applied to the gantry crane may vary according to the position of the bars and loads. And in the verification of its stability, the following forces are considered for each bar: Normal Force (N), Shear Force (V) and Bending Moment (M). A stable gantry has all the members stable with respect to the three forces.

TEMPERATURE EFFECTS

Azkune, Puente and Insausti (2007) define the variation of ambient temperature as the key factor for the redistribution of stresses in the structure, especially during construction. Due to the interaction with the climatic conditions of the environment, the temperature of fresh concrete suffers a great reduction and its volume decreases, that is, the piece contracts, originating thermal stresses that will pull it, generating cracking of thermal origin (Santos, 2012).

These variations can be uniform or even present thermal gradients and, consequently, the lattice elements of a structure can be subject to different temperature variations for its upper and lower faces (Martha, 2017).

In more critical situations, concrete structures can be accidentally subjected to high temperatures or these can make up part of their usual working conditions. For most structural materials, thermal deformation is proportional to the temperature difference (Gere, 2003; Hibbeler, 2004).

The strength of concrete when subjected to considerable temperatures occurs due to the thermal characteristics of the materials that compose it, among which the expansion coefficient, among others, stands out. However, the increase in temperature in the concrete elements causes a reduction in the modulus of elasticity and resistance characteristic of its constituent materials, with losses in the rigidity of the element. These thermal effects are associated with thermal expansion and degradation of mechanical properties due to increased temperature (Rigobello, 2011).

A solution to these problems is to perform a mathematical operation that takes into account the material of the structure and the temperature variation, better known as linear thermal expansion, which consists of the following formula:

$$\Delta \mathrm{L} = L_i \cdot \alpha \cdot \Delta T \qquad \text{Equation } \mathbf{(1)}$$

Where:

ΔL = Length change

Li = Initial length

α = Coefficient of expansion of the material

ΔT = Temperature variation.

In cases where there is no application of force, it becomes necessary to make the deduction that relates the deformation by force to temperature, as follows:

$\delta = \delta\Delta T$

$\frac{F \times L}{E \times A} = \alpha \times \Delta T \times L$

$$F = \alpha \times \Delta T \times E \times A$$

MATRIX ANALYSIS

With advances in structural technology, there is an increasing demand for faster analysis and accuracy in structural systems, due to the need to demonstrate structural safety, with this, there was a need to develop more accurate methods of analysis, since conventional methods are satisfactory when used in simple structures, but when applied in complex structures they are no longer satisfactory, There is a need for new methods that establish with greater precision any structural modifications that may occur in the project.

The matrix methods meet both the speed requirement and the accuracy of analysis for complex structures, where the digital computer is used not only for the solution of simultaneous equations, but also for the entire process of structural analysis, from the initial input data to the final output, which represents stress and force distributions, deflections, coefficients of influence, physical frequency characteristics and modal shapes.

Matrix methods are based on the concept of replacing the actual continuous structure with a mathematical model composed of finite-sized structural elements (also called discrete elements) with known elastic and inertial properties that can be expressed in matrix form. The matrices representing these properties are considered building blocks that, when fitted together according to a set of rules derived from elasticity theory, provide the static and dynamic properties of the actual structural system. In matrix methods, particles are finite in size and have a specified shape. These finite-sized particles are called structural elements, and they are specified somewhat arbitrarily by the analyst in the process of defining the mathematical model of the continuous structure. The properties of each element are calculated, using the theory of half-elastic continuity, while the analysis of the entire structure is performed for the assembly of the individual structural elements.

When the size of the elements decreases, the deformational behavior of the mathematical model converges to that of the continuous structure (Przemienicki, 1968).

The main function of any structure is to support and transfer loads applied externally to the reaction points and, at the same time, be subject to some specific constraints and a known temperature distribution. In civil engineering, reaction points are those points in the structure that are fixed to a rigid foundation. The structural designer is therefore primarily concerned with the analysis of known structural configurations that are subject to known distributions of static or dynamic loads, displacements, and temperatures. From his point of view, however, what is really needed is not the analysis, but the structural synthesis that leads to the most efficient design (optimal design) for the specified load and ambient temperature. Consequently, the ultimate goal of structural design should not be the analysis of a particular structural configuration, but the automated generation of a structure, i.e. the structural synthesis with its efforts, displacements, and reactions, which will satisfy the specified design criteria.

According to Przemienicki (1968), the methods of structural analysis can be divided into two groups: analytical methods and numerical methods. The limitations imposed by analytical methods are well known. Only in special cases are closed solutions possible. Approximate solutions can be found for some simple structural configurations, but in general, for complex structures analytical methods cannot be used, and numerical methods must invariably be employed. Numerical methods of structural analysis can be subdivided into two types, (1) numerical solutions of differential equations for displacements or stresses and (2) matrix methods based on the idealization of discrete elements.

Two complementary matrix methods of formulating any structural problem are possible: the method of displacements (method of stiffness), where displacements are chosen as unknowns, and the method of forces (method of flexibility), where the forces are unknown. In both methods, the analysis can be thought of as a systematic combination of individual structural elements disassembled into an assembled structure in which the conditions of equilibrium and combatibility are satisfactory.

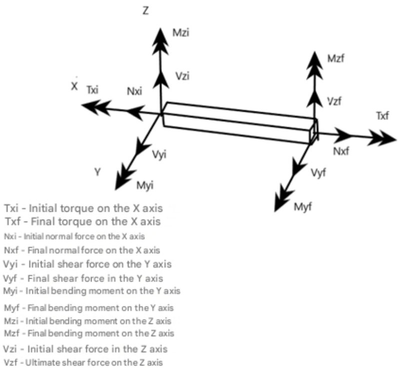
RIGIDITY METHOD

Although there are other methods for solving matrix analyses using computational implementation, the Rigidity method is the most used due to its ability to apply numerical methods for resolution.

The stiffness method uses the superposition of displacements applied to the boundary conditions that guarantee the balance of the structure. Thus, structural analysis in the static situation presents itself as a boundary problem that must satisfy the balance of the analyzed structure (Martha, 2017).

The application of this method requires the structure to be divided into parts, according to Hibbler "the application of the stiffness method requires subdividing the structure into a series of discrete finite elements and identifying their extreme points as nodes". In each element we have initial nodes and end nodes and each node has 6 degrees of freedom, as shown in the image below:

Figure 2 – Three-dimensional element and Degrees of Freedom.



Source: Author

As an example of this method, we take a bar, as in the image above, where we have six degrees of freedom or nodal displacements (u) that can be related to external nodal forces (f) through their stiffness matrix (k), thus obtaining the following equation:

$$(f) = [k] \cdot (u)$$ Equation **(2)**

Where (k) is the stiffness matrix whose function is to establish the compatibility relationship between forces and displacements in the analyzed element. As explained earlier, in the element there are 6 initial degrees and 6 degrees, so the matrix (k) is formed by 12 columns and 12 rows (12x12).

The forces (f) refer to the translational movement in the direction and direction of the displacement, there will then be a force f1 corresponding to the normal effort. Thus, through the definitions of stress in relation to the strain force, and the relationship between normal force and axial displacement, the equation is obtained.

$$F_1 = \left(\frac{EA}{L}\right) \times u_1$$ Equation **(3)**

Where:

E = Modulus of elasticity of the material;

A = Cross-sectional area of the element;

L = Length.

This equation can vary due to nodal forces, in the end we have the 12x12 matrix that represents the displacements that occurred in the element.

Figure 3 – Stiffness Matrix for Three-Dimensional Frame element.

$$K = \begin{bmatrix}
\frac{EA}{L} & 0 & 0 & 0 & 0 & 0 & -\frac{EA}{L} & 0 & 0 & 0 & 0 & 0 \\
0 & \frac{12EI_z}{L^3} & 0 & 0 & 0 & \frac{6EI_z}{L^2} & 0 & -\frac{12EI_z}{L^3} & 0 & 0 & 0 & \frac{6EI_z}{L^2} \\
0 & 0 & \frac{12EI_y}{L^3} & 0 & -\frac{6EI_y}{L^2} & 0 & 0 & 0 & -\frac{12EI_y}{L^3} & 0 & -\frac{6EI_y}{L^2} & 0 \\
0 & 0 & 0 & \frac{GI_X}{L} & 0 & 0 & 0 & 0 & 0 & -\frac{GI_X}{L} & 0 & 0 \\
0 & 0 & -\frac{6EI_y}{L^2} & 0 & \frac{4EI_y}{L} & 0 & 0 & 0 & \frac{6EI_y}{L^2} & 0 & \frac{2EI_y}{L} & 0 \\
0 & \frac{6EI_z}{L^2} & 0 & 0 & 0 & \frac{4EI_z}{L} & 0 & -\frac{6EI_z}{L^2} & 0 & 0 & 0 & \frac{2EI_z}{L} \\
-\frac{EA}{L} & 0 & 0 & 0 & 0 & 0 & \frac{EA}{L} & 0 & 0 & 0 & 0 & 0 \\
0 & -\frac{12EI_z}{L^3} & 0 & 0 & 0 & -\frac{6EI_z}{L^2} & 0 & \frac{12EI_z}{L^3} & 0 & 0 & 0 & -\frac{6EI_z}{L^2} \\
0 & 0 & -\frac{12EI_y}{L^3} & 0 & \frac{6EI_y}{L^2} & 0 & 0 & 0 & \frac{12EI_y}{L^3} & 0 & \frac{6EI_y}{L^2} & 0 \\
0 & 0 & 0 & -\frac{GI_X}{L} & 0 & 0 & 0 & 0 & 0 & \frac{GI_X}{L} & 0 & 0 \\
0 & 0 & -\frac{6EI_y}{L^2} & 0 & \frac{2EI_y}{L} & 0 & 0 & 0 & \frac{6EI_y}{L^2} & 0 & \frac{4EI_y}{L} & 0 \\
0 & \frac{6EI_z}{L^2} & 0 & 0 & 0 & \frac{2EI_z}{L} & 0 & -\frac{6EI_z}{L^2} & 0 & 0 & 0 & \frac{4EI_z}{L}
\end{bmatrix}$$

Source: Gere and Weaver Junior (1987)

For support reactions and requesting efforts, for example, it should be noted that the boundary condition cannot be used when multiplying the matrices.

ROTATION MATRIX

In structural analysis, the rotation matrix is used to transform the local stiffness and mass properties of structural elements into the global coordinate system. This matrix is obtained from the use of its directing cosines, taking into account the global coordinates of its initial and final nodes, so that the transformation of the solutions of a member in a local three-dimensional system to a global one, that is, it transforms local coordinates into global ones, defined as:

Figure 4 – Summarized rotation matrix

$$R = \begin{bmatrix} R_{3x3} & 0 & 0 & 0 \\ 0 & R_{3x3} & 0 & 0 \\ 0 & 0 & R_{3x3} & 0 \\ 0 & 0 & 0 & R_{3x3} \end{bmatrix}$$

Source: Author

Figure 5 – Expanded rotation matrix

$$R = \begin{bmatrix} R & R & R & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ R & R & R & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ R & R & R & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & R & R & R & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & R & R & R & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & R & R & R & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & R & R & R & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & R & R & R & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & R & R & R & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & R & R & R \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & R & R & R \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & R & R & R \end{bmatrix}$$

Source: Author

R3x3 is formed by a 3x3 matrix with the coordinates of the X, Y, and Z axis as follows:

R3x3 = [CxX CxY CxZ
CyX CyY CzY
CzX CzY CzZ]

The rotation matrix is composed of nine elements, in which each member of the matrix is obtained through the directing cosines of the local coordinate axis with respect to the global coordinate axis. The first line of the matrix represents the directing cosines of the local x-axis, and can be analyzed using the global coordinates of the two ends at which the member is fixed, as can be seen in the equations:

$$C_xX = Cos\theta_xX = (Xf – Xi)/ L$$
$$C_xY = Cos\theta_xY = (Yf – Yi)/ L$$
$$C_xZ = Cos\theta_xZ = (Zf – Zi)/ L$$

Where, L is the length of the bar is can be described as follows:

$$L = \sqrt{(Xf - Xi)^2 + (Yf - Yi)^2 + (Zf - Zi)^2} \quad \text{Equation } \mathbf{(4)}$$

COMPUTATIONAL IMPLEMENTATION

Technology has contributed to better resolutions and more agility in several sectors, including Civil Engineering with new platforms, whether calculations, drawings, preparation of spreadsheets for budgets, planning of works, meetings, among others, and as technological advancement occurs, these software are increasingly sophisticated, both in the labor market, as in teaching in universities.

METHODOLOGY

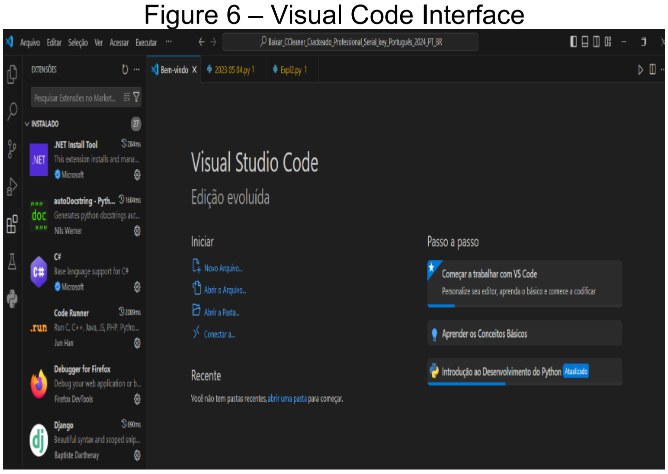
PYTHON LANGUAGE

Python is a language aimed at projects, facilitating stability control when they start to take on large proportions, but Python has a versatility of applications.

In order to meet the proposed objectives, the work was developed through the use of methodological procedures: at first, a literature review was carried out on the matrix analysis of three-dimensional frame structures and the effects of temperature on these structures. In a second stage, a research was carried out in order to understand the reciprocal transformations between local coordinates and global coordinates. Subsequently, a programming language was adopted for the computational implementation of a program capable of determining deformations and stresses requesting three-dimensional gantry.

An algorithm was developed containing all the steps for the elaboration of the structure stiffness matrix, temperature force vector, boundary conditions and post-processing. In the next step, an open source computational language Python was used in the Visual Code platform (Figure 6), being the most effective language for this project.

Figure 6 – Visual Code Interface

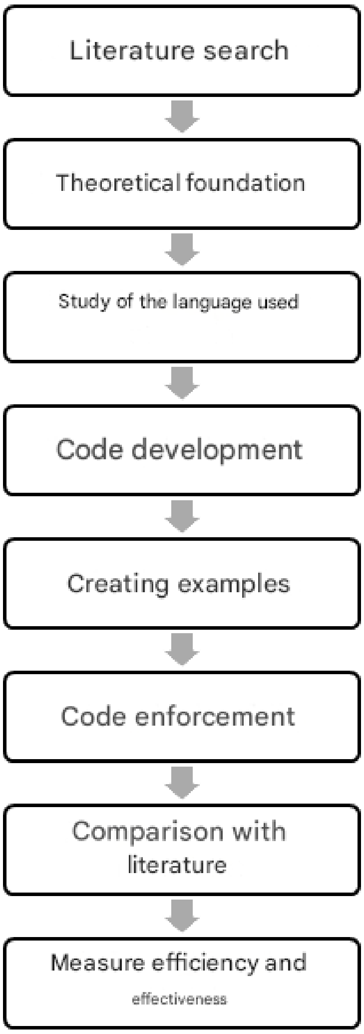


Source: Author.

And as a last step, generate a code to insert the temperature variation values in the mathematical operations and run the program to generate the values and check their veracity.

The flowchart below demonstrates the entire process carried out to write Python code.

Methodological structure adopted.



Source: Author.

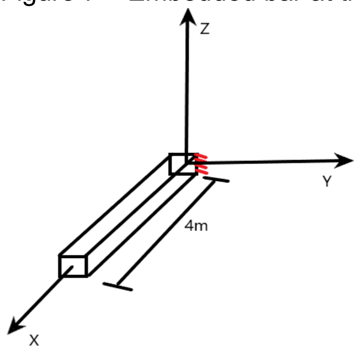
RESULTS AND DISCUSSION

To demonstrate the effectiveness of the code, five examples were solved, with different characteristics, focused on the effect that temperature causes on the steel bar.

EXAMPLE 1

A steel bar is set at one end and free at the other. This bar is subjected to a temperature of 20 °C. What is the displacement suffered by the bar due to the effect of temperature?

Figure 7 – Embedded bar at the origin.

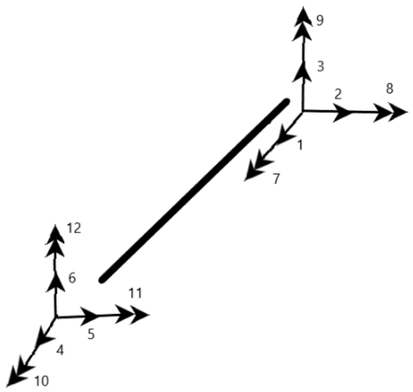


Source: Author

Where we can see that: Length (L): 4m Young's Modulus (E): 205x106 Area (A): 0.01m2 Thermal Coefficient (α): 12x10-3

To begin with, it should be remembered that a bar has degrees of freedom that will be influenced by the actions on it. There are a total of 12 degrees of freedom, 6 initial and 6 final as shown in the figure below:

Figure 8 – Representation of the Degrees of Freedom of the Bar



Source: Author

Based on the 12 degrees of freedom, the rigidity matrix is used, a matrix consisting of 12 rows and 12 columns.

Figure 9 – Three-dimensional stiffness matrix

$$
\begin{bmatrix}
\frac{EA}{L} & 0 & 0 & 0 & 0 & 0 & \frac{-EA}{L} & 0 & 0 & 0 & 0 & 0 \\
0 & \frac{12EI_z}{L^3} & 0 & 0 & 0 & \frac{6EI_z}{L^2} & 0 & \frac{-12EI_z}{L^3} & 0 & 0 & 0 & \frac{6EI_z}{L^2} \\
0 & 0 & \frac{12EI_y}{L^3} & 0 & \frac{-6EI_y}{L^2} & 0 & 0 & 0 & \frac{-12EI_y}{L^3} & 0 & \frac{-6EI_y}{L^3} & 0 \\
0 & 0 & 0 & \frac{GI_x}{L} & 0 & 0 & 0 & 0 & 0 & \frac{-GI_x}{L} & 0 & 0 \\
0 & 0 & \frac{-6EI_y}{L^2} & 0 & \frac{4EI_y}{L} & 0 & 0 & 0 & \frac{6EI_y}{L^2} & 0 & \frac{2EI_y}{L} & 0 \\
0 & \frac{6EI_z}{L^2} & 0 & 0 & 0 & \frac{4EI_z}{L} & 0 & \frac{-6EI_z}{L^2} & 0 & 0 & 0 & \frac{2EI_z}{L} \\
\frac{-EA}{L} & 0 & 0 & 0 & 0 & 0 & \frac{EA}{L} & 0 & 0 & 0 & 0 & 0 \\
0 & \frac{-12EI_z}{L^3} & 0 & 0 & 0 & \frac{-6EI_z}{L^2} & 0 & \frac{12EI_z}{L^3} & 0 & 0 & & \frac{-6EI_z}{L^2} \\
0 & 0 & \frac{-12EI_y}{L^3} & 0 & \frac{6EI_y}{L^2} & 0 & 0 & 0 & \frac{12EI_y}{L^3} & 0 & \frac{6EI_y}{L^2} & 0 \\
0 & 0 & 0 & \frac{-GI_x}{L} & 0 & 0 & 0 & 0 & 0 & \frac{GI_x}{L} & 0 & 0 \\
0 & 0 & \frac{-6EI_y}{L^2} & 0 & \frac{2EI_y}{L} & 0 & 0 & 0 & \frac{6EI_y}{L^2} & 0 & \frac{4EI_y}{L} & 0 \\
0 & \frac{6EI_z}{L^2} & 0 & 0 & 0 & \frac{2EI_z}{L} & 0 & \frac{-6EI_z}{L^2} & 0 & 0 & 0 & \frac{4EI_z}{L}
\end{bmatrix}
$$

Source: Kassimali, 2012.

The first step is to find out the force caused to generate the deformation in the bar, in this case it is the temperature variation, which causes the expansion of the bar, which can be calculated as follows:

$$\delta = \delta\Delta T$$
$$\frac{F\times L}{E\times A} = \alpha \times \Delta T \times L$$
$$F = \alpha \times \Delta T \times E \times A$$
$$F = (12x10\text{-}3)x20x(205x106)x00.1$$
$$F = 492\ kN$$

With the calculated force, the matrix for the solution of the displacement is developed using the equation below:

K x {d}= {F} Equation **(5)**
Where:
F = Strength
K = Stiffness Matrix
d = Displacement

In the rigidity matrix, it should be noted that 6 degrees of freedom are embedded; Therefore, they must be removed from the matrix, making it a 6x6 matrix, so we have:

Stiffness matrix after deleting rows and columns in the embedded degrees

$$\begin{bmatrix} \frac{EA}{L} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{12EI}{L^3} & 0 & 0 & 0 & \frac{6EI}{L^2} \\ 0 & 0 & \frac{12EI}{L^3} & 0 & \frac{-6EI}{L^2} & 0 \\ 0 & 0 & 0 & \frac{GI}{L} & 0 & 0 \\ 0 & 0 & \frac{-6EI}{L^2} & 0 & \frac{4EI}{L} & 0 \\ 0 & \frac{6EI}{L^2} & 0 & 0 & 0 & \frac{4EI}{L} \end{bmatrix} x \begin{bmatrix} d1 \\ d2 \\ d3 \\ d4 \\ d5 \\ d6 \end{bmatrix} = \begin{bmatrix} F1 \\ F2 \\ F3 \\ F4 \\ F5 \\ F6 \end{bmatrix}$$

After assembling the equation, it will substitute the values in the force and displacement:

$$\begin{bmatrix} \frac{EA}{L} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{12EI}{L^3} & 0 & 0 & 0 & \frac{6EI}{L^2} \\ 0 & 0 & \frac{12EI}{L^3} & 0 & \frac{-6EI}{L^2} & 0 \\ 0 & 0 & 0 & \frac{GI}{L} & 0 & 0 \\ 0 & 0 & \frac{-6EI}{L^2} & 0 & \frac{4EI}{L} & 0 \\ 0 & \frac{6EI}{L^2} & 0 & 0 & 0 & \frac{4EI}{L} \end{bmatrix} x \begin{bmatrix} d1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 492 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

The multiplication of the stiffness matrix with the displacement matrix must be performed, obtaining the following:

$$\begin{bmatrix} \frac{EA}{L} . d1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 492 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

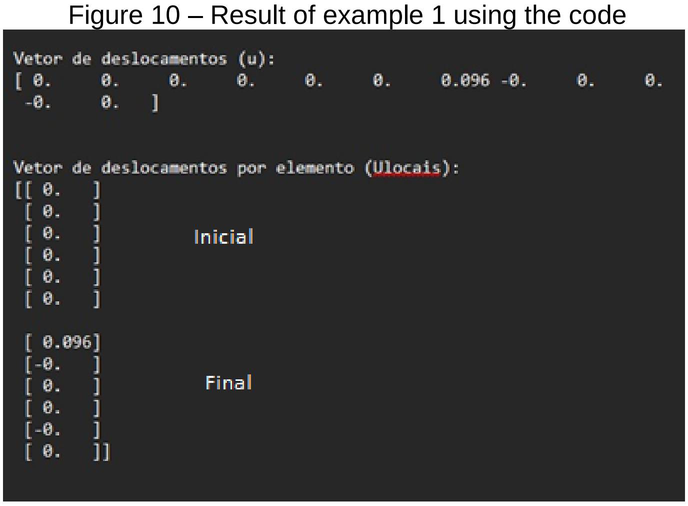
Finally, force is equated with displacement:

$$492 = \frac{EA}{L} \times d1$$
$$d1 = \frac{492 \times 4}{0{,}001 \times 205 \times 10^6}$$

d1 = 0.096m

As observed, the bar shifted 0.096m in the direction of the X axis. With this value, just use the program to compare the values, the result of the program coincided with those obtained previously, as can be seen in the image below.

Figure 10 – Result of example 1 using the code



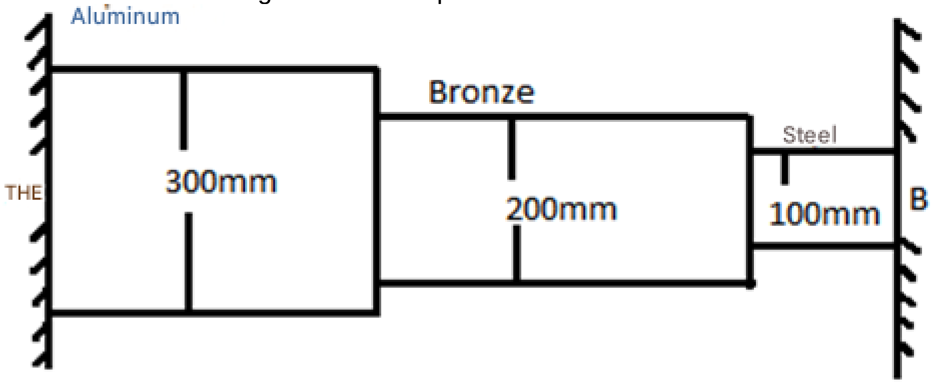
Source: Author

As can be seen in the image above, the results coincided, the value of 0 in the initial vector demonstrates that this part is set, so it only has a value in the final X, where the structure is free is positive, because the signal is in accordance with the orientation of the axis.

EXAMPLE 2

A group of materials composed of aluminum, bronze and steel, are adjusted between the fixed supports with a temperature of 40°C. Determine the expanded length generated in the bezels

Figure 11 – Group of materials in contact



Source: Author

Data:

Temperature (T)=40 °C; α1 = 23. (10^{-6}) 1/°C; E1= 73.1GPa

Aluminum Length(L1)= 1.2m; d1=300mm; α1=23. (10^{-6}) 1/°C; E1= 73.1GPa

Bronze Length(L2)= 1.8m; d2=200mm; α2=17. (10^{-6}) 1/°C; E2 = 103GPa

Steel Length (L3)= 0.9m; d3=100mm α3 = 1. (10^{-4}) 1/°C; E3 = 193GPa

$$A = \left(\frac{\pi}{4}\right) \cdot d^2$$ Equation **(4)**

You should start by transforming the effort generated by the deformation of the elements into just two forces, as shown in the figure below:

Figure 12 – Diagram of forces of the group of materials.

Source: Author

Thus, we have the following equation of balance of forces:

$$\sum F_x = 0 \Rightarrow F_a - F_b = 0 \Rightarrow F_a = F_b$$ Equation **(6)**

Although the element is set, obtaining the expanded length is necessary to identify what force the sockets would have to exert to resist this expansion:

$$\delta_T = a_1 \cdot (\Delta T) \cdot L_1 + a_2 \cdot (\Delta T) \cdot L_2 + a_3 \cdot (\Delta T) \cdot L_3$$

$$\delta_T = 23 \cdot 10^{-6} \cdot (20) \cdot 1{,}2 + 17 \cdot 10^{-6} \cdot (20) \cdot 1{,}8 + 17 \cdot 10^{-6} \cdot (20) \cdot 0{,}9$$
$$\delta_T = 1{,}47mm$$

The module result is used for both the initial and final setting, as they are at the same temperature and the materials are together. Thus, using the program to solve, the following result is obtained.

```
 Reacoes  :
[-1.45702488
     0.
     0.
     0.
     0.        Início
     0.

 1.45702488
     0.
     0.
     0.        Fim
     0.
     0.       ]
```

The result of the program was a little different, and may be related to the approximation of the values in the accounts. The first result being in the first node is negative because it is contrary to the convention, where the positive would be to the right, that is, it would be in the opposite direction to FA, while the second node would be to the right and opposite to FB

Table 1 - Results of Example II.

Length	Results of Literature	Results of the Program	E(%)
X	1.4700	1.4570	1.38

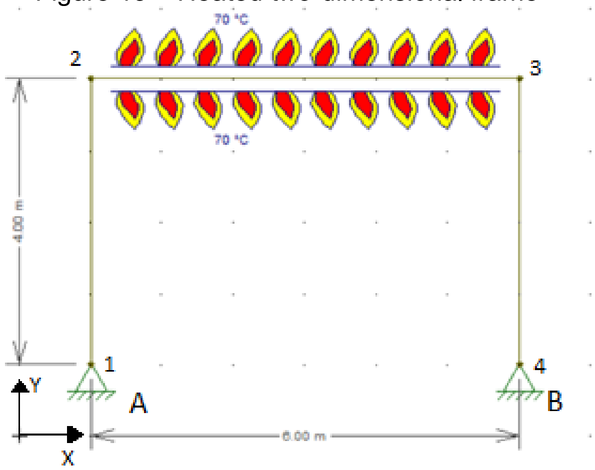
Source: Author.

The table above, despite having a small margin of error due to rounding, proves the efficiency and effectiveness of the program, achieving the main objective of this work.

EXAMPLE 3

As a third example, he calculated the horizontal displacement of point B due to temperature variation. Its structure has a material that has a thermal coefficient of 10-5/ºC and its bars have a rectangular section of 0.5m².

Figure 13 – Heated two-dimensional frame



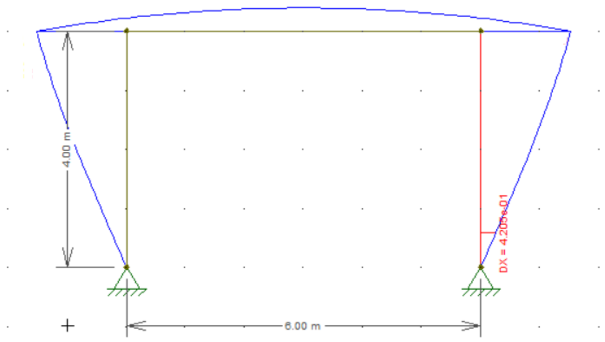
Source: Author.

To calculate the expansion due to temperature variation, we use the following formula:

$$\delta = \alpha \times tg \times AN$$
$$\delta = 10^{-5} \times (70) \times (6 \times 1)$$
$$\delta = 0.0042\text{m}$$

δ =0.0042m, i.e., as the value was positive, according to the determined orientation, point B shifted 4.2 cm to the right.

Figure 14 – Displacements due to Temperature Variation (Ftool).



Source: Author

With the execution of the code, it obtained the displacement generated by the heating of the beam of the structure, or element 2, generating the following values:

```
Vetor de deslocamentos (u):
[-4.20000000e-03   X1
  0.00000000e+00   Y1
  0.00000000e+00   Z1
 -8.90000000e-03   X2
  1.29073982e-17   Y2
  0.00000000e+00   Z2
  8.90000000e-03   X3
  1.29073982e-17   Y3
  0.00000000e+00   Z3
  4.20000000e-03   X4
 -0.00000000e+00   Y4
 -0.00000000e+00   Z4
```

As observed in the results, the negative values refer to the orientation, contrary in the adopted case. The value obtained in X at point B, which is the objective of the example, coincided with the one obtained in the literature taken from the book Structural Analysis of Kassimali, generating the table below.

Table 2 - Results of Example **III**.

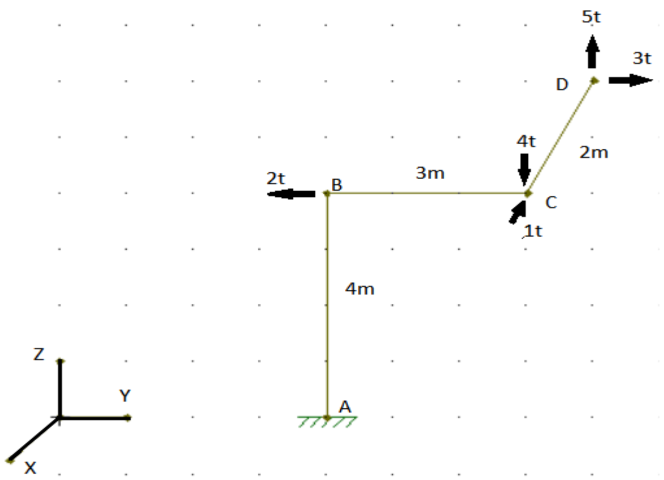
Degrees of freedom	Results of Literature	Results of the Program	E(%)
δ (m)	0.0042m	0.0042m	0

Source: Author.

EXAMPLE 4

Calculate the support reactions in the setting A of the three-dimensional structure, whose angle between the members is 90º. Although the example does not contain the temperature variation, it was used to demonstrate that the program can calculate moments and forces in the structure.

Figure 15 – Three-dimensional frame with forces.



Source: Author

The graph above shows a three-dimensional gantry with forces, where we have the following forces acting:

$$\sum X = 0 \Rightarrow X_A = 1t$$
$$\sum Y = 0 \Rightarrow Y_A = -1t$$
$$\sum Z = 0 \Rightarrow Z_A = -1t$$

Moment in X:

$$\sum M_x = 0 \Rightarrow M_{Xa} + 2 \times 4 - 4 \times 3 + 5 \times 3 - 3 \times 4 = 0 \Rightarrow M_{Xa} = 1m$$

Y-moment:

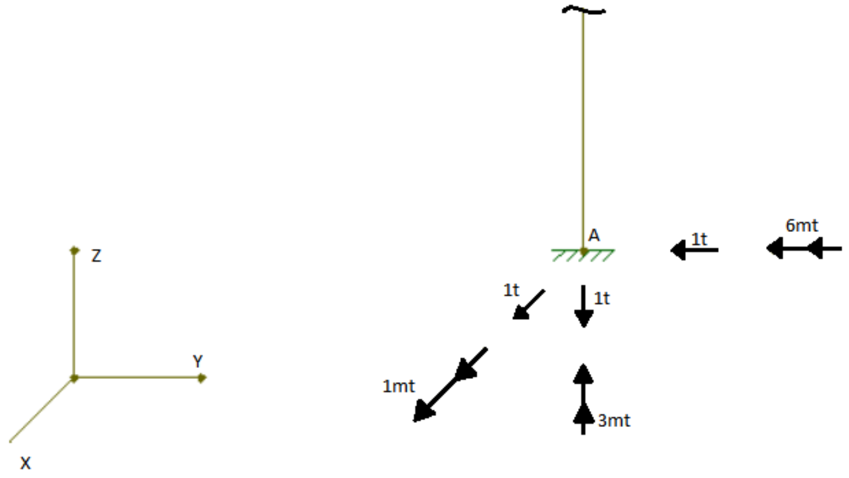
$$\sum M_Y = 0 \Rightarrow M_{Ya} - 1 \times 4 + 5 \times 2 = 0 \Rightarrow M_{Ya} = -6m$$

Moment in Z:

$$\sum M_Z = 0 \Rightarrow M_{Za} + 1 \times 3 - 3 \times 2 = 0 \Rightarrow M_{Za} = 3m$$

With this, we have the following configuration:

Figure 16 – Forces at point A



Source: Author.

The graph represents the results of the forces acting at point A.

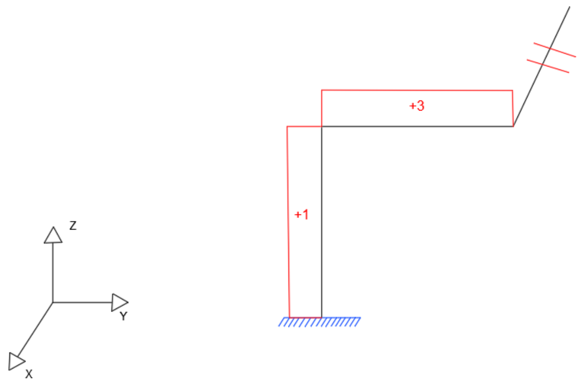
Table 3 - Results of the forces at point A.

Degrees of freedom	Results of Literature	Results of the Program	E(%)
MXA (mt)	1.00	-1.000	0
∑MYA (mt)	-6.00	6.000	0
MZA (mt)	3.00	-3.000	0

Source: Author.

Although the results obtained have signs opposite to those obtained in the literature, in modules they have the same value, this occurs because the orientation of signs in the literature is different from that adopted by the program, that is, if in the literature the clockwise moment is positive in the program it is negative, with the results for educational purposes, we have the diagrams of normal force, cutting, bending moment, the most important for the example and torsing moment, as it is the objective of the example, according to the figures below (Figures 17, 18, 19 and 20):

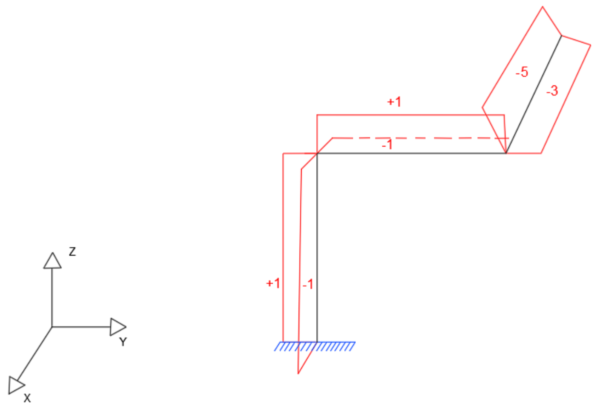
Figure 17 – Normal Diagram



Source: Author

The figure above represents normal strength.

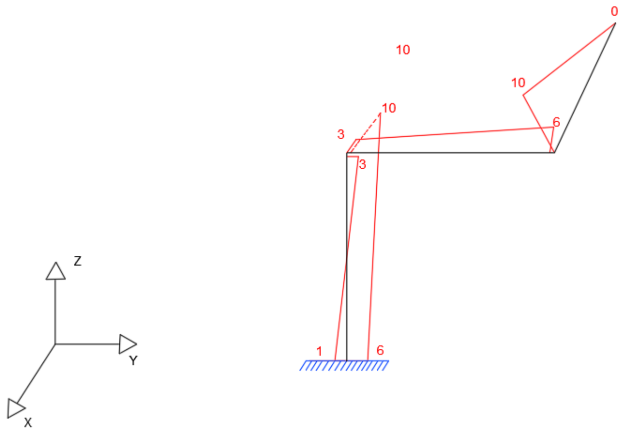
Figure 18 – Cutting Diagram



Source: Author

We observe the diagram of the shear force.

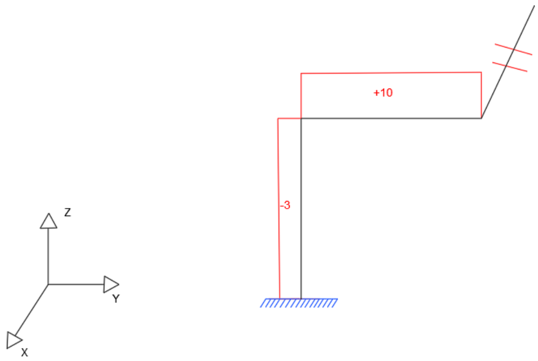
Figure 19 – Bending Moment Diagram



Source: Author

The figure above represents the diagram of the Bending Moment.

Figure 20 – Torsor Moment Diagram



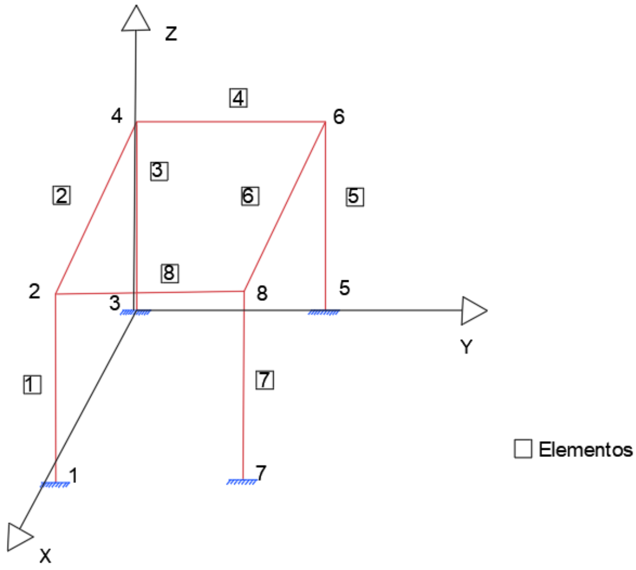
Source: Author

The Figure above represents the Torsor Moment strength of the structure.

EXAMPLE 5

The upper part of a "table" shaped gantry was heated to 20 °C, the bars have the same measurement and metallic material. Determine the displacement generated by this temperature increase.

Figure 21 – Three-dimensional "table" shaped gantry(2)



Source: Author

The figure above shows the displacement, due to the increase in temperature.

The table below contains information that is necessary for the sample to run in the program.

Table 4 – Portico Information

Property	Value	Unit
Length (L)	4	m
Young (L)	100	Mpa
Poisson (υ)	0,25	-
Iy	5000000	mm4
Iz	5000000	mm4
Temperature	20	°C
Area(A)	5000	mm²

Source: Author

This example will be solved using only the program for two reasons: the first due to its complexity, doing it otherwise would be extremely complicated and would be deviating from the proposal of the work; Second, based on the previous examples, the program is fulfilling its objective.

First, you must enter the data corresponding to the example and then execute the same, below is the image.

Figure 22 – Code Data Entry

```
# Entrada de dados
elemento = 'Portico3D'
Nele = 8
Nnos = 8
Nvinc = 24
Nf = 0
Nm = 0
Nenc = 4
Ntg = 0

Vele = numpy.zeros((Nele)) # cria um vetor/lista nulo de tamanho Nele
Vele = [ 1 , 2 , 3 , 4 , 5 , 6 , 7 , 8 ]  # ordem crescente sempre!

noi = numpy.zeros((Nele))
nof = numpy.zeros((Nele))
noi = [  1 , 2 , 3 , 3 , 5 , 5 , 7 , 2 ]
nof = [  2 , 3 , 4 , 5 , 6 , 7 , 8 , 7 ]

Young = numpy.zeros((Nele))
poison = numpy.zeros((Nele))
Young = [ 100.0 , 100.0 , 100.0 , 100.0 , 100.0 , 100.0 , 100.0 , 100.0 ]
poison = [ 0.25 , 0.25 , 0.25 , 0.25 , 0.25 , 0.25 , 0.25 , 0.25 ]
alfa = [ 0.0001 , 0.0001 , 0.0001 , 0.0001 , 0.0001 , 0.0001 , 0.0001 , 0.0001 ]

A = numpy.zeros((Nele))
Iy = numpy.zeros((Nele))
Iz = numpy.zeros((Nele))
A = [ 50.0 , 50.0 , 50.0 , 50.0 , 50.0 , 50.0 , 50.0 , 50.0 ]
Iy = [ 500.0 , 500.0 , 500.0 , 500.0 , 500.0 , 500.0 ,500.0 , 500.0 ]
Iz = [ 500.0 , 500.0 , 500.0 , 500.0 , 500.0 , 500.0 ,500.0 , 500.0 ] # 800.0 , 800.0 , 800.0 , 800.0 , 800.0 , 800.0 ,800.0 , 800.0
# Entrada do ângulo em graus
Irot_deg = [ 0.0 , 0.0 , 0.0 , 0.0 , 0.0 , 0.0 , 0.0 , 0.0 ]
```

Source: Author

The first result that is obtained are the support reactions at nodes 1, 3, 5 and 7, as they are the ones that support the structure as shown in the figure in the example. What is noticed is that as the temperature is not being applied to the elements belonging to these nodes that are embedded, the values are very low or practically zero

Table 5 – Supporting Reactions from the example above

Apoio	Valor
X 1	-3.74999297e-05
Y 1	3.74999297e-05
Z 1	-8.94412719e-16
θX 1	-9.99998125e-02
θY 1	-9.99998125e-02
θZ 1	-3.52881146e-13
X 3	3.74999297e-05
Y 3	3.74999297e-05
Z 3	7.82438643e-16
θX 3	-9.99998125e-02
θY 3	9.99998125e-02
θZ 3	-3.52875069e-13
X 5	3.74999297e-05
Y 5	-3.74999297e-05
Z 5	1.81273174e-15
θX 5	9.99998125e-02
θY 5	9.99998125e-02
θZ 5	-3.52883895e-13
X 7	-3.74999297e-05
Y 7	-3.74999297e-05
Z 7	-1.70072178e-15
θX 7	9.99998125e-02
θY 7	-9.99998125e-02
θZ 7	-3.52881830e-13

Source: Author

The second result that is obtained is the soliciting efforts. Note that table 6 describes the results of the degrees of freedom of the initial and final nodes for the eight elements. , it can be noted that in the nodes where the elements do not have heating, that is, those that are embedded mentioned in the table above, the value is basically zero as observed in the table below

Table 6 – Efforts Requesting the example

	ELEMENTS								
Degrees of freedom	And	1	2	3	4	5	6	7	8
	Xi	-8.94412719e-16	-1.99999625e+01	7.82438643e-16	-1.99999625e+01	1.81273174e-15	-1.99999625e+01	-1.70072178e-15	-1.99999625e+01
	Yi	3.74999297e-05	-7.99524165e-16	-3.74999297e-05	-9.75962316e-16	3.74999297e-05	7.99509738e-16	3.74999297e-05	-9.75966187e-16
	Zi	3.74999297e-05	7.02928926e-16	-3.74999297e-05	7.94923480e-17	-3.74999297e-05	1.89219029e-15	3.74999297e-05	-1.91476880e-16
	θXI	-3.52881146e-13	7.22581717e-14	-3.52875069e-13	3.17142146e-13	-3.52883895e-13	7.22442939e-14	-3.52881830e-13	3.17138676e-13
	θYi	-9.99998125e-02	4.99999062e-02	4.99999063e-02	-4.99999063e-02	4.99999063e-02	-4.99999063e-02	-4.99999062e-02	-4.99999062e-02
	θZi	9.99998125e-02	-1.59905868e-12	-4.99999063e-02	-1.95193984e-12	4.99999062e-02	1.59902007e-12	4.99999063e-02	-1.95194501e-12
	Xf	8.94412719e-16	1.99999625e+01	-7.82438643e-16	1.99999625e+01	-1.81273174e-15	1.99999625e+01	1.70072178e-15	1.99999625e+01
	Yf	-3.74999297e-05	7.99524165e-16	3.74999297e-05	9.75962316e-16	-3.74999297e-05	-7.99509738e-16	-3.74999297e-05	9.75966187e-16
	Zf	-3.74999297e-05	-7.02928926e-16	3.74999297e-05	-7.94923480e-17	3.74999297e-05	-1.89219029e-15	-3.74999297e-05	1.91476880e-16
	θXf	3.52881146e-13	-7.22581717e-14	3.52875069e-13	-3.17142146e-13	3.52883895e-13	-7.22442939e-14	3.52881830e-13	-3.17138676e-13
	θYf	-4.99999062e-02	-4.99999063e-02	9.99998125e-02	4.99999063e-02	9.99998125e-02	4.99999062e-02	-9.99998125e-02	4.99999063e-02
	θZf	4.99999062e-02	-1.59906314e-12	-9.99998125e-02	-1.95191332e-12	9.99998125e-02	1.59901368e-12	9.99998125e-02	-1.95192365e-12

Source: Author

The third and most important result, which is the displacement caused by the increase in temperature, is worth noting that those with zero are where the elements are restricted by the settings. It can be seen that some nodes have the same value, but with inverted signs, such as node 2 and node 4, which indicates that they expand to opposite directions, remembering that the fact that the signs represent the direction in favor or against the orientation adopted in the example. The other values that are too low will be disregarded, due to the dimensions of the object as shown in the table below.

Table 7 – Example offsets

We	Value (cm)
1 uX	0
1 uY	0
1 uZ	0
1 θx	0
1 θy	0
1 θz	0
2 uX	7.99998500e+00
2 uY	-7.99998500e+00
2 uZ	7.15530175e-16
2 θx	1.99999625e-03
2 θy	1.99999625e-03
2 θz	3.52881146e-14
3 uX	0
3 uY	0
3 uZ	0
3 θx	0
3 θy	0

3 θz	0
4 uX	-7.99998500e+00
4 uY	-7.99998500e+00
4 uZ	-6.25950914e-16
4 θx	1.99999625e-03
4 θy	-1.99999625e-03
4 θz	3.52875069e-14
5 uX	0
5 uY	0
5 uZ	0
5 θx	0
5 θy	0
5 θz	0
6 uX	-7.99998500e+00
6 uY	7.99998500e+00
6 uZ	-1.45018539e-15
6 θx	-1.99999625e-03
6 θy	-1.99999625e-03
6 θz	3.52883895e-14
7 uX	0
7 uY	0
7 uZ	0
7 θx	0
7 θy	0
7 θz	0
8 uX	7.99998500e+00
8 uY	7.99998500e+00
8 uZ	1.36057742e-15
8 θx	-1.99999625e-03
8 θy	1.99999625e-03
8 θz	3.52881830e-14

Source: Author

In summary, elements 2, 4, 6 and 8 expand in X and Y approximately 8 cm according to the orientation of the signs. To prove this value in a simple way, just use the thermal expansion formula, as follows:

$$\delta\Delta T = \alpha \times \Delta T \times L$$
$$\delta\Delta T = 0.001 \text{ x } 20 \text{ x } 4$$
$$\delta\Delta T = 0.08 \text{ m or } 8 \text{ cm}$$

This result serves as a module for nodes 2, 4, 6 and 8, since they have the same qualities, differing only in the orientation explained in table 7. Thus, we have the following table:

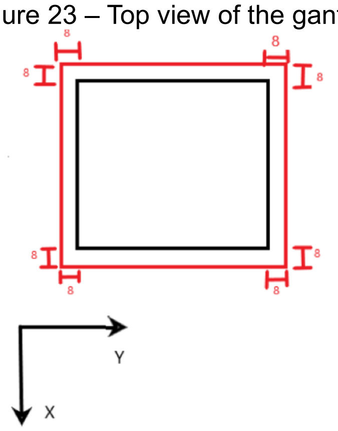
Table 8 - Results example 5

Thermal Strain	Literature	Program	E(%)
δΔT	8,000 cm	7.999cm	0.013%

Source: Author.

The result of the program showed a variation of 0.013% in relation to the manual one, which is due to the lower number of decimal places used in the calculations made by the reference value. The figure below shows the representation of the effect generated by the temperature in the upper part of the structure.

Figure 23 – Top view of the gantry



Source: Author

FINAL CONSIDERATIONS

From the results presented, it can be concluded that the developed program is effective for the analysis of spatial frames considering the uniform variation of temperature given structure, with an insignificant margin of error

The use of software to solve calculations in any area of study is essential to reduce the time spent on its execution and reliability, in this sense the Python language demonstrated efficiency in the analysis of three-dimensional frames with uniform temperature influence by the rigidity method, and the frame can be uniform or not according to the results obtained and based on the literature.

The Python language provides greater knowledge and applicability in the use of computational languages, and can be an educational tool in the teaching and learning process for students of the Civil Engineering Course, in addition to enabling the expansion to other topics such as non-symmetrical structures, non-uniform temperature variations.

REFERENCES

1. Azkune, M., Puente, I., & Insausti, A. (2007). Effect of ambient temperature on the redistribution of loads during construction of multi-storey concrete structures. Engineering Structures. Elsevier. (Nota: Como "Science Direct" e "San Sebastian" não são necessários no APA, foram omitidos; presume-se que seja um artigo de periódico, mas sem volume/páginas especificados, mantive como está).

2. Hibbeler, R. C. (2019). Resistência dos materiais [Strength of materials] (10th ed.). Pearson Prentice Hall.

3. Kassimali, A. (2012). Matrix analysis of structures (2nd ed.). Stamford.

4. Kassimali, A. (2015). Análise estrutural [Structural analysis] (5th ed.) (L. A. V. Carneiro, Trans.). Cengage Learning. (Original work published in English; tradução por Noveritis do Brasil).

5. Kimura, A. (2018). Informática aplicada a estruturas de concreto armado [Applied informatics to reinforced concrete structures] (2nd ed.). Oficina de Textos.

6. Martha, L. F. (2017). Análise de estruturas: Conceitos e métodos básicos [Structural analysis: Basic concepts and methods] (2nd ed.). Elsevier.

7. McGuire, W., Gallagher, R. H., & Ziemian, R. D. (2000). Análise estrutural de matriz [Matrix structural analysis] (2nd ed.). Bucknell University Faculty Books. Available at: https://digitalcommons.bucknell.edu/books/7

8. Nussenzveig, H. M. (2014). Curso de física básica: Fluidos, oscilações e ondas, calor [Basic physics course: Fluids, oscillations and waves, heat] (Vol. 2, 5th ed.). Editora Edgard Blucher.

9. Pravia, Z. M. C., Pasquetti, E., & Chiarello, J. A. (2001). Visualbarras: Um software didático para o ensino de análise matricial de estruturas reticuladas planas [Visualbarras: A didactic software for teaching matrix analysis of planar reticulated structures]. XXIX Congresso Brasileiro de Ensino de Engenharia, 10.

10. Przemienicki, J. S. (1968). Theory of structural analysis. McGraw-Hill.

11. Santos, M. R. G. (2012). Deterioração das estruturas de concreto armado – Estudo de caso [Deterioration of reinforced concrete structures – Case study] [Unpublished undergraduate thesis]. Universidade Federal de Minas Gerais, Minas Gerais. Available at: http://pos.demc.ufmg.br/novocecc/trabalhos/pg2/88.pdf. Retrieved on November 7, 2016.