THE METHOD OF DISCARDING: A PROPOSAL FOR THE TEACHING OF GEOMETRY

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ABSTRACT

Analytical geometry makes it possible to analyze, organize and systematize spatial knowledge, favoring understanding and admiration for the natural environment. It also stimulates creativity and a positive attitude towards mathematics in students and, in teachers, the use of approaches and strategies that enrich the processes in the classroom. The teaching of geometry requires encouraging the student to develop processes of concentration, attention, patience, among others, so that he learns to think and understand abstract concepts that can have meaning and utility within the educational process through the analytical method of Descartes, based on the method of solving geometric problems focusing on the description and understanding of referential elements. thus allowing the generation and deepening of the reasoned knowledge.

Keywords: Geometry. Descartes' method. Didactics. Teaching. Problem solving.

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INTRODUCTION

This article is based on a documentary review of Descartes' Method: Conics, in such a way that it facilitates the teaching-learning process of mathematics, especially geometry. This technique allows the didactic strengthening of teachers for the development of geometric contents, resulting in a pedagogical strategy that is beneficial in the teaching and learning process of students. The importance of the study is based on the intention of introducing techniques with a new vision to the didactics of geometry teaching, conceived as classroom action, thus contributing to reduce failure rates in the area.

BACKGROUND

Rene Descartes, French philosopher and mathematician, considered the father of modern philosophy, counted as one of the main exponents of rationalism. He imposed a style to address problems in such important fields of knowledge as physics, medicine and theology, among others, a style very imbued with reason.

As fundamental elements of analytical geometry, there are, according to González Urbaneja (2017, p. 241):

a) Cartesian coordinate system: Composed of the rectangular coordinate system and the polar coordinate system. It is named in honor of René Descartes, since he was the first to talk about coordinates with positive numbers, thus allowing future studies to be completed.

b) Rectangular coordinate system: This is the name given to the plane formed by the drawing of two number lines perpendicular to each other, the horizontal line or axis of the abscissas and the vertical line or axis of the ordinates and where the cut-off point coincides with the common zero. $(X)(Y)$

c) Polar coordinate system: It is responsible for verifying the relative position of a point in relation to a fixed line and a fixed point on the line.

d) Cartesian equation of the line: This equation is obtained from a line when two points where it passes are known.

e) Straight line: It is one that does not deviate and, therefore, has neither curves nor angles.

f) Conics: These are curves defined by the line that pass through a fixed point and the points of a curve: ellipse, circumference, parabola and hyperbole.

As formulas of analytical geometry, geometric figures are presented and their basic equations, according to González Urbaneja (2017, p. 25) are:

a) The lines are described by the formula (1)$ax + bx = c$

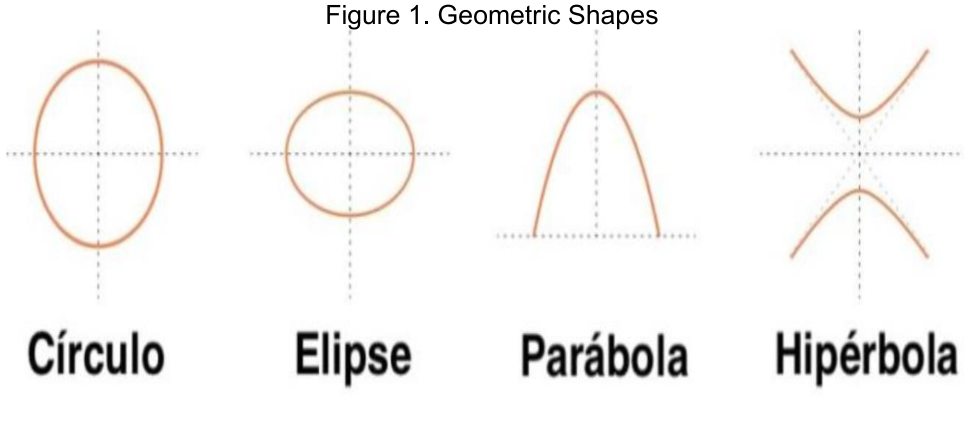
b) The circles are described by the formula (2)$x^2 + y^2 = 4$

c) Hyperbolas are described by the formula (3)$xy = 1$

d) The parables are described by the formula (4) $y = ax^2 + bx + c$

e) Ellipses are described using formula (5)$\left(\frac{x^2}{a^2}\right) + \left(\frac{y^2}{b^2}\right) = 1$

Figure 1. Geometric Shapes

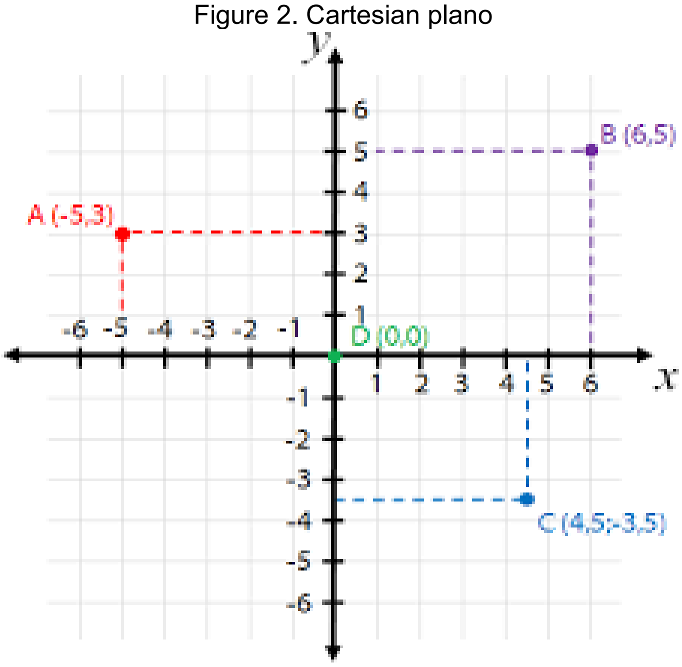


Source. González Urbaneja (2017)

The Cartesian plane is formed by two perpendicular number lines, one horizontal and one vertical that intersect at a point. The horizontal line is called the axis of the abscissas or of the x's, and the vertical line, the axis of the ordinates or yes, (y); The point where they are cut is called the source. In such a way that the purpose is to describe the position of points, which are represented by their coordinates or ordered pairs (González Urbaneja, 2017).

In Figure 2, a Cartesian plane is diagrammed:

Figure 2. Cartesian plano



Source. *González Urbaneja (2017)*

It has wide applications in life, both directly and indirectly. It has been used in medicine, power generation, and construction. It is also one of humanity's most useful conceptual tools, making itself manifest:

a) The circumference: A closed curved line whose points are equidistant from another located in the same plane that is called the center (Santiago Vegas, 2017). The application has been used since ancient times in prehistory, with the creation of the wheel, in music with CDs, musical drums with drums and cymbals. In weapons, in all types of transport in tires and rims for different types of vehicles: bicycles, cars, passenger or cargo trucks. The use in the time system, clocks and their gears, also in some sports the circumference of the balls and balls that are used. In the economic system, with coins (Santiago Vegas, 2017).

Its general equation is given as follows:

$$(x-a)^2+(y-b)^2=r^2 \qquad (6)$$

Figure 3. Circumference

Source. *Google, 2022*

b) The parabola: Open curve formed by two lines or branches symmetrical with respect to an axis and in which all its points are at the same distance from the focus (one point) and the guideline (line perpendicular to the axis) (Santiago Vegas, 2017). In satellite dishes and radio telescopes such as: Radar to receive and send signal beams, vehicle headlights or house lamps to concentrate and send energy as light beams, solar parabolic concentrators for the concentration of solar energy and its storage. Some constructions on bridges or architectural designs for buildings or houses. The ideal trajectories of bodies that move under the influence of gravity such as: throwing some balls, throwing arrows, a ballistic trajectory, throwing objects in some mechanical and children's games are even made exclusive designs of objects for their aesthetics (Santiago Vegas, 2017).

Its general equation is given as follows:

$$(x-h)^2 = 2p(y-k) \quad (7)$$

Figure 4. Parabolic Trajectory

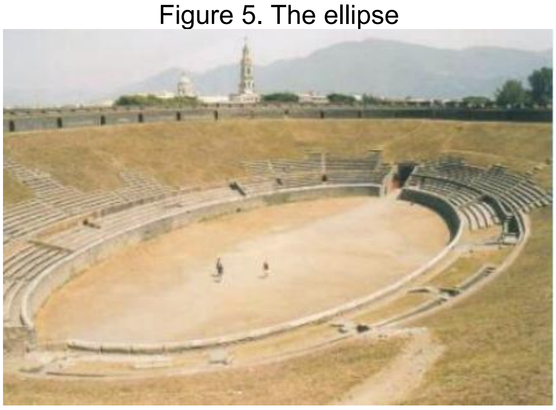
Source. *Google, 2022*

c) The ellipse: Curved and closed geometric figure, with two unequal perpendicular axes, which results from cutting the surface of a cone by a plane not perpendicular to its axis, and which has the shape of a flattened circle (Santiago Vegas, 2017). One of the first applications is in the solar system, since the relationship between the elliptical orbits of the planets and the sun with the sun that represents their focus was determined. Some sports stadiums have an elliptical shape, for athletics, bicycle, speed, among others (Santiago Vegas, 2017).

Its general equation is given as follows:

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1 \quad (8)$$

Figure 5. The ellipse



Source. *Google, 2022*

d) The hyperbola: It is a flat, open curve, with two branches; It is defined as the geometric place of the points whose distances differ from two other fixed points, called foci. Its usefulness, in astronomy, the orbits of some comets are hyperbolas, these comets only approach the sun once, which is one of their focuses in their trajectory. The intersection of a wall and the cone of light emanating from a table lamp with a truncated conical shade is hyperbola (Santiago Vegas, 2017).

Its general equation is given as follows:

$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1 \quad (9)$$

Figure 6. The hyperbola

Source. *Google, 2022*

The Cartesian method expounded by Descartes also provides a set of rules or procedures to deduce and achieve knowledge more precisely. These are summarized in four fundamental statements, expressed by Descartes in his *Discourse on Method,* cited by Arellano Oktac (2019, p. 79):

Rule 1: Evidence

Never to admit anything as true without knowing with evidence that it was so: that is to say, to avoid precipitation and prevention with all care, and not to understand in my judgments anything more than that which presented itself so clearly and distinctly to my mind that I had no occasion to doubt it (Descartes quoted by Arellano Oktac, 2019).

The author accepts as true only what is evident. Understanding that evidence occurs only in intuition, that is, in a purely rational act by which the mind immediately and simply grasps an idea.

Rule 2: Analysis

The second rule of the method is stated "to divide each of the difficulties I examine into as many parts as possible and as required to solve them better" (Descartes cited by Arellano Oktac, 2019). It is interpreted that any problem is nothing more thana vertebrate set of complex ideas. To analyze consists of decomposing the complex into its simple elements, which can be intuited as clear and distinct, that is, evident, ideas.

In this sense, the complex is reduced to the simple and, in it, from the unknown to the known, innate ideas, are accessed. Hence, analysis produces a chain of inferences that leads from a premise of unknown value

Rule 3: Deduction

The third, in leading thoughts in order, starting with the simplest and easiest to know objects to ascend little by little, as if by degrees, to the knowledge of the most compound, even assuming an order among those that naturally precede each other" (Descartes cited by Arellano Oktac, 2019).

Once he has arrived at the simple elements of a problem, he must reconstruct it in all its complexity, deducing all the ideas and consequences that derive from those absolutely certain first principles. Synthesis is an orderly process of deduction, in which some ideas are necessarily linked to others.

Rule 4: Enumeration (Revision)

It corresponds to the last standard, it is about checking and checking that there has been no error in the entire analytical-synthetic process. Verification tries to cover in a single moment and intuitively the globality of the process being studied to be sure of its certainty (Descartes cited by Arellano Oktac, 2019).

In relation to the most important contributions of Descartes in terms of the notion of curve, there are, cited by Arellano Oktac (2019), the classification by genres, the introduction of the generalized compass tool; which allows us to obtain the polynomial equation associated with a curve and the classification of geometric problems by classes, as follows:

Class 1: Those that lead to quadratic equations and can be constructed by means of lines, circles, parabolas, hyperbolas, or ellipses.

Class 2: Those that lead to cubic and quartic equations, whose roots can be constructed by means of the conchoid.

Class 3: Those that lead to equations of degrees five or six, which can be constructed by introducing an auxiliary cubic curve, such as the trident or cubic parabola.

In the same way, Descartes, by introducing the new curves he needed for his geometric constructions beyond the fourth degree, adds a new principle to the axioms of geometry:

> Two or more straight lines (curves) can move over each other, determining new curves through their intersections. (R. Descartes, 1947).

He then made a distinction between curves of two types:

Type 1: Geometric curves; they are those that are described in an exact way, that is, they are polynomial equations. These are: the straight lines, the circles, the conics, the trident, among others.

Type 2: Mechanical curves; They have to be imagined as described by two separate and independent motions, whose relationship does not admit of an exact determination, that is, a polynomial equation. These are: The quadratrix, the spiral, among others.

Descartes proposes a form of problem solving based on the application of analysis through the performance of algebra, which assumes the problem solved and establishes an orderly dependence between the known and the unknown, until the desired result is found, so that the rules of the Cartesian method acquire the mathematical sense of rules for the solution of geometric problems by means of equations (Terrado Juez, 2017).

TEACHING THE CONIC SECTIONS

DIDACTIC PROPOSAL 1

Approach to the activity

Didactic proposal. Tapered Sections

Mathematical concepts involved	Conic sections: circumference, ellipce, parabola, hyperbola
Materials	cardboard, ruler, scissors, pilot, glue.
Objectives	Construct the conical sections through didactic material
Proposal for the student	

1. A cone must be made first
2. Make a cut to the cone horizontally to the plane, then we have the cut in circular sections.
3. Make a diagonal cut to the cone and we would have the cut in the shape of an ellipse.
4. The cone makes a cut parallel to the generatrix (the generatris is one of the sides of the triangle) and we would be left with the cut in the shape of a parabola.
5. Make a cut to the cone vertically to the plane, then we are left with the cut in the form of a hyperbola.

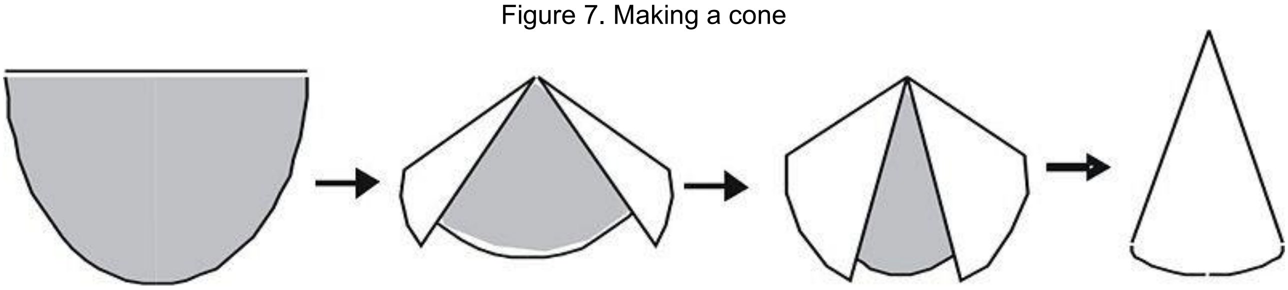
Source. Yacsury Montilla (2022)

DEVELOPMENT

Steps to follow:

1. The first thing we must do is make a cone. The simplest way to make a cone is to start with a semicircle and then overlap the straight edges until they achieve the desired shape.

Figure 7. Making a cone

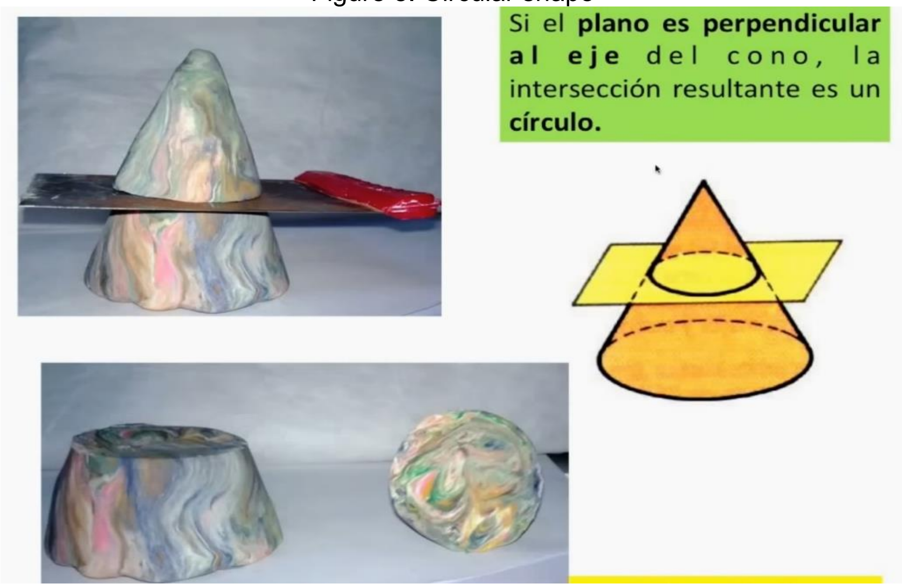


Source. *Google, 2022*

2. Then make the correct cut, according to the figure we want to obtain. To do this, follow the instructions below:

a) Make a cut to the cone horizontally to the plane, then we have the cut in circular sections.

Figure 8. Circular shape



Source. Google, 2022

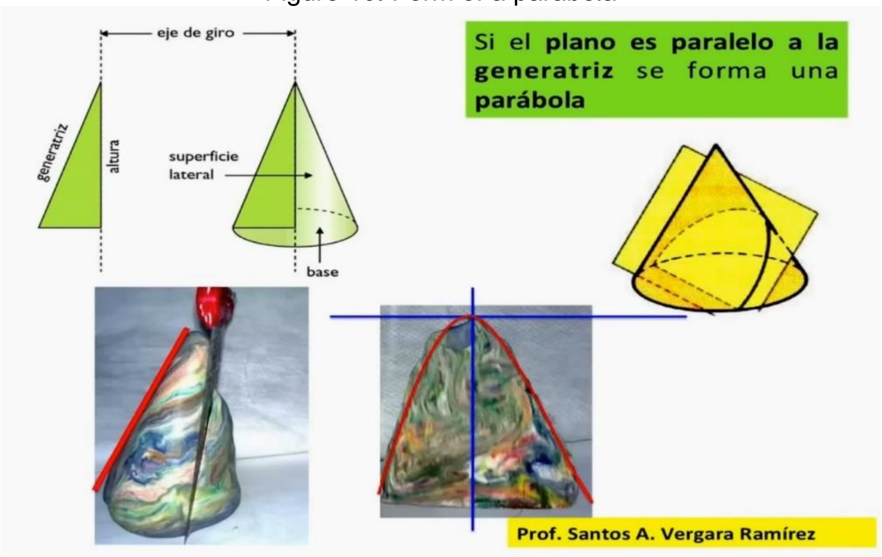
b) Make a diagonal cut to the cone and we would have the cut in the shape of an ellipse.

Figure 9. Shape of an ellipse

Source. Google, 2022

c) The cone makes a cut parallel to the generatrix (the generatris is one of the sides of the triangle) and we would be left with the cut in the shape of a parabola.

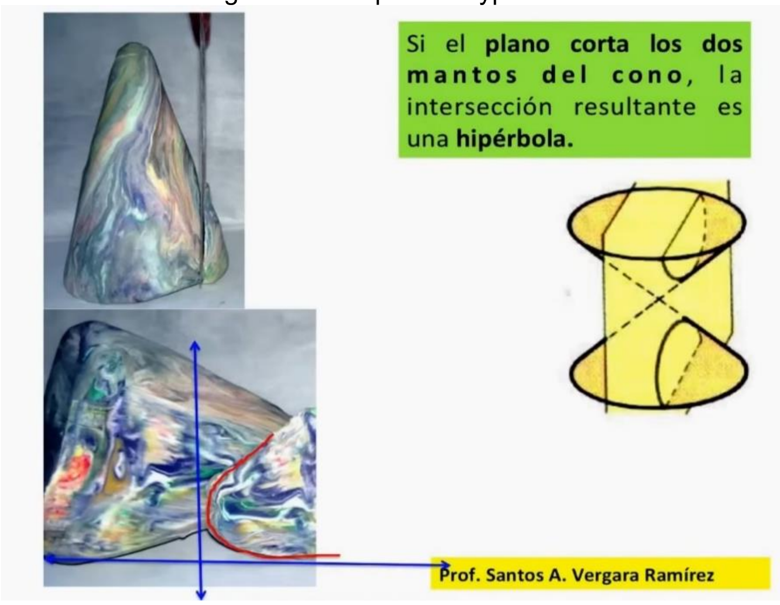
Figure 10. Form of a parabola



Source. *Google, 2022*

d) Make a cut to the cone vertically to the plane, then we are left with the cut in the form of a hyperbola.

Figure 11. Shape of a hyperbola



Source. Google, 2022

DIDACTIC PROPOSAL 2

Approach to the activity

Didactic proposal. Conical sections through soap bubbles

Mathematical concepts involved	Conic sections: circumference, ellipce, parabola, hyperbola
Materials	Water, liquid soap and glycerin, 10 straws, 10 skewer sticks, 5 meters of alyric thread (grues strand) galvanized wire N°18, rubber washer, plastic bucket 30 cm deep.
Objectives	Construct the conical sections through didactic material
Proposal for the student 1. Build a circular cone 20 cm in diameter and 25 cm in height using galvanized wire, 10 skewer sticks, 10 straws and a rubber washer. 2. Secure the cone structure with lyrical thread. 3. Immerse the cone structure in a mixture of water, liquid soap and glycerin	

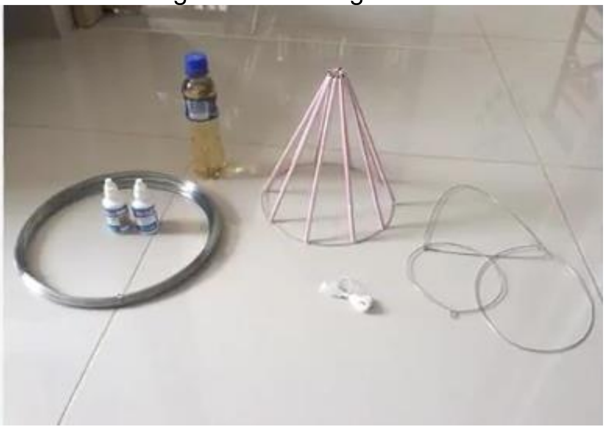
Source. Yacsury Montilla (2022)

DEVELOPMENT

Steps to follow:

1. Let's build the cone with the materials already mentioned.

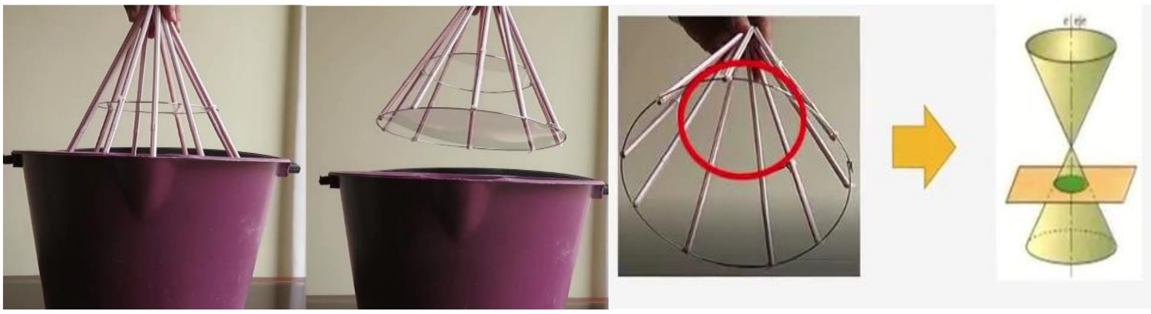
Figure 12. Making a cone



Source. Google, 2022

2. If we intersect a plane perpendicular to the axis of the cone, we obtain a circular surface whose edge is a circumference parallel to the base of the cone, for this it immerses the structure of the cone in the plastic bucket that contains the mixture of water, liquid soap and glycerin, in order to appreciate the circumference.

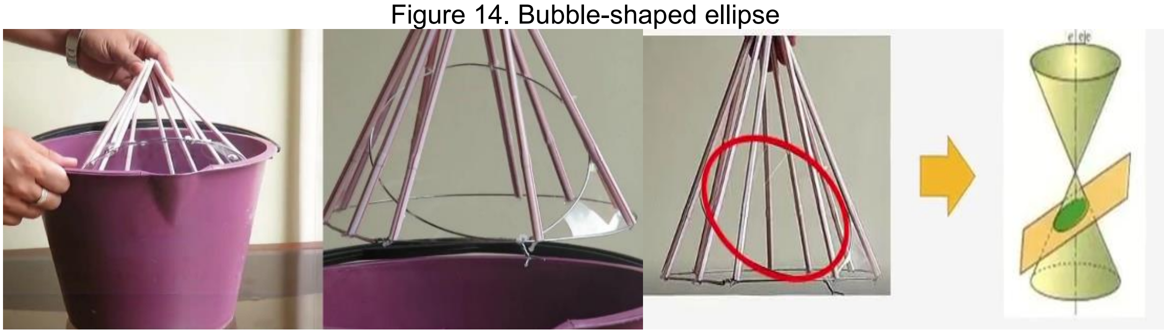
Figure 13. Bubble-shaped circumference



Source. Google, 2022

3. It intersects the structure with a plane oblique to the axis of the cone, in which we obtain a surface whose edge is a closed curve called an ellipse.

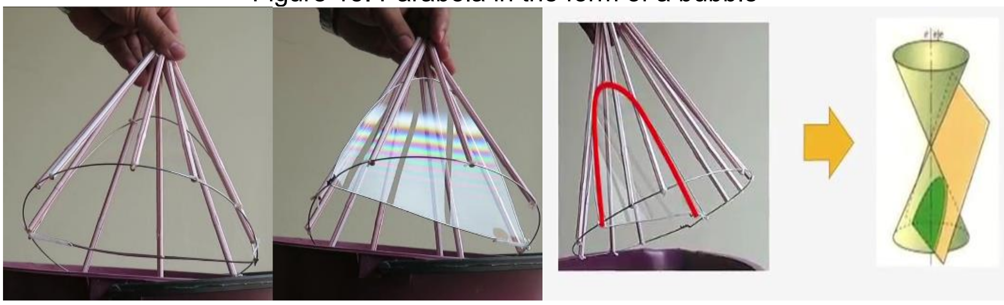
Figure 14. Bubble-shaped ellipse



Source. Google, 2022

4. Submerge the structure of the cone by tilting it so that it is palalelo to a generatrix, resulting in a surface whose edge is a parabola. Remember that the generator is an inclined line that rotates around the cone. In this case, the generator of the cone is represented by each of the straws.

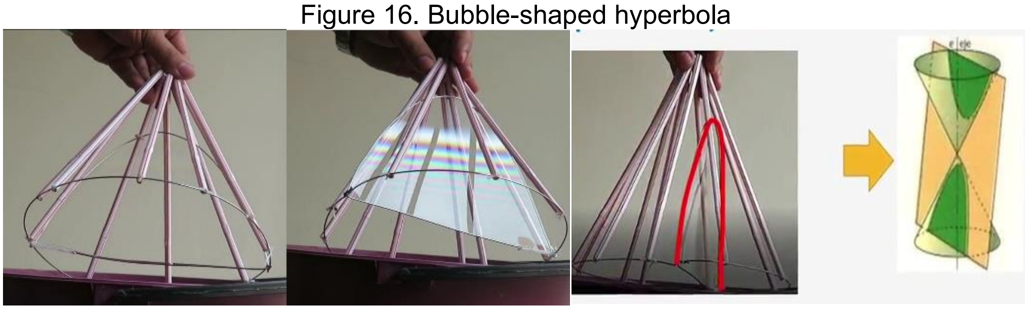
Figure 15. Parabola in the form of a bubble



Source. Google, 2022

5. Finally, if we cut a cone with a plane parallel to its axis, we obtain a surface whose edges are a hyperbola. It is important to clarify that a hyperbola is an open curve of two branches that forms on the surface of two cones joined by their vertex.   Only one of the branches of the hyperbola is shown in the images.

Figure 16. Bubble-shaped hyperbola



Source. Google, 2022

As a synthesis of this chapter, it is expressed that different problematic situations arise in the field of analytical geometry, in particular about the quadratic function, its graph and associated equation, in which a reorganization of contents and the form of teaching is sought, which enables the construction of meanings by the student.

The examples of the activities presented are based on theory, in each of these the mathematical concept that needs to be addressed, the objective, the procedure that the student must develop to achieve the proposed purpose has been indicated (Vergaras Salas, 2017).

It is important to note that the orientation of the student must be clear so as not to generate confusion and achieve the development of their creative thinking by incorporating the skills required for this purpose in each face of the process (Terrado Juez, 2017)

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