Building the Tower of Hanoi with recyclable materials: A methodological proposal for teaching empowerment in Elementary School classes

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ABSTRACT

This work presents an innovative methodological proposal for the teaching of empowerment in Elementary School II classes, using the construction of the Tower of Hanoi with recyclable materials. The main objective is to facilitate the understanding of mathematical concepts related to empowerment through a practical and playful approach, while promoting awareness of the importance of recycling and sustainability. The methodology involves several steps, starting with the selection and preparation of recyclable materials, which are transformed into components of the Tower of Hanoi game. Then, students actively participate in the construction of the game, which arouses their interest and creativity. During the assembly, concepts of recycling and reuse of materials are discussed, encouraging reflection on sustainable practices in everyday life. After construction, the game is used as a didactic tool to teach potentiation. Practical activities and complementary exercises are developed to reinforce learning, promoting a more contextualized understanding of the concepts involved. Undoubtedly, this approach tends not only to increase student engagement, but also to significantly improve content assimilation. Students are expected to demonstrate greater interest and participation in class, as well as evident progress in understanding empowerment. Another relevant aspect of this proposal is the promotion of interdisciplinarity, integrating knowledge of Mathematics with environmental themes. In conclusion, the methodological proposal of using the construction of the Tower of Hanoi with recyclable materials for the teaching of empowerment proves to be efficient and enriching. It is recommended to continue and expand this type of initiative, exploring other forms of integration between mathematics education and environmental education, aiming at the formation of individuals more prepared for the challenges of the future.

Keywords: Tower of Hanoi, Methodology, Empowerment, Teaching.

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INTRODUCTION

Because it is a complex science and treated in such a formalistic way throughout the history of mathematics education, there are those who think that it is impossible or, at least, very difficult, to relate Mathematics with other areas of knowledge.

However, as we set out to give a more palpable meaning to those contents studied, we realized that a large part of it is present, directly or indirectly, in the lives of the different individuals that make up society. Thus, there is a lot of mathematical knowledge in the daily work, for example, of the engineer, the historian, the geographer, the biologist, the anthropologist, as well as those individuals who did not even attend university, such as the bricklayer, the seamstress, the baker, etc. Thus, Mathematics is also present, as simple as it may be, in the daily lives of all teachers and, in order to further improve our knowledge, it is essential to incorporate a close relationship with other areas into our teaching practice.

This interdisciplinary approach will enhance our knowledge, enriching the educational experience and providing students with a more comprehensive and integrated view of knowledge. In this perspective, this work aims to present a methodological proposal to work Mathematics from the perspective of environmental education in elementary school classes, through the production of the Tower of Hanoi game with recyclable materials, considering the importance of preserving the environment. In addition to the environmental relevance, there will be a significant pedagogical contribution to Mathematics classes, which almost always follow that traditional teaching script, basically composed of the exposition of the content, presentation of examples and application of a list of exercises, in which there is no space to contextualize that knowledge, which generates, unfortunately, little interest on the part of the students, ending up in an unsatisfactory level of learning. In other words, such a proposal will enable an interdisciplinary and playful teaching of Mathematics, which motivates students to get actively involved, which will provide more meaningful learning.

As for the structure of this work, initially a bibliographic review is presented, which will deal with mathematics education, playfulness, interdisciplinarity and environmental education, as well as the history and rules of the game in question. Later, the description of the methodological proposal will be made in detail. Finally, we will make some observations that may be useful during an eventual implementation.

OBJECTIVES

GENERAL

Present a proposal for a non-conventional methodology for the teaching of Mathematics, which will integrate mathematical concepts and environmental issues through the creation and use of games made with recyclable materials.

SPECIFIC

- Reflect on the importance of learning mathematical content through play, as well as contribute to the strengthening of environmental awareness through the reuse of recyclable materials;
- Explore the content of the Towers of Hanoi with the students;
- Stimulate logical-mathematical reasoning during the practice of the game.

THEORETICAL FOUNDATION

MATHEMATICS EDUCATION

In the early days of humanity, Mathematics was used by different peoples only to solve everyday situations, such as dividing land and counting animals. Over time, great characters emerged and made this science more and more systematized. Thus, man has made the applicability of this knowledge increasingly complex, not limiting himself, then, simply to the solution of problems that basically concerned survival.

What can be seen, therefore, is that Mathematics, without a doubt, has had and will continue to have a primordial role in the construction of society. It is inconceivable to think about the evolution of humanity without attributing to it, among others, of course, the existence of this science, as D'Ambrósio (1999, p. 97) brings us, when he states that "mathematical ideas appear in the entire evolution of humanity, defining strategies to deal with the environment [...]".

Because it is such an important area of knowledge for us, because it is so complex from a certain point and because it is also so demanded in a general way, it is essential that there is extra attention to the formulation of its teaching process, because "currently, it is perceived that, in the educational context, universality, objectivity, the sharpness and perfection of the languages used in mathematics do not guarantee its relationship with society." (CORREIA; SILVA, 2021, p. 62-63).

Like this

> In the teaching of Mathematics, in particular, education restricted to the presentation of contents formalized in textbooks and/or that require the student to reproduce processes for the resolution of activities, provokes the conception that Mathematics is restricted to calculations and formulas. This invariably reinforces the students' demotivation. In this sense, research in Mathematics Education has pointed to the effectiveness of developing activities that reward a context of "discovery". Individually or in a team, the provocation of

> situations that present playful and recreational aspects emerges as motivating elements in the approach to mathematical concepts. When learning is pleasurable, it becomes a relatively simple process. Students, when motivated, have their curiosity aroused and learning becomes something natural (LONDERO, 2020, p. 12).

From this perspective,

> In order for our students to develop knowledge, acquire intellectual autonomy and have the capacity for critical reflection, educational processes must reward challenging experiences and situations. The student must find out for himself and not just know. He must feel like a citizen, be critical, responsible, competent and respect the limits of living in the community. When the student is encouraged to use their investigative and/or creative skills, the process of reproducing content is minimized, favoring sustainable learning (LONDERO, 2020, p. 12).

Following this line of reasoning,

> The naturalness of understanding the mathematical exercise awakens in the student the desire and curiosity to identify the Mathematics around them, making them recognize themselves as mathematicians. Realizing that Mathematics is not as complicated as one imagines, leads the student to understand the importance of Mathematics in his life, feeling its naturalness and being able to master it without major complications. Mathematics is part of the world and the world is present in it (CORREIA; SILVA, 2021, p. 63).

To make happen, in fact, what is proposed by the authors is obviously not easy. The efforts of teachers alone are not enough. However, if we want to at least start this process, it is up to us, teachers, to kick things off, since we are the ones who recognize the relevance of the link between scientific knowledge and popular knowledge, in order to create significant knowledge for the formation of a sensitive, reflective and engaged individual in his environment.

In this way, "both the school and the teacher must become observers so that they can seek resources that help in the teaching and learning process [...]" (CORREIA; SILVA, 2021, p. 63). One of the methodological resources that can be used is playfulness.

PLAYFULNESS

In an attempt to make the teaching and learning process more pleasurable, meaningful and connected with reality, playfulness emerges, which "stimulates the child while working with concrete material, games, that is, everything he can handle, reflect and reorganize [...]" (MODESTO; RUBIO, 2014, p. 4).

"Playfulness is part of the children's universe and can contribute to the child's learning and development." (BIZERRA, 2017, p. 13). According to Santos (2000) *apud* Bizerra (2017), "the word Playful means to play. Play happens from the beginning of existence to the end of life."

We can understand then that

> The game is a very significant pedagogical tool. In the cultural and biological context, it is a free, joyful activity that encompasses a meaning. It is of great social value, offering numerous educational possibilities, as it favors bodily development, stimulates psychic life and intelligence, contributes to adaptation to the group, preparing the child to live in society, participating and questioning the assumptions of social relations as they are set (KISHIMOTO, 2006, p. 26).

Complementing this idea,

> Playfulness is a human need at any age and cannot be seen only as fun. The promotion of playful activities, whether individual or group, provides privileged situations that lead to learning and personal, social and cultural development, producing knowledge and experiences that will be incorporated into the student's life, opening up possibilities for them to be free and to make decisions according to their own conscience (FREIRE, 2008).

Thus, Mafra (2008) emphasizes that playfulness plays a fundamental role for teachers who wish to promote transformations in their educational practice, since it is a flexible and adaptable tool that can be incorporated into the methodological proposal.

In the teaching of mathematics, playfulness, in addition to bringing a feeling of satisfaction, opens space for imagination. Playfulness in mathematics education brings the student closer to scientific knowledge, introducing a language that "little by little will be incorporated into formal mathematical concepts, by developing the ability to deal with information and by creating cultural meanings for mathematical concepts and the study of new contents." (MOURA, 2009, p. 95).

So

> The learning of Mathematics consists of creating strategies that enable the student to attribute meaning and construct meaning to mathematical ideas in order to become capable of establishing relationships, justifying, analyzing, discussing and creating. In this way, it surpasses teaching based only on developing skills, such as calculating and solving problems or fixing concepts through memorization or lists of exercises (PARANÁ, 2008, p. 45).

This alerts us to the need to overcome teaching based only on memorization and the mechanical practice of exercises. Instead, it is more interesting to seek active learning, in which the student becomes an active agent in the construction of knowledge, exploring, investigating and understanding mathematical concepts in a deeper way. This pedagogical approach that values the understanding, reflection and application of mathematical concepts, allows students to develop critical thinking, problem-solving and creativity skills. In this way, the learning of Mathematics becomes more meaningful and prepares students to face the challenges of the real world.

INTERDISCIPLINARITY

Interdisciplinarity is of paramount importance, as it helps students in their difficulties by providing more meaningful learning, which ends up building a logical and critical sense about what is studied. Disciplinary approaches create a broad learning, thus facilitating the resolution of various

problems, however, it is not possible to do interdisciplinarity without integration. This allows the collaboration of ideas simultaneously, by connecting the knowledge of one discipline to another with the aim of solving a problem. Complementing this idea,

> We live and work in a world of academic disciplines that alone are ill-equipped to deal with the complexity of society's problems, leaving us with something of a conundrum: disciplinary approaches are an essential and ever-present part of any problem-solving strategy. However, without integration, they are insufficient to solve problems in all their complexity. Interdisciplinarity enables integration by providing ways and means to combine wisdom and science, to expand and refine information, and to build a tradition of problem-solving that mixes old and new ideas, while maintaining the capacity for cumulative learning that refines, clarifies, and simplifies (KOBARG, 2021, p.87).

Interdisciplinarity brings, therefore, the conception that it is increasingly necessary to unite two or more disciplines, seeking to get out of the traditional way, through the various approaches focused on a single subject, as explained below:

> Mutual exchange and interaction of diverse knowledge in a reciprocal and coordinated manner; a methodological perspective common to all; integrate the results; the interests of each discipline remain, however, they seek solutions to their own problems through articulation with the other disciplines (KOBARG, 2021, p.86).

What is noticeable, fortunately, is that currently educators have, as far as possible, tried to work in a more interdisciplinary way, since it is perceived that there have been profound changes in various aspects related to teaching and learning in our country. Adding to this idea,

> Nowadays, interdisciplinarity and contextualized practices have been part of the routine of most educators, since such an attitude generates a broad construction of knowledge, breaking with the limits of the disciplines, as just integrating the disciplines would not be satisfactory. It should, then, be part of the Elementary School routine, teachers encouraging students to build relationships between the different contents present in the various subjects (KOBARG, 2021, p.134).

Thus, simply integrating disciplines is not enough, but promoting a broad construction of knowledge, which goes beyond the boundaries of individual disciplines.

ENVIRONMENTAL EDUCATION

The human being has been producing, transforming and building the geographical space without feeling an integral part of this environment. As a result, we are now experiencing environmental destruction and irrational use of natural resources, causing a series of socio-environmental problems that affect the global community. To understand the concept of the environment is to create conditions for human beings to feel morally responsible for their behavior in relation to nature [...]. (COSTA; VIVIANE, 2012, p. 15).

The preservation of the environment is fundamental, as it is in the environment that we have the natural resources for our survival. One of the ways to take care of it is through the recycling of materials.

As we live and "wait" for new global environmental transformations, we have a duty to use all the intelligence and technology achieved throughout our evolution to mitigate our influence on these climate changes. How can we do this? As mentioned by Costa and Viviane (2012, p. 64), it is recommended to acquire knowledge about recycling, reducing the use of fossil fuels, exploring alternative energy sources and taking advantage of renewable natural resources, such as wind energy. In addition, it is important to demonstrate respect for the environment and promote environmental education in an interdisciplinary way, covering everyone.

Another very important point is the role of the school for the environment: it is there where pedagogical plans aimed at environmental development are created. At school, this knowledge is learned and discussed and so we can apply it in our daily lives.

According to Costa and Viviane (2012, p. 88), the relationship between the environment and the individual is not restricted only to the community sphere, but also extends to the school context. In this sense, it is essential that the pedagogical work in the classroom is constant and based on previously established objectives and knowledge, according to the context in which it is inserted.

In this sense, Costa and Viviane (2012, p. 89) explain that, by considering the environment within the school environment, we provide an environmental education that is meaningful, concrete and endowed with meaning for the individuals for whom it is intended. Thus, all those involved in the educational process play an integral and fundamental role in ensuring that theory is contemplated in practice and vice versa.

The edition of volume 9 of the PCNs will discuss the issue of the environment and health, making analyses of the modes of interaction between man and nature:

> […] to deal with issues related to the environment in which we live, considering its physical and biological elements and the modes of interaction of man and nature, through work, science, art and technology (BRASIL, 1997, p. 15).

Human beings have contributed significantly to the pollution of the environment, mainly through large companies. We need to take a more critical look at the consequences we may face if we do not take action. Through environmental education, it will be possible to promote the change of our habits. By each one doing their part, we can build a better world.

A HANOI TOWER

The following is a brief history of the games, as well as their rules.

The Tower of Hanoi is a famous mathematical puzzle that was invented by the French mathematician named Édouard Lucas (1842-1891) in the nineteenth century. The name was given in reference to the city of Hanoi, the capital of Vietnam, where there is a legend associated with the tower.

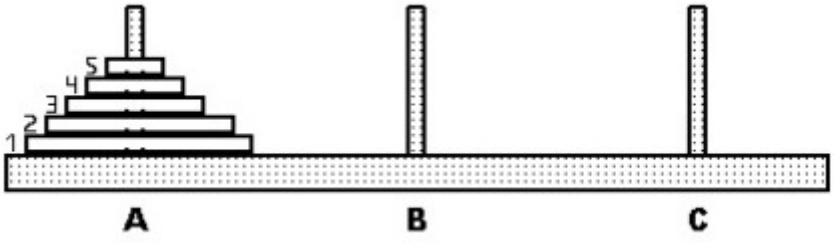
Figure 1: Édouard Lucas.

Source: MacTutor History of Mathematics Archive. Disponível em: <https://mathshistory.st-andrews.ac.uk/Biographies/Lucas/pictdisplay/>.

The legendary history of the Tower of Hanoi says that in the temple of Hanoi, there is a tower with 64 (sixty) gold discs, initially stacked in ascending order of size, on one of the three pins of the temple. Legend states that when all the disks are moved to the third pin, the world will come to an end.

The puzzle consists of a base with three pins and a set of discs of different sizes. The disks are stacked from largest to smallest on one of the pins (see image below), and the goal is to transfer the entire stack of disks to another pin, following a few rules.

Figure 2: Tower of Hanoi.



Source: UNESP. Available at: <https://www.ibilce.unesp.br/Home/Departamentos/Matematica/labmat/torre_de_hanoi.pdf>.

The rules of the puzzle are simple: only one disk can be moved at a time, and a larger disk can never be placed on top of a smaller one. Therefore, in each movement, a disc is taken from the top of a pile and placed on top of another pile. The challenge lies in figuring out the sequence of movements that allows you to transfer all the discs to the target pin, using the middle pin, without violating the rules.

While the legend of the Tower of Hanoi may just be a fictional story, the puzzle itself is an interesting mathematical challenge and has been the subject of study in many fields. There are several strategies and algorithms that can be used to solve the puzzle efficiently, regardless of the number of disks.

The Tower of Hanoi is considered a classic example of recursion, and its solution requires 2N − 1 (two to the nth power, minus one) moves, where N is the number of disks. Therefore, for the initial stack with 64 (sixty-four) disks mentioned in the legend, it would take 264 – 1 (two to the sixtieth power, minus one) moves, which is an astronomical amount.

Nowadays, the Tower of Hanoi is popular as a tabletop puzzle as well as a computational challenge. It is used as a teaching tool to develop problem-solving, logical thinking, and mathematical skills, as well as provide fun and entertainment for people of all ages.

METHODOLOGY

The initial stage of the research that resulted in this TCC began with the choice of the theme. Once defined, the objectives were established and the methodology was selected, opting for bibliographic research.

According to Gil (2002, p. 44), the bibliographic research "[...] it is developed based on material already prepared, consisting mainly of books and scientific articles". Complementing this, Severino (2007, p. 122) tells us that bibliographic research is carried out by:

> [...] available record, resulting from previous research, in printed documents, such as books, articles, theses, etc. Data from theoretical categories already worked on by other researchers and duly recorded are used. The texts become sources of the themes to be researched. The researcher works from contributions from the authors of the analytical studies contained in the texts. (SEVERINO, 2007, p.122).

In this way, the texts chosen by the researcher play a fundamental role in providing the basis and support necessary for the elaboration of the work. They not only broaden the researcher's understanding of the topic, but also clarify the limits and possibilities of research, allowing for a more in-depth and comprehensive analysis.

In this sense,

> This research aims to know the different forms of scientific contribution already made on a given subject, aiming to enter current and relevant data on the investigated theme. It uses exclusively material already prepared and available, in particular books and scientific articles, and is the basis for any type of research [...] (METRING, 2010, p. 64).

After the bibliographic survey, a selection of the most relevant texts to the research theme was made. With the systematic reading of this material completed, the writing of this TCC was carried out, integrating the readings related to the theme studied.

RESULTS AND DISCUSSIONS

To achieve the objectives, we will describe the activities to be carried out in elementary school classes.

For the making of the game, the predominant material will be cardboard. The use of scissors, ruler and glue is essential. For a better finish, some extra materials will also be needed, such as paint and tape. The applicator (teacher, intern, etc.) will guide each phase of the game production process.

Now, we will introduce the steps to follow:

1st pedagogical action – **Introduction** (10 to 15 minutes): Initially make a discussion, through images printed on A4 sheet, of how various mathematical concepts are, even if in a simple and sometimes almost imperceptible way, present in people's daily lives. These images may be of some professionals performing their activities. To finish this first part, discuss the importance of preserving the environment, emphasizing recycling as one of the resources, as well as how Mathematics can relate to environmental issues.

2nd pedagogical action - **Production** (60 minutes): This moment will be dedicated to the production of the game. Depending on the number of students present, divide the class into groups of 4 or 5 members, so that each of these groups will produce a unit of the game. Before the day of application, ask the classes to bring cardboard and, as far as possible, other materials. It is important that the applicator in charge also gets these materials, since some may not take them. Due to time, it is recommended to provide some disc molds to speed up the manufacturing process. It is important to emphasize that Styrofoam, which is 100% recyclable, is an alternative to cardboard. However, it is harder to find, which could cause some to buy, and that is not the idea, at least at first. Specifically with regard to production, this process will consist essentially of three phases: the first is the construction of a firm base, through the gluing of some rectangular layers of cardboard (it may be more practical to use adhesive tape to join the layers); after that, the production phase of the discs begins, which will be similar to the previous one, which will also be formed by overlapping cardboard (we reinforce the importance of making molds available); Finally, the finishes must be made, which will depend on the creativity of the groups and the material available. For example, plastic straws can be used as rods.

3rd pedagogical action – **Practice** (25 minutes): After this practical moment, talk about the mathematical nuances present in the game. Explore the potentiation content from the fact that the minimum number of moves to solve the game is given by a power, 2N-1, where N is the number of discs. Propose to the students that they try to solve the puzzle, at first, with a small number of discs, 3 or 4. As they succeed, gradually increase this amount, always paying attention to the number of movements performed, in order to verify whether or not it matches the 2N-1 relationship. It can also be proposed that, once the number of movements is known, determine the number of existing disks.

For example: If someone solved the puzzle with 255 moves, how many discs were there in the tower? In this case, it would be enough to solve the equation 2N – 1 = 255, where N = 8. This example would be for an initial approach. The problems can and should gradually be improved.

4th pedagogical action – **Reflection** (10 minutes): Dedicate these final minutes to listening to them. To do this, ask some questions, such as "What is your opinion about the use of games in Mathematics classes?". Throughout the implementation of the project, carefully monitor the students' receptivity to the proposal, evaluating whether they are real interest in creating the resources and if, by presenting the mathematical content in a different way, they are more engaged in learning.

FINAL CONSIDERATIONS

By using recyclable materials in the construction of the game, students are encouraged to explore their creativity and develop practical skills, actively collaborating in the learning process. This methodology is capable of engaging students, making learning more dynamic and meaningful. In addition, the playful nature of the activity contributed to the creation of a lighter and more motivating learning environment.

From this perspective, the use of practical and concrete tools, such as the Tower of Hanoi, can significantly improve the understanding of abstract concepts of Mathematics. Students tend to show greater interest and participation in classes, in addition to a better assimilation of the contents covered.

Another relevant aspect is the interdisciplinarity promoted, by integrating knowledge of Mathematics with environmental themes. This holistic approach not only enriches the school curriculum, but also forms more conscious and responsible citizens. Thus, the methodological proposal presented in this work has a great potential for improvement both in the mediation of mathematical knowledge and in the promotion of sustainable values. It is recommended that future research and projects explore other forms of integration between mathematics education and environmental education, further expanding the benefits for students and society. The continuity and expansion of this type of initiative can contribute significantly to a more integral and transformative education.

REFERENCES

1. Bizerra, E. T. C. (2017). *A ludicidade na relação ensino-aprendizagem: O papel do professor de uma escola de educação infantil de Castanhal – PA* (TCC, Licenciatura em Pedagogia). Faculdade de Pedagogia do Campus de Castanhal da Universidade Federal do Pará. Castanhal.

2. Brasil. (1997). *Parâmetros Curriculares Nacionais*. Brasília: Ministério da Educação. Disponível em http://portal.mec.gov.br/seb/arquivos/pdf/livro091.pdf. Acesso em 18 jan. 2024.

3. Correia, D. M. das N., & Silva, G. L. B. (2021). Contribuições da ludicidade no processo de inclusão no ensino de Matemática no quinto ano do Ensino Fundamental de uma escola municipal de Nova Olinda-PB. *Revista Principia, 58*, 60-68. Disponível em https://periodicos.ifpb.edu.br/index.php/principia/article/viewFile/5661/1832. Acesso em 07 fev. 2024.

4. Costa, C. A. P. da, & Viviani, D. (2012). *Meio ambiente e saúde*. Indaial: UNIASSELVI.

5. D'Ambrósio, U. (1999). A história da Matemática: Questões historiográficas e políticas e reflexos na Educação Matemática. In M. V. Bicudo (Org.), *Pesquisa em Educação Matemática: Concepções e perspectivas* (pp. 11-24). São Paulo: UNESP.

6. Freire, P. (2008). *Pedagogia do oprimido* (47a ed.). São Paulo: Paz e Terra.

7. Gil, A. C. (2002). *Como elaborar projetos de pesquisa*. São Paulo, SP: Atlas.

8. Kishimoto, T. M. (2006). O jogo e a educação infantil. In T. M. Kishimoto (Org.), *Jogo, brinquedo, brincadeira e a educação* (pp. 13-43). São Paulo: Cortez.

9. Kobarg, A. P. R. (2021). *Prática pedagógica interdisciplinar: Escola e sociedade*. Indaial: UNIASSELVI.

10. Londero, E. F. (2020). *Matemática lúdica*. Indaial: UNIASSELVI.

11. Mafra, S. R. C. (2008). *O lúdico e o desenvolvimento da criança deficiente intelectual*. Paraná: Secretaria de Estado da Educação.

12. Metring, R. A. (2010). *Pesquisas científicas: Planejamento para iniciantes*. Curitiba: Juruá.

13. Modesto, M., de Alcântara, J., & Rubio, S. (2014). A importância da ludicidade na construção do conhecimento. *Revista Eletrônica Saberes da Educação, 5*. Disponível em http://docs.uninove.br/arte/fac/publicacoes_pdf/educacao/v5_n1_2014/Monica.pdf. Acesso em 21 jan. 2024.

14. Moura, M. O. de. (2009). *A séria busca no jogo: Do lúdico na matemática*. São Paulo: Cortez.

15. Paraná. Secretaria de Estado da Educação. Superintendência da Educação. (2008). *Diretrizes Curriculares da Educação Básica Matemática*. Curitiba: SEED.

16. Severino, A. J. (2007). *Metodologia do trabalho científico*. São Paulo, SP: Cortez.