Analysis of student performance using the Fuzzy intuitionist model

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ABSTRACT

In the Public Education Network, the Mathematics teacher from the 6th to the 9th grade is faced with the abyss between pedagogical planning and learning. Evaluations plus bonuses for complementary activities can be considered extra-class excesses, as they generate unrealistic results for the education system. The Fuzzy Intuitionist modeling allows the recognition of patterns for the evaluation of learning using degrees of pertinence and non-pertinence as a function of the results of the three mandatory measurements in the course. Personalized assessment would be more coherent with the student's profile, combating the discouragement generated to "school failure", formative assessments are suggested, which can provide the student with greater feedback, directing him to structured knowledge. It is vital to help a teacher feel good about himself and his performance, not minimizing the emotional balance and psychological resilience to live with the stress generated by the current building and educational conditions, avoiding psychosomatic effects. In this domain, it is essential that pedagogical processes are in an appropriate position to contribute to functional learning, that is, skills (activities) that encompass self-care, hygiene habits, school attendance, commitment to tasks and interpersonal interaction.

Keywords: Fuzzy Set, Intuitionistic Fuzzy Sets, Similarity, Functional Learning, Fuzzy Adaptive Weighter for Assessment scenarios.

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INTRODUCTION

The Mathematics teacher in the Public Education Network, when working from the 6th to the 9th grade, is faced with the abyss between the current school pedagogical planning and learning, often there is a disparity between what is planned and what is actually accomplished.

The evaluations of the students, when added to bonuses for activities complementary to the activities in the classroom, can be considered true extra-class excesses, as they often mask the gaps in knowledge, generating unrealistic results for the education system. It is not uncommon to find students who arrive at high school with difficulty in basic operations, especially in relation to Division, the basic operation of Mathematics.

It is noteworthy that the importance of teaching focused on the themes of the National Common Curricular Base (BRASIL, 2018) should be recognized, with regard to universality, plurality and fluidity between disciplines. It is noteworthy that despite so many efforts, learning problems, especially in Mathematics, persist over the years. The experience in Youth and Adult Education in terms of the basic operations of Fundamental Mathematics, allows us to perceive the extreme difficulty in the algorithm regarding division (getting help in my master's work and other works). It is worth reflecting on the importance of its use in financial planning, especially in the notions of family budget, home economics, where the mastery of at least the four arithmetic operations are fundamental. The abyss between the political pedagogical project and curricular learning masks the problem, since extra-class activities generate bonuses for basic curricular subjects, often without any connection in relation to the teaching of Mathematics, since the bonus is only due to the student participating in the quadrilha of the June festivals without there being any correlation with the concepts of fundamental measures referring to the average time of executions during the square rehearsals and geometric shapes in the operational of dance.

This reflection motivates the use of Intuitionist Fuzzy Modeling in the evaluations applied in the first quarter of 2023 regarding one of the 8th grade classes in a municipal school to confront the performance standard as a way to outline strategies that can improve student learning in order to optimize school management, mitigating possible deficiencies of students, which can support future innovative pedagogical and curricular strategies to improve the teaching and learning process. The general objective is to show that Intuitionistic Fuzzy Modeling can provide pertinence values that help in verifying the student's response to teaching strategies. To this end, an analysis of the first trimester of an eighth-grade class from a municipal public school system was carried out. In this segment of education, the evaluation takes place in a triple way under the criterion of three learning evaluations, hereinafter AV1, AV2 and AV3, where each one has a score of 10, but each one with its own proposal and approach. AV1 is directed to extra-class activities that directly or indirectly involve the concepts portrayed in the classroom, unlike AV2 and AV3 that are restricted to the contents taught in class. The student's final grade is the sum of the three evaluations. Within this scope, the following specific objectives are sought: to analyze the performance of students according to the grades assigned to the evaluations of the syllabus related to AV2 and AV3, adding or not the evaluation AV1 considered extra-class. The results can be considered diagnostic tools, both for the teacher and for the institution regarding the ongoing teaching and learning process.

OBJECTIVE

Propose a method that seeks to analyze student performance to contribute to the proposal, with the goal of a fair evaluation in the Elementary School cycle.

Within this scope, the following specific objectives are sought to be met:

a) to analyze the performance of the students according to the Fuzzy Intuitionist modeling of the AV1, AV2 and AV3 evaluations.

b) To compare the results of these evaluations as a simultaneous diagnostic tool of the scenarios of the teaching process.

METHODOLOGY

Initially, a literature review of articles published in electronic databases, from 1990 to 2023 made available by the Sirius Network of UERJ, Scientific Electronic Library Online – Google Scholar, using the descriptors: Fuzzy Sets, Intuitionist Fuzzy Sets, Similarity, Fuzzy Activation Functions and the Department of Education of the Municipality of Maricá, Rio de Janeiro, Brazil.

DEVELOPMENT

CLASSIC SETS

In a *crisp set,* there are only two possibilities regarding the inclusion of an element x in the setA, in a universe of discourse, in which or . The following characteristic function represents the approach in a classical set (SZMIDT, 2014):$X x \in A' x \notin A' \varphi_A(x)$

$\varphi_A(x) = \begin{cases} 1, & \text{se } x \in A \\ 0, & \text{se } x \notin A \end{cases}$, being the notation Characteristic function:

$$A = \{ < x,\ \varphi_A(x) > / \ x \ \in X \} \qquad (1)$$

FUZZY SETS

A Fuzzy set in a universe of discourse is characterized by a function of pertinence where each element of the set is associated with a real value of pertinence in the universe of discourse (ZADEH, 1965). $\mathrm{A}'\mathrm{X}\mu_{\mathrm{A}'}(\mathrm{x}) \in [0,1], xA'$

$$\mathrm{A}' = \{<\mathrm{x}, \mu_{\mathrm{A}'}>/\ \mathrm{x} \in \mathrm{X}\}, \qquad (2)$$

where , is the pertinence function of the Fuzzy Set . $\mu_{\mathrm{A}'}: \mathrm{X} \rightarrow [0,1]A'$

The Intuitionistic Fuzzy Set was introduced by Atanassov (1983), a generalization of Zadeh's Fuzzy Sets, such that they denote, respectively, the degrees of pertinence and non-pertinence of an element, in a set.$\mu_A(x)\nu_A(x)x \in XA$

$$\mathrm{A} = \{<\mathrm{x}, \mu_{\mathrm{A}}(\mathrm{x}),\ \nu_{\mathrm{A}}(\mathrm{x})>/\ \mathrm{x} \in \mathrm{X}\}, \qquad (3)$$

where

$$\mu_{\mathrm{A}}: \mathrm{X} \rightarrow [0,1]$$
$$\nu_{\mathrm{A}}: \mathrm{X} \rightarrow [0,1],$$

such that:

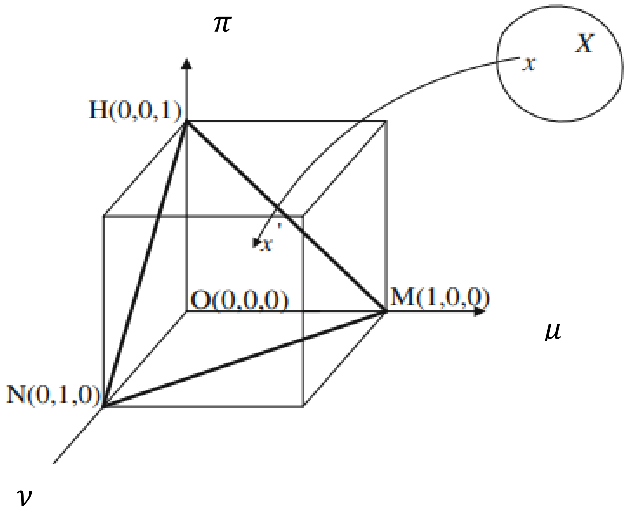
$$0 \leq \mu_{\mathrm{A}}(\mathrm{x}) + \nu_{\mathrm{A}}(\mathrm{x}) \leq 1, \forall \mathrm{x} \in \mathrm{X} \qquad (4)$$

The margin of hesitation in a Fuzzy Intuitionist Set, $\pi_{\mathrm{A}}(\mathrm{x})$, defines the lack of information about the degree of belonging of the element in the set, according to the expression:xA

$$\pi_{\mathrm{A}}(\mathrm{x}) = 1 - (\mu_{\mathrm{A}}(\mathrm{x}) + \nu_{\mathrm{A}}(\mathrm{x})), \text{ where} \qquad (5)$$
$$0 \leq \pi_{\mathrm{A}}(\mathrm{x}) \leq 1, \text{ for everything.} x$$

Figure 1 geometrically represents the sets, *crisp*, Fuzzy and Fuzzy Intuitionist (SZMIDT; KACPRZYK, 2000). The triangle MNH represents the surface for the coordinates of the Intuitionistic Fuzzy set, where the points M and N indicate the (μ, ν, π)*Crisp set*, where M and N are the maximum values of pertinence and non-pertinence, respectively. Point H registers the maximum hesitation and represents the complete inability to determine the pertinence or non-pertinence of a given element. The MN segment refers to the Fuzzy Set, where there is zero hesitation ().$\pi = 0$

Figure 1: Geometric representation of the Intuitionistic Fuzzy Model.



Fonte: Distances between intuitionistic fuzzy sets, p.14.

ACTIVATION FUNCTION

The pre-processing of the dataset for the fuzzification process is performed through a vector approach, $V_i = (n_{i,1}, ,n_{i,2} n_{i,3}, \dots, n_{i,h})$ in which it denotes the jth component of this data vector. The Fuzzy Intuitionistic conceptual representation of a data set for the vector would be defined as , normalized the result of the expression below:$n_{i,j}$ $hV_i A_i = \{< x_1, \mu_1(x), \nu_1(x) >, \dots, <$ $x_h, \mu_h(x), \nu_h(x) >\}.\, n_{i,1}$

$$z_{i,j} = \frac{n_{i,j} - X_j}{s_j}, \qquad (7)$$

where and , are the mean and standard deviation of the sample, respectively.$X_j s_j$

Hesitation was fixed, the pertinence and non-pertinence functions adopted for this project were the weighted sigmoid functions ($\pi = 0{,}1$INTARAPAIBOON, 2016), given by the following expressions:

$$\begin{cases} \mu_{i,j} = \dfrac{r_j}{1 + e^{-z_{i,j}}} \\ \nu_{i,j} = \dfrac{r_j}{1 + e^{z_{i,j}}} \end{cases} \qquad (8)$$

Where, $r_j = 1 - \pi, \ \forall j \in [1, k]$. Thus, the following were adopted:

$$\begin{cases} \mu_{i,j} = \dfrac{0{,}9}{1 + e^{-z_{i,j}}} \\ \nu_{i,j} = \dfrac{0{,}9}{1 + e^{z_{i,j}}} \end{cases} \qquad (9)$$

SIMILARITIES

The problem of quantitative properties is best addressed by the *metric conception* of similarity, which postulates that there are certain dimensions of similarity in relation to different aspects, for example, color, shape, or weight, which constitute the axes of a unified metric-space (BLUMSON, 2018). Similarity is a measure widely applied in fields where uncertainty must be considered. Within the scope of Fuzzy Logic, similarity is a measure of equivalence between Fuzzy Sets. Considering two Fuzzy Intuitionistic sets A and B, in a universe of discourse X, in this work several measures of similar measures were obtained, which will be listed below:

Cosine (C_{IFS}):

According to Salton and McGill (1983), the cosine similarity model follows the following representation:

$$C_{IFS}(A,B) = \frac{1}{n}\sum_{i=1}^{n}\frac{\mu_A(x_i)\mu_B(x_i) + \nu_A(x_i)\nu_B(x_i)}{\sqrt{\mu_A{}^2(x_i) + \mu_B{}^2(x_i)}.\sqrt{\nu_A{}^2(x_i) + \nu_B{}^2(x_i)}} \tag{10}$$

Weighted cosine (W_{IFS}):

In this similarity proposed by Li and Cheng (2002), a weighting is used for each belonging to the Fuzzy Intuitionistic sets A and B, where:w_i x_i

$$\sum_{i=1}^{n} w_i = 1 \tag{11}$$

Like this:

$$W_{IFS}(A,B) = \sum_{i=1}^{n} w_i \,.\, \frac{\mu_A(x_i)\mu_B(x_i) + \nu_A(x_i)\nu_B(x_i)}{\sqrt{\mu_A^2(x_i) + \mu_B^2(x_i)}.\sqrt{\nu_A^{2(x_i)} + \nu_B^2(x_i)}} \tag{12}$$

P. Shi e Z. Liang (S_c):

The expression suggested by Shi and Liang (2003) is represented below:

$$S_C(A,B) = 1 - \frac{\sum_{i=1}^{n}|S_A(x_i) - S_B(x_i)|}{2n}, \tag{13}$$

where: $S_A(x_i) = \mu_A(x_i) - \nu_A(x_i)$ and ;$S_B(x_i) = \mu_B(x_i) - \nu_B(x_i)$

H. B. Mitchell (S_H):

Mitchel (2003) proposes the following expression for the calculation of similarity:

$$S_H(A,B) = 1 - \frac{\sum_{i=1}^{n} |\mu_A(x_i) - \mu_B(x_i)| + |\nu_A(x_i) - \nu_B(x_i)|}{2n} \qquad (14)$$

E. Szmidt e J. Kacprzyk (S_O):

Szmidt and Kacpryzic (2005) propose the calculation of the following expression for a similarity measure:

$$S_O(A,B) = 1 - \sqrt{\frac{\sum_{i=1}^{n} (\mu_A(x_i) - \mu_B(x_i))^2 + (\nu_A(x_i) - \nu_B(x_i))^2}{2n}} \qquad (15)$$

H.W. Liu (S_{HB}):

Liu (2005) suggests for the calculation of similarity:

$$S_{HB}(A,B) = \frac{\rho_\mu(A,B) - \rho_\nu(A,B)}{2}, \qquad (16)$$

where $\rho_\mu(A,B) =$ and $= 1 - \sqrt[p]{\frac{\sum_{i=1}^{n} |\mu_A(x_i) - \mu_B(x_i)|^p}{n}} \rho_\nu(A,B) 1 - \sqrt[p]{\frac{\sum_{i=1}^{n} |\nu_A(x_i) - \nu_B(x_i)|^p}{n}}$

W. L. Hung e M. S. Yang (S_e^p):

Hung and Yang (2007) present as a measure of similarity:

$$S_e^p(A,B) = 1 - \sqrt[p]{\frac{\sum_{i=1}^{n} (\phi_\mu(x_i) + \phi_\nu(x_i))^p}{n}}, \qquad (17)$$

Where = and = $\phi_\mu(x_i) |\mu_A(x_i) - \mu_B(x_i)|/2 \phi_\mu(x_i) |(1 - \nu_A(x_i))/2 - (1 - \nu_B(x_i))/2|$

L. A. Zadeh (S_{HY}^1):

Zadeh (1965) expresses the following expression for similarity:

$$S_{HY}^1(A,B) = 1 - d_H(A,B) \qquad (18)$$

T. Gerstenkorn e J. Manko (S_{HY}^2):

Gerstenkorn and Manko put forward the following proposition:

$$S_{HY}^2(A,B) = (e^{-d_H(A,B)} - e^{-1})/(1 - e^{-1}) \qquad (19)$$

I. K. Vlachos e G. D. Sergiadis (S_{HY}^3):

Vlachos and Sergiadis (2007) denote the following result for the calculation of similarity:

$$S_{HY}^3(A,B) = (1 - d_H(A,B))/(1 + d_H(A,B)) \qquad (20)$$

DEVELOPMENT

After obtaining results from the AV1, AV2 and AV3 evaluations for one of the eighth-grade classes of a Municipal Public Education Network, the grades of each student were standardized in units of the standard deviation (), for each of the 28 students ordered alphabetically. In this study, three curricular evaluations will be addressed, AV1, AV2 and AV3, which were submitted to the fuzzification process according to the Intuitionist Fuzzy modeling to obtain pertinences and non-pertinences using the "weighted sigmoid functions" (INTARAPAIBOON,x_i z_i 2016), with hesitation being fixed. $\pi = 0{,}1$

Subsequently, the similarity measures (S) mentioned above were calculated for these evaluations combined in pairs, S(AV1, AV2), S(AV1, AV3) and S(AV2, AV3). When it came to the Cosine Weighted Similarity, 6 different weights were used. A weighting based on the student's performance in each assessment, average weighting obtained through the arithmetic average of the weights assigned to the three assessments and the other weights, mean square, $(\mathrm{W_{IFS}})(\overline{w_{AV}})(\overline{w_{AV^2}})$ mean cubic $(\overline{w_{AV^3}})$, based on the arithmetic quadratic and cubic averages of the assessments, respectively. Simulations of the various similarities discussed above were performed, and in the specific case of the models proposed by Hung and Yang (2007) and Liu (2005), they were calculated for three different sensitivities, with p ranging from 1 to 3. The results of these similarities are shown in Table 1 and were compared with the six Cosine Weighted Similarities to evaluate the best weighting that translates the profile of performance in Mathematics in Elementary School.

Table 1: Comparison similarities between the AV1, AV2 and AV3 evaluations.

SIMILARITIES	AV1, AV2	AV1, AV3	AV2, AV3
Ponderada Cosseno (ponderador av1)W_{IFS} −	0,799	0,860	0,943
Ponderada Cosseno (ponderador av2)W_{IFS} −	0,918	0,911	0,949
Ponderada Cosseno (ponderador av3)W_{IFS} −	0,871	0,911	0,945
Cosine Weighted (W_{IFS} - weighter)$\overline{w_{AV}}$	0,863	0,894	0,946
Cosine Weighted (weighting)$W_{IFS} - \overline{w_{AV^2}}$	0,862	0,898	0,943
Cosine Weighted (W_{IFS} - weighter)$\overline{w_{AV^3}}$	0,862	0,902	0,941
Cosine (C_{IFS})	0,861	0,888	0,953
P. Shi e Z. Liang (S_c)	0,790	0,805	0,875
Similaridade H. B. Mitchell (S_H):	0,790	0,805	0,875
E. Szmidt e J. Kacprzyk (S_O)	0,718	0,750	0,844
H.W. Liu usando p=1(S_{HB})	0,790	0,805	0,875
H.W. Liu usando p=2(S_{HB})	0,718	0,750	0,844
H.W. Liu usando p=3(S_{HB})	0,665	0,712	0,822
W.L. Hung, M.S. Yang (usando p = 1S_e^p)	0,790	0,805	0,875
W.L. Hung, M.S. Yang (S_e^p) usando p = 2	0,718	0,750	0,844
W.L. Hung, M.S. Yang (usando p = 3S_e^p)	0,665	0,712	0,822
L.A. Zadeh (S_{HY}^1)	0,895	0,902	0,937
T. Gerstenkorn e J. Manko (S_{HY}^2)	0,842	0,853	0,904
I. K. Vlachs e G. D. Sergiadis (S_{HY}^3)	0,810	0,822	0,882

Table 1 shows the comparison of the similarities of the evaluations expressed in units of the standard deviation. The highest degree of similarity corresponds to the AV2 and AV3 evaluations, indicating that there is greater similarity for the written evaluations. On the other hand, in these measurements, it indicated that the lowest degree of equivalence was between the AV1 and AV2 evaluations, signaling a greater discrepancy than the previous comparison. It should be noted that AV1 has a quantitative evaluation, but with qualitative items. AV2 is based on an exam with questions inherent to the concepts of Mathematics. The difference between these two evaluative proposals influences the students' performance due to contextual diversity, a factor that probably influenced the results obtained. The AV3 evaluation, when compared with AV2 and AV1, showed that the values of the similarities can become differentiated according to the approach of the different authors, which influences the interpretation. In S_{HY}^1 Zadeh's model, the degree of similarity of AV1 in relation to AV2 and AV3 are similar. It is noteworthy that in the Hung and Yang (2007) model, the similarity when using p = 3, there is a more expressive discrepancy. It was observed that there was an identical measure for the similarities attributed to Shi and Liang (2003), Mitchell (2003) and Liu (2005), the latter using the exponent p = 1.

In this table, cosine-weighted similarities were treated according to weights. Initially, three weights were calculated, one for each respective evaluation, obtained by dividing the evaluation score expressed in unit of standard deviation, ordered in ascending order. The "unit order weight" was obtained by the ratio between the standardized score and the sum of the same, which resulted in the weighting for each student, whose sum was equal to 1, a condition previously expressed. The list of students for each evaluation was ordered alphabetically, with the respective weights of each evaluation, with these weights being added per student and this value divided by the sum of the weights of the three evaluations, which resulted in the average weighting for each student. Similar procedures were adopted, but standardized quadratic and cubic values were used, resulting in quadratic mean and cubic mean weights.

By using weightings inherent to cosine weighted similarities, greater sensitivity is obtained in the measurement of the knowledge acquired by the student. The unit weight, when applied to the AV3 evaluation, showed proximity in relation to the other similarities in the table, which indicated that this evaluation in relation to the others carried out was able to translate with more coherence the level of learning acquired over the period, about the contents taught. On the other hand, the unit weighting for evaluation AV1 resulted in indices with greater disagreement with the other similarities, that is, this test was not very representative about the acquisition of specific knowledge of Mathematics. The quadratic and cubic mean weights did not reflect great sensitivity in relation to the unit mean weight.

FINAL CONSIDERATIONS

The Fuzzy modeling proved to be an optimized tool for the analysis of school performance in Elementary School, providing important excerpts regarding the first trimester of an eighth-grade class in the municipal public network. Although schools have managers and pedagogues, there is no concern in the logical analysis of pedagogical processes in the scenario of Elementary Education, contributing to the compromise of the education chain, dragging on to High School and College.

The learning gaps inherent to the fundamental knowledge of Mathematics should not be neglected, because the extra-class evaluations, designated as events, have not contributed in a strong way to recover learning deficiencies, masking the learning deficit that accumulates, especially in the concepts of Mathematics that will be the foundations of disciplines of the Exact Sciences.

The proposed method that deals with cosine-weighted similarity using the unit weighting in AV3, provided to differentiate the adoption of the basic concepts of mathematics when the AV2 and AV1 assessments are applied, since AV2 does not make the full curricular assessment. Therefore, it is necessary to take a more refined look at the treatment present in these activities examined outside the classroom, so that they have greater affinity with the mathematical knowledge addressed within the classroom. In cosine weighted similarity there is a reinforcement when the unit weighting is used in which AV3 is highlighted as the option with the highest sensitivity for knowledge acquisition.

The weighting associated with the students' results in AV1 was the one that most differed from all the similarity measures performed. This fact indicates that this is, in itself, an assessment that does not reflect the reality of the students' knowledge in the period analyzed. Another significant fact was that the exam with the largest scope of contents covered, AV3, was the one through which the best weighting was obtained, that is, the one whose measurements found among the tests was the one that came closest to the other measurements made. No less important, its results for cosine similarity are close to those found with the use of an average weighting between the three evaluations, indicating that the written evaluation with the greatest magnitude of subjects addressed, better expresses the reality of the class in the period in question, better exposing its deficiencies. It is understood that AV3 translates with good reality, the performance of students in the assessments, and the relationship between the performances in the different tests for each student. Finally, it is suggested for future approaches, different types of weights, seeking the one that best approximates the other measurements, believing that the weighted cosine similarity provides, in addition to the measures of similarities themselves, depending on the weighting used, additional information about each fuzzy set in relation to the others.

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