Origami: A path to successful teaching and learning of geometric concepts in basic education

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ABSTRACT

The teaching of geometry requires encouraging the student to develop concentration processes. Attention, patience, creativity, among others, so that they learn to think and understand abstract concepts that can have meaning and usefulness within the educational process and daily life, analyzing the origami technique for the successful teaching and learning of geometric concepts.

Keywords: Basic education, Didactics, Geometry, Origami, Origami and origami, Abstract concepts.

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INTRODUCTION

This article is based on a documentary review of the origami technique in such a way that it facilitates the teaching-learning process of mathematics, especially geometry. This technique allows the didactic strengthening of teachers for the development of geometric contents, resulting in a pedagogical strategy that is beneficial in the teaching and learning process of students. The importance of the study is based on the intention of introducing techniques with a new vision to the didactics of geometry teaching, conceived as classroom action, thus contributing to reduce failure rates in the area.

BACKGROUND

Origami is the Spanish term for paper folding or paper folding, it is a synonym of Origami referring to the Japanese art of paper folding, which has become a technique incorporated in education in various areas, which facilitates the teaching and learning process in students.

Its history begins in China in the first or second century like paper, and reaches Japan in the sixth; It was used as an exclusive pastime for the upper classes, considering that only they could have access to paper due to its high cost, since it was considered a luxury item. By 1338, the Muromachi Period, paper was more accessible, so origami ornaments emerged to highlight social class.

However, in the year 1603, the Tokugawa Period, the democratization of origami took place, giving way to the emergence of a great culture; giving basis to the bird and the crane, which is the most popular figure in Japan.

At the beginning of the 20th century, the promoter of origami was Miguel de Unamuno y Jugo, who discovered it in an exhibition while visiting Paris for the inauguration of the Eiffel Tower, later creating his school of folders. (Prieto Bustamante, 2017). This has been the most important contribution to origami since the invention of paper, since it has allowed the international dissemination of the different creations.

Thus paper folding appears more frequently in the East, specifically,

> in Japan, a technique that is transmitted through generations within the Shinto religion; one of the oldest expressions developed in the seventeenth century were the symbols of fortune called Noshis and the poetics of Haiku, used in the teachings of Zen Buddhism and the Jöruri puppet theater made with paper folds (Fernández Arevalo, 2017).

Another aspect of interest, it is observed that the ceremonial and symbolic character of paper folding was lost over time and, reborn in 1878 in the Froebelian works, it is thus, that the activity of folding paper resurfaces with pedagogy and didactics, being Friedrich Fröebel who incorporates manual work into the educational system (Santillana Mujica, 2018). In this sense, the author highlights that:

> Folding paper, starting from a square, had the objective of the intuitive teaching of geometry, using its shapes to bring knowledge closer and generate a question and answer activity between the teacher and the student, thus developing the student's sense of observation and criticality (p.12).

Another important contribution to this is made by the pioneer of modern origami, the Japanese Akira Yoshizawa, to whom we owe the current symbology of the folding instructions of the models, which allowed the international dissemination of the different creations, regardless of the language in which these developments are written.

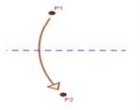
For Royo Prieto (2002) "origami creations, according to the laws of nature, require the use of geometry, science and physics. They also cover religion, philosophy, among others; the possibility of job creation is infinite" (p.45). Certainly, the author affirms, folding is a dialogue between the artist and the paper, a manual work, of spirit and creation, which must be carried out with a state of positive energy that radiates from the creativity and innovation of the being.

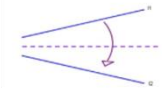
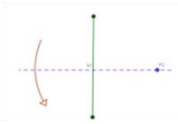
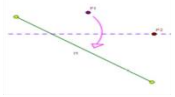
In the face of these useful approaches to origami, two currents have been established today, the non-scientific school, where philosophy consists of expressing the essence of what is to be represented with the minimum of folds, and the scientific school, where folding has been developed fundamentally by mathematicians, engineers and technicians. pursuing accuracy with a number of mathematical and algorithmic methods.

We have taken into account Humiaki Huzita's proposal as a reference to the six axioms and Koshiro Hatori's the seventh axiom of paper folding, as cited by Santa Jaramillo (2017).

Table 1 presents the axiomatic postulates that provide the theoretical basis for the use of the origami of paper folding as a didactic tool for the teaching of geometric content in basic education, allowing a dynamic learning of geometry, where concepts appear and reappear integrating manipulation, theory and art. as well as, facilitating the consolidation and stimulation of the highest levels of abstraction.

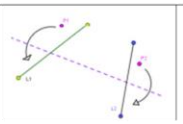
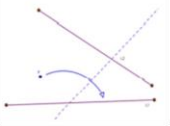
Cuadro 1
Definiciones de Axiomas de la papiroflexia

Axioma	Gráfico	Objeto matemático
Dados dos puntos P1 y P2, se puede realizar un pliegue que los conecte		Recta que pasa por dos puntos
Dados dos puntos P1 y P2, se puede realizar un pliegue que los conecte P1 sobre P2		Mediatriz del segmento P1 y P2

Dadas dos rectas l1 y l2, se puede plegar l1 sobre l2		Bisectriz del ángulo formado por las rectas l1 y l2
Dado un punto P y una recta l, se puede hacer un pliegue perpendicular a l que pase por P.		Recta perpendicular a otra que pasa por p. Segmento de longitud mínima que une un punto de l y p. Distancia de p a l
Dados dos puntos P1 y P2, y una recta l, podemos hacer un pliegue que haga corresponder a P1 con un punto de l y que pase por P2		Repitiéndolo, se obtiene la envolvente de una parábola. Es equivalente a encontrar la Intersección de una línea con un círculo, por lo que puede tener 0, 1 o 2 soluciones

Fuente. *Santa Jaramillo (2017)*

Cuadro 1
Definiciones de Axiomas de la papiroflexia (Cont...)

Axioma	Gráfico	Objeto matemático
Dados dos puntos P1 y P2, y dos rectas l1 y l2, se puede hacer un pliegue que haga corresponder a P1 con un punto de l1 y P2 con un punto de l2.		Permite resolver ecuaciones cúbicas (ecuaciones de tercer grado).
Dado un punto P y dos líneas l1 y l2, hay un pliegue que coloca p en l1 y perpendicular a l2.		Solución de una ecuación de segundo grado, por lo que puede tener dos soluciones reales distintas, dos soluciones reales iguales o no tener solución en los reales

Fuente. *Santa Jaramillo (2017)*

- Two different dots on the same sheet of paper.
- A point on a fold.
- In summary, the axioms take as their theoretical basis:
- Two folds of the same sheet.
- Two angles and their congruence if they overlap when they overlap.
- Two segments and their congruence if they overlap when they overlap.

Based on this, origami, in addition to creating its own rules, also provides education with an important tool to improve the capacities of concentration, memory, analysis and development of geometric concepts through the activation of logical-spatial thinking and the development of psychomotor skills.

Origami as a technique for teaching geometry is governed by rules, even if:

1. Vergara Soler (2017) explains that the choice of paper is the first rule to be followed. It must be perfectly cut out into a square and the size will depend on the dimensions that you want the figure to have. If the measurements are not adequate, the piece may go wrong and you may have to repeat steps that can leave noticeable marks on the paper.
2. The second rule mentioned by Vergara Soler (2017) is that you must work carefully and neatly, so it is necessary to work on smooth and stable surfaces. It is worth mentioning

washing your hands before forming a figure, cleaning the furniture to be used and making sure that it does not have unevenness in order to have exact folds.

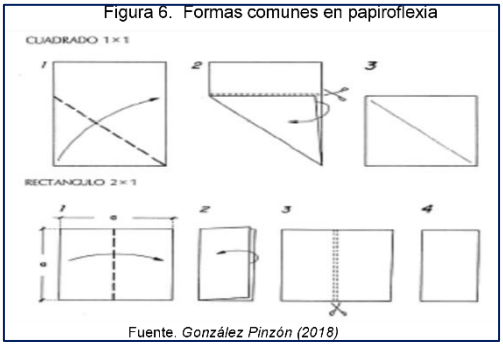
3. The third rule is that it is necessary to go over the folds and folds with the thumbnail, these will be better marked and thus the following steps are easier (González Pinzón, 2018).
4. At the same time, González Pinzón (2018) explains as a fourth rule that you must know the set of international symbols to make a move, because each step shown has an order that must be followed.
5. As a fifth rule, Vergara Soler (2017), explains that those who have never practiced the origami technique should start doing it with the base figures.

The size and shape of a paper to be used depends on the shape and size that is sought in the final result. As Corrado Núñez (2018) explains, "the paper for most origami must be square, but some models are made from rectangles (...). Others use paper in the shape of a triangle, rhombus; five-, six-, or eight-sided paper, and

> even round" (p.18). In any case, its measurement must be proportional, both in height and width.

Thus, González Pinzón (2018) indicates that "the three most common forms in origami are: the 1 x 1 square, the 2 x 1 rectangle, and the folio, which does not require an exact measurement" (p.19). Sometimes, the paper you have does not have the desired measurements, which is why the same author points out that "often, the first step is to fold the paper in order to cut it into the desired shape without the need to make measurements" (p.20).

Figura 6. Formas comunes en papiroflexia



Fuente. *González Pinzón (2018)*

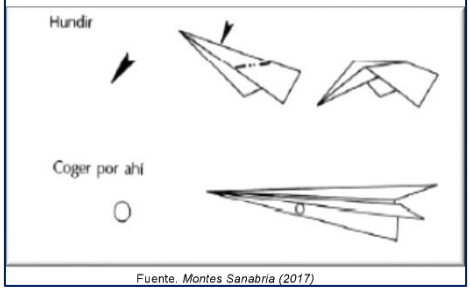
In this sense, to obtain an adequate size, it is necessary to fold the paper and thus cut it. For his part, González Pinzón (2018, p.29) mentions relevant aspects of the role that:

- The size of the paper is not important, unless it is so small or large that it cannot be handled. In most folds, a piece between fifteen and thirty centimeters wide can be used.

- The paper used in origami must be resistant, thin and of the right color. But if you can't get a special paper, the truth is that you can use just about any other.
- The type of paper is not a limitation for anyone who wants to create origami figures. It doesn't matter if it is a gift, office, transparent or a more specialized one, it only needs to be resistant to make several folds and have a passion for folding.

In short, regarding the paper, any type can be used, since each one will give character and its own personality to the piece. What should be taken into account is that some are more opaque and others that transmit light better. Similarly, consider the thickness, since for greater folds it is advisable to use a lower thickness than for less complex pieces.

Figura 9. Tipos de flechas o signos



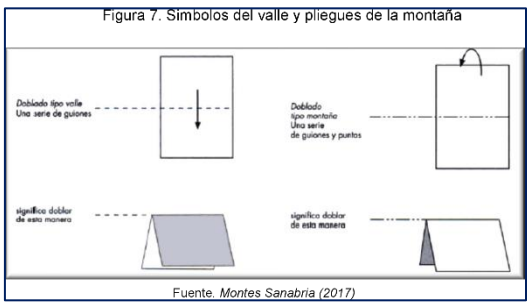
Fuente. *Montes Sanabria (2017)*

In origami, a common language has been created developed by Akira Yoshizawa, who in the sixties created a set of symbols that are currently used internationally. For his part, Montes Sanabria (2017) indicates that:

> The symbols necessary to learn are those of the valley and the folds of the mountain. The former are folded inwards and are indicated by dashes (---), the latter outwards and are indicated by a series of dashes and dots (-···-···-). It is important to take into account the directions to achieve them.

Figure 7 shows symbols of the valley and folds of the mountain.

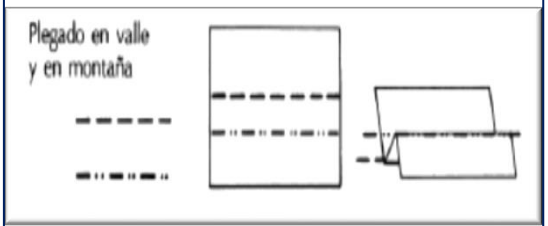
Figura 7. Símbolos del valle y pliegues de la montaña



Fuente. *Montes Sanabria (2017)*

When creating a figure, it is likely to be composed of these types of folds, so it is important to recognize when it is inward or outward, since figures can have both movements.

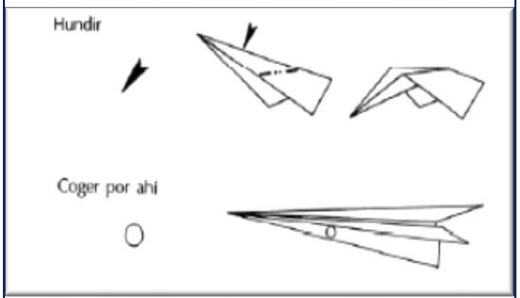
Figura 8. Plegado hacia adentro y afuera



Fuente. *Montes Sanabria (2017)*

Other types of arrows or signs according to Montes Sanabria (2017) "(...) indicates that it must sink, press, or squeeze at certain points."

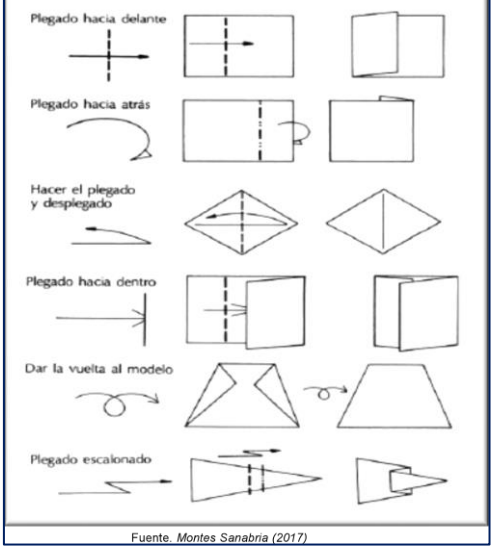
Figura 9. Tipos de flechas o signos



Fuente. *Montes Sanabria (2017)*

Montes Sanabria (2017) indicates that "the arrows show the direction in which we must turn: left, right, up, down, forward, backward and inside".

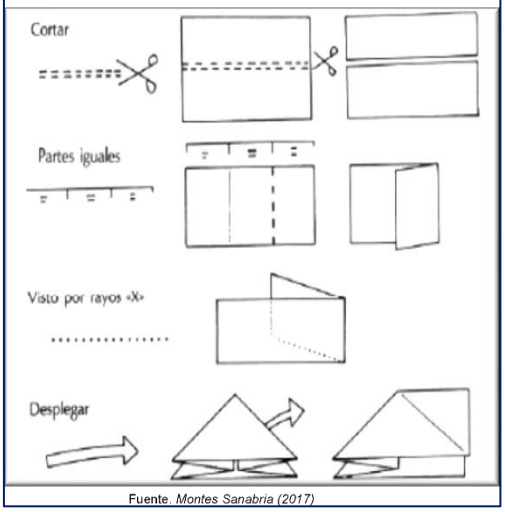
Figura 10. Dirección de los plegados



Fuente. *Montes Sanabria (2017)*

In addition to the direction or direction of the fold, there are also complicated processes that can only be performed if you know the origami symbols well. These can range from cutting, unfolding, folding to fold again, repeating a step, to seeing the result of the figure on both sides.

Figura 11. Otros procesos



Fuente. *Montes Sanabria (2017)*

It is understood that to make the figures you must always follow the instructions correctly and look at the drawings on the diagrams. This allows for an easier, more accurate and quality process.

Figura 12. Síntesis de Pliegues y líneas básicas

Tipo de línea.	*Proceso de plegado.*	*Explicación.*
Pliegue valle.		Consiste en doblar hacia delante, llevando un lado del papel sobre el otro.
Pliegue Monte.		Consiste en doblar hacia atrás, llevando un lado del papel sobre el otro.
Plegar y desplegar.		Esto en realidad no es un pliegue, son dos que se hacen uno tras otro. Consiste en doblar, bien sea en monte o en valle y acontinuación desdoblar. El resultado que queda es una marca.
Marca.		Las marcas son siempre el resultado de plegar y desplegar algo.
Rayos X.		Este tipo de línea, puede representar pliegues que se están haciendo en alguna capa de nuestro modelo que no podemos ver o bien marcarnos alguna línea del borde de la figura que está oculta.

Fuente. *Montes Sanabria (2017)*

The different types of origami are classified as follows:

Cuadro 2
Clasificación según finalidad, tipo de papel y cantidad de piezas utilizadas

Aspectos	Tipos y clasificación
Finalidad	Artístico: construcción de figuras de la naturaleza Educativo: construcción de figuras para el estudio de propiedades geométricas y desarrollar diferentes habilidades
Forma del papel	Tiras: se parte de una tira de papel Papel completo: se parte de un trozo de papel en forma cuadrangular, rectangular o triangular
Cantidad de trozos	Tradicional: un solo trozo de papel inicial Modular: varios trozos de papel inicial simples que se pliegan o superponen para formar unidades, generalmente iguales, que se unen para dar lugar a una figura compleja

Fuente. *Grados Pinto (2019)*

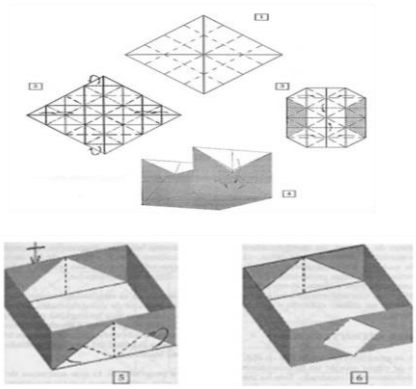
Cuadro 3
Clasificación según tipos

Tipos	Características
Papiroflexia de acción	Estáticas: figuras inmóviles Móviles: figuras con las que se puede emular movimientos de objetos/animales.
Papiroflexia modular	Se crean figuras más complejas colocando varias piezas iguales juntas.
Papiroflexia con plegado húmedo	Se humedece el papel de figuras ya creadas para moldearlo a la forma deseada. Con esto se consigue mayor realismo a las figuras
Papiroflexia pura	Únicamente se realiza mediante pliegues
Papiroflexia teselada	Composición de figuras que se repiten para cubrir totalmente una superficie.

Fuente. *Vásquez Fuerte (2018)*

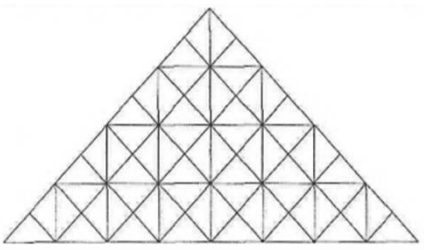
The origami technique is part of a whole arsenal of functional and economic educational aids that a teacher can incorporate into the work within a classroom at any of the school levels, it only has one limitation: the imagination or creativity of those who use it. Demonstrations and applications developed through the origami technique are presented, illustrated by González Pinzón (2018):

Figura 13. Básico. Caja rectangular básica



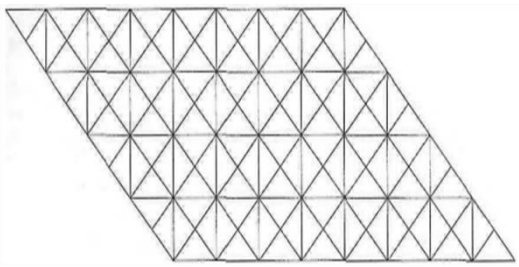
Fuente. *González Pinzón (2018)*

Figura 14. Triangulo isosceles igualmente rectangulo isosceles con la misma área cuadrarda



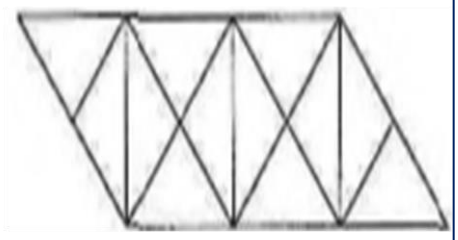
Fuente. *González Pinzón (2018)*

Figura 15. Construir un paralelogramo de idéntica área al cuadrado original



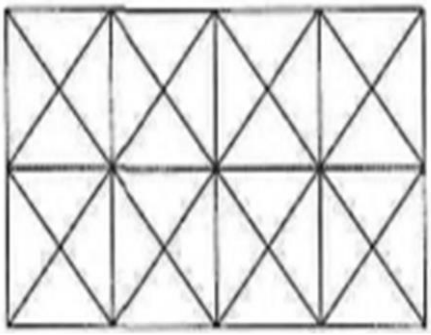
Fuente. *González Pinzón (2018)*

Figura 16. Paralelogramo con dobleces de la figura anterior



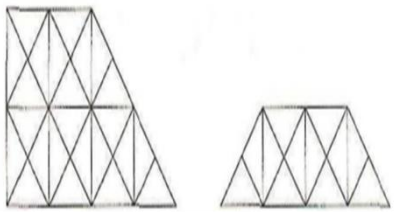
Fuente. *González Pinzón (2018)*

Figura 17. Boceteo de un rectángulo



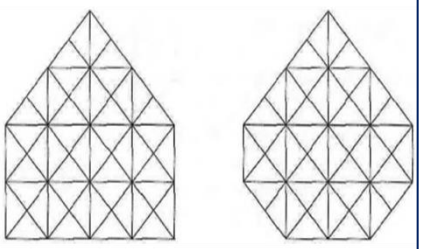
Fuente. *González Pinzón (2018)*

Figura 18. Trapecio rectángulo e isósceles



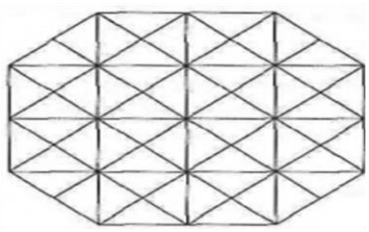
Fuente. *González Pinzón (2018)*

Figura 19. Pentágono y heptágono



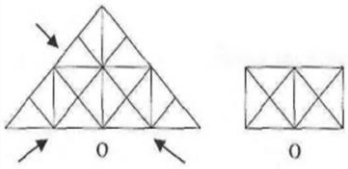
Fuente. *González Pinzón (2018)*

Figura 20. Octógono con sus lados iguales cuatro a cuatro



Fuente. *González Pinzón (2018)*

Figura 21. Suma ángulos interiores de un triangulo



Fuente. *González Pinzón (2018)*

It is shown that the sum of the interior angles of any triangle is 180° To do this, the square is divided into two triangles again. If you bring the three vertices to the common point 0, the three angles are in a straight line forming a flat angle or 180°.

As a synthesis, it is considered that the technique of origami or origami, is understood as the art of making paper figures, it is an ancient activity that endures today as an educational instrument. Its qualities are positive, as it favors concentration and attention.

As a technique used for the teaching of geometry, from a didactic point of view, the folding process involves and motivates the student, and favors the development of skills such as precision and spatial vision, being able to have an objective perspective to manipulate it and study the geometric properties in situ.

The technique contains seven defined axioms, each of which is associated with the resolution of mathematical objects that are configured in the teaching of geometry. By labeling an origami structure with length, width, and height, students will learn key terms and ways to describe a shape, and can use it to determine the area by applying a formula to a real-world structure.

Therefore, in some ways, it is an innovative resource to complement instruction in geometric construction, determine geometric and algebraic formulas and increase manual dexterity. In addition, it is a great way to be able to merge different areas such as science, technology, engineering, art, and mathematics.

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