The classification of distances and groupings of various musical elements through three methods of evaluation of similarities developed in cognitive psychology: Contrast Model, Structural Alignment and Transformation

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ABSTRACT

The study aims to use three methods of evaluation of similarities developed in cognitive psychology in the classification of distances and groupings of diatonic chords in major and minor keys, taking into account the four tonal scales (Ionian, Aeolian, Harmonic Minor and Melodic Minor), of the twenty-four tonalities, also taking into account the four tonal scales, of the forty-eight series of Alban Berg's Violin Concerto and of some sets representing scales in music theory. The methods employed are: the Contrast Model, which weights the common and distinct characteristics between the objects compared (Tversky 1977); the Structural Alignment, which classifies similarities by distinguishing between two types of differences and two types of commonalities (Gentner; Markman 1995, 1997; Goldstone 1994; Markman; Gentner 1990, 1993a, 1993b, 1993c, 1996); and Transformation, which evaluates similarities through the number of operations required to transform one object into another (Chater; Hahn 1997; Hahn; Chater; Richardson 2003; Hahn; Richardson; Chater 2001).

Keywords: Similarities, Distances, Groupings.

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INTRODUCTION

One of the important questions in psychology is to investigate how people judge similarities in order to try to understand the functioning of various cognitive processes, such as categorization. According to the similarity view, when new items appear, their characteristics are compared with the already stored characteristics of other items or of a representative prototype of the category (e.g., Goldstone 1994b; Hampton 1995; Nosofsky 1986; Reed 1972; Rosch 1975; Rosch; Mervis 1975; Smith; Medin 1981). In choice situations, decisions can be made based on similarities with previous situations (e.g., Kahneman; Tversky 1984; Lindemann; Markman 1996; Markman; Medin 1995; Medin; Goldstone; Markman 1995; Slovic; Macphillamy 1974; Smith; Osherson 1989; Tversky 1972). In problem-solving situations, new problems can be solved using procedures used in previous problems (e.g., Bassok 1990; Gick; Holyoak 1980; Novick 1990; Ross 1987, 1989). Three methods developed in cognitive psychology that attempt to predict and explain how people make judgments of similarities are the Contrast Model, Structural Alignment, and Transformation.

Psychologists who have formulated methods to try to understand how people make judgments of similarities point out that these theories must be tested in various domains of human knowledge in order to evaluate their effectiveness in terms of what they propose. Hahn, Chater, and Richardson (2003, p.26), for example, "recommend the transformational approach for further study and they suggest that the development of more detailed transformational models of similarity might be worthwhile." Later, Hahn, Chater and Richardson (2003, p.28) comment that "a general theory of similarity cannot restrict itself to demonstrations in any single context. Consequently, future research must also seek to apply the transformational approach to different domains." For Hodgetts, Hahn and Chater (2009, p.76), "to be useful as a cognitive account, a transformational account such as RD will need to be validated in a wide range of domains.". [3] For Hahn, Richardson and Chater (2001, p.398), "another potentially interesting area is to apply the approach to different domains, particularly those that appear to require structured representations where RD can be utilized in a straightforward way.". According to Markman and Gentner (1993c, p.464), "the mechanism that determines psychological similarity is a natural and seemingly effortless process that can operate across a wide range of stimulus types.". A similar point of view can also be found in Gentner and Markman (1994, p.157). Thus, this article proposes to apply the three methods of evaluation of similarities developed in cognitive psychology in the classifications of distances and groupings of diatonic chords in both major and minor keys, taking into account the four tonal scales, of the twenty-four major and minor keys, also taking into account the four tonal scales, of the forty-eight series of Alban Berg's Violin Concerto and of some sets representing scales.

[3] RD is short for *Representational Distortion*, which is one of the names given in English to the transformation model.

METHODS EMPLOYED

Tversky (1977) proposed that similarities between objects should be classified according to a principle of weighting their common and distinct characteristics that he called **the Contrast Model**. The Contrast Model is based on a three-argument equation that measures the similarity between objects A and B, expressed by S(A, B), as follows:

$$S(A, B) = \theta f(A \cap B) - \alpha f(A\text{-}B) - \beta f(B\text{-}A)$$

(A∩B) means the characteristics shared by A and B (see fig.1)

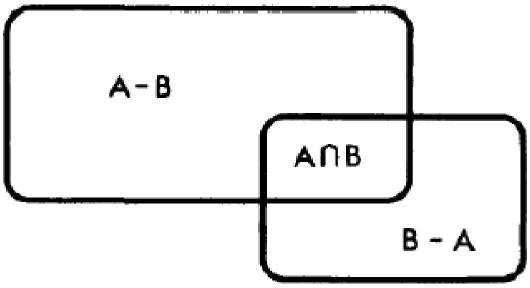
(A-B) means the characteristics of A that B does not possess

(B-A) means the characteristics of B that A does not possess

θ, α, β are parameters that represent their common and distinct characteristics

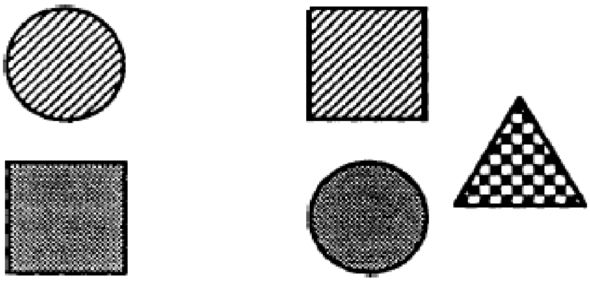
f is a measure of the salience of the features

Figure 1 (Tversky 1977, p. 330, fig.1): (A∩B) means the characteristics shared by A and B; (A-B) means the characteristics of A that B does not possess; (B-A) means the characteristics of B that A does not possess.



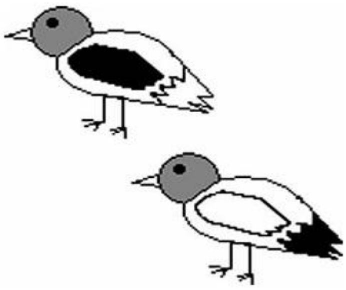
Structural alignment (Markman; Gentner 1990, 1993a, 1993b, 1993c, 1996; Gentner; Makman 1995, 1997; Goldstone 1994) classifies the similarities between objects by distinguishing between two types of differences called alignable and non-alignable differences (Markman; Gentner 1993a), and two types of commonalities called Match in *Place (MIP)* and Match out *Place (MOP)* (Goldstone 1994). Alignable differences are those attributes or characteristics that two objects have that are different, but that correspond through some relationship. Non-alignable differences are those attributes or characteristics that only one object has and the other does not, that is, that do not correspond relationally. For example, if the two settings (left and right) of fig.2 are matched based on the "above" relationship, the fact that there is a circle at the top of one image and a square at the top of the other is considered an alignable difference because they are different elements based on the same relationship. The same can be said about the "below" relationship between the bottom square and circle. On the other hand, the triangle on the right side, which doesn't correspond to anything on the left side, is an non-alignable difference because there is no corresponding element.

Figure 2 (Markman; Gentner 1993a, p. 683, fig.1): example of alignable and non-alignable differences.



Match in *Place (*MIP) is a feature shared by the compared objects that is in exactly the same place, i.e., as shown in fig.3, the gray color of the birds' heads is in the same place, i.e., in the heads, an MIP, while the black color of the wing and tail is an out-of-place match. i.e. a MOP (*Match out Place*).

Figura 3: (Hodgetts; Hahn; Chater 2009, p. 63, fig.1): exemplo de MIP e MOP.



According to Hahn, Richardson and Chater (2001, p.393), Chater and Hahn (1997) relied on a branch of mathematics called Kolmogorov's Complexity Theory to measure the transformations that evaluate psychological similarities (Li; Vitányi 1997). According to this theory, the complexity of a representation is measured by the length of the computer program that distorts one representation into another. The representations that can be generated by shorter programs are simple, and those that are generated by longer programs are complex (Chater 1999). Therefore, Kolmogorov's complexity theory serves as a measure of similarity. The degree to which two representations are similar is determined by how many instructions must be followed to transform one representation into another.

The **transformation method**, therefore, evaluates similarities through the number of operations required to transform one object into another. In the case of the two sequences XOOO and OXOO, to transform the first into the second, simply move the X one position to the right. In the case of the two sequences XOOO and OOXO, in order to transform the first into the second, in addition to moving the X one position to the right, it is necessary to reverse the order. Thus, the two sequences in the first example are more similar than the two sequences in the second example because it took only one operation to transform one into the other. The operations generally used in the transformation process are reversal, deletion, insertion, mirroring, and phase change (see Imai 1977).

ASYMMETRY IN SIMILARITY ASSESSMENTS: REFERENCE POINT VERSUS NON-REFERENCE POINT

According to Rosch (1975), some objects have a different psychological status in perception. These are called reference points and are prototypes against which the other members of the set are compared and evaluated. Rosch demonstrated in his 1975 paper *Cognitive Reference Points* that colors, horizontal, vertical, and diagonal lines, and numbers multiples of 10 can play the roles of reference points. Referential colors such as red were prototypes in relation to which the other colors when compared were less red or not red. Horizontal, vertical, and diagonal lines were prototypes against which other types of lines were compared. The multiples of 10 were prototypes against which the other numbers were also compared. These reference points are those that the non-reference points are seen 'in relation to'.

Empirical work has been carried out on reference points or prototypes in a variety of domains, including visual objects, colors, numbers, faces, and descriptions of personalities. These investigations have shown that cognitive reference points are given priority in processing, are more stable in memory, and have a special role in linguistic descriptions. The studies of Rosch and Mervis (1975), for example, demonstrated that the most prototype members of the categories "are those which bear the greatest family resemblance to other members of their own category and have the least overlap with other categories" (Rosch; Mervis 1975, p.598-599). His experiments "attempted to provide a structural principle for the formation of the prototypes" (Rosch; Mervis 1975, p. 600). According to the authors (Rosch; Mervis 1975, p.573), "six experiments explored the hypothesis that the members of categories which are considered most prototypical are those with most attributes in common with other members of the category and least attributes in common with other categories.". According to Krumhansl (1978, p.448), the differentiated status of these reference points is due to the fact that they are central or have a greater number of characteristics in common with the other members of the set.

Rosch (1975) also demonstrated the existence of **asymmetry** in judgments of similarities and distances between reference and non-reference points. She concluded in her empirical experiments that non-referential stimuli were judged to be more similar to referential stimuli than the other way around, i.e., the psychological distance between an element 'a' and an element 'b' was not the same as in the reverse order, i.e., between 'b' and 'a'. In the case of the number similarity experiments, subjects said that "103 is virtually 100" and not the other way around. In the experiment between referential and non-referential places, the distances were asymmetrical, varying according to which of the two stimuli was placed at the fixed point. She applied a task in which subjects placed a stimulus in physical space to indicate their psychological distance from a second stimulus fixed in a given position. She considered that the distance was shorter when the fixed position contained a referential stimulus than when it contained a non-referential stimulus. Thus, Rosch concluded that non-referential stimuli were judged to be more similar to referential stimuli than the other way around. In the sentence where 'a' is 'b', the prototype was always in the second position and there was asymmetry in the relationship between the reference and non-reference points, that is, the distance between 'a' and 'b' was different from the distance between 'b' and 'a'. For Tversky (1977), the asymmetry is explained by the salience of the objects:

> Similarity judgments can be regarded as extensions of similarity statements, that is, statements of the form "a is like b." Such a statement is directional; it has a subject, a, and a referent, b, and it is not equivalent in general to the converse similarity statement "b is like a." In fact, the choice of subject and referent depends, at least in part, on the relative salience of the objects. We tend to select the more salient stimulus, or the prototype, as a referent, and the less salient stimulus, or the variant, as a subject. We say "the portrait resembles the person" rather than "the person resembles the portrait." We say "the son resembles the father" rather than "the father resembles the son." We say "an ellipse is like a circle," not "a circle is like an ellipse," and we say "North Korea is like Red China" rather than "Red China is like North Korea." (Tversky 1977, p.328).[4]

For Tversky (1977, p.333), there is symmetry in similarity whenever the objects under consideration are equally salient, $f(A) = f(B)$, or the task is not directional, $\alpha = \beta$. Note examples of the two types of judgments given by Tversky (1977, p.333):

1) Assess the degree to which 'A' and 'B' are similar

2) Evaluate the degree to which 'A' is similar to 'B'.

In (1), the task is formulated in a non-directional manner, so the similarity between 'a' and 'b' is equal to the similarity between 'b' and 'a'. In (2), the task is directional, so the similarity between 'a' and 'b' may be different from the similarity between 'b' and 'a', i.e., asymmetric. If the similarity between 'a' and 'b' is interpreted as the degree to which 'a' is similar to 'b', then 'a' is the subject of the comparison and 'b' is the referent. In this task, the focus is naturally on the subject, so its characteristics are more weighted than those of the referent. Consequently, similarity is reduced more by the distinct characteristics of the subject than by the referent. As an example from Tversky and

[4] According to Tversky (1977, p.328-329), the directionality and asymmetry in the relations of similarities are particularly perceptible in similes and metaphors. We say that "the Turks fight like tigers" and not "the tigers fight like Turks", since it is the tiger that is famous for its fighting spirit, it that is used as a referent and not as a subject. The poet writes "my love is as deep as the ocean" and not "the ocean is as deep as my love", since it is the ocean that symbolizes depth. Sometimes both directions are used, but they convey different meanings. "A man is like a tree" means that man has roots. "A tree is like man means that the tree has a life history." Life is like a play" means that people have roles. 'A piece is like life' means that the piece can capture essential elements of life.

Gati (2004 [1978], p.81), a toy train is more similar to the real train because many features of the toy train are included in the real train. On the other hand, the real train is not as similar to the toy train because many of the features of the real train are not included in the toy train. Thus, according to Tversky's (1977, p.333) focus hypothesis, the asymmetry is determined by the salience of the stimulus or prototype, so that the less salient stimuli are more similar to the more salient stimuli and not vice versa, i.e., S(A, B) > S(B, A) whenever *f*(B) > *f*(A) (Tversky 1977, p.388). The similarity of (a, b) is greater than the similarity of (b, a) whenever the characteristics of 'b' are more salient than the characteristics of 'a' or whenever 'b' is more prominent than 'a', i.e., S(A, B) = S(B, A) if *f*(A – B) = *f*(B – A) or α = β (Tvesky; Gati 2004 [1978], p.81).

In Tversky and Gati (2004 [1978], p.82), the hypothesis of directional asymmetry, derived from the contrast model, was tested using semantic (countries) and perceptual (figures) stimuli. The two studies employed essentially the same design. Pairs of stimuli that differed in salience were used to test for the presence of asymmetry in the choice of similarity statements and in direct similarity assessments. According to Tversky and Gati (2004 [1978], p.85), studies using countries and figures revealed the presence of systematic and significant asymmetries in the judgments of similarities. The results support the theory based on the contrast model and the focus hypothesis, according to which the characteristics of the subject are more weighted than the characteristics of the referent. Subsequently, Sadalla, Burroughs and Staplin (1980, p.516 and 526) also found asymmetries in their experiments with spatial stimuli, that is, between referential and non-referential places: "since a reference point is considered as a place that defines the position of other adjacent places, it is concluded that other places should be easily seen 'in relation to' a reference point than vice versa" (Sadalla; Burroughs; Staplin 1980, p.517). That is, the adjacent places were judged to be closer to the reference places than the other way around. Asymmetric features were also found by Tversky and Hutchinson (1986, p.4), where they studied the relationships between nearest neighbors from one hundred datasets of similarities. It was assumed that 'i' was the nearest neighbor of 'j', but 'j' was not the nearest neighbor of 'i'. Similarly, a tonal context assigns to a note, chord, and to the central key itself, the status of reference point in relation to which the other notes, chords, and keys are viewed "in relation to". According to Krumhansl and Cuddy (2010, p.81):

> Despite the common principle of reference points, tonal hierarchies appear to be unique to music. Nothing analogous appears, for example, in language or in other perceptual domains. This raises the possibility that tonal hierarchies are especially important in music because most listeners (those without absolute pitch, the ability to name tones in isolation) process music relatively. In other words, musical tones do not have inherent qualities that are invariant across contexts. Instead, pitches are heard in context, and related to one another in that context. The tonal hierarchy provides a stable framework for establishing these relationships.[5]

[5] According to Krumhansl and Kessler (1982, p.363), "according to this view there are certain members of natural categories that function as cognitive reference points, or prototypes, for the category as a whole. These elements are described as the

Asymmetries were also found in an earlier study by Krumhansl (1979) where there were differences in the judgments of pairs of musical notes according to the order of presentation to the listeners and these differences were attributed to the hierarchy manifested by the tonal context. Two-note sequences that ended on a stable note were preferred by listeners over those that ended on a unstable note. These temporal effects were found when one of the test notes was a component of the tonic triad and the other was chromatic in relation to the established key and, consequently, less stable [6]. The asymmetry was lower when the notes occupied similar positions in the tonal hierarchy. Thus, it is possible to consider asymmetrical relationships in the distances between musical elements as well. A note, a chord, or a key 'a' may be closely related to a note, chord, or key 'b', but the reverse is not true, i.e., the distance between 'a' and 'b' is different from the distance between 'b' and 'a', depending on the reference or salient point which, in this case, is determined by the key. The central key determines the referential and non-referential points in comparisons of similarities and distances between any musical elements that occur within a tonal context.

According to Rosch (1975, p.546): "people in everyday life may well actually navigate through those distances as if they were asymmetrical. If use of reference points is a general cognitive strategy it should be applicable in many domains of human activity". In our case, to music. The studies by Krumhansl (1979), Krumhansl and Kessler (1982), Krumhansl *et al.* (1982) and Bharucha and Krumhansl (1983), for example, proposed "that stable notes and chords also function as cognitive reference points, and that the perception of melodic and harmonic events in relation to reference events is a crucial component of the perception of coherence in music" (Bharucha; Krumhansl 1983, p.93).

> We suggest that not only do cognitive reference points function similarly in music, but also they may be especially important there. This is because music does not provide fixed reference tones except as determined by the music itself. Thus, unlike other domains in which cognitive reference points are defined independently of the category (red is perceptually red whether it is or is not thought of in terms of the category of colors), the function of a tone depends entirely on the musical context. Another way to express this is that for most listeners relational processing (relative pitch) predominates over absolute pitch (with pitches having fixed labels independent of context). At a general level, the importance of musical reference points is not merely that they exist, but also that they guide musical perception, memory, thought, and understanding (Krumhansl; Cuddy 2010, p.53).

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most representative of the category, in relation to which all other category members are seen. Krumhansl (1979) noted the applicability of this description to the most structurally stable pitches in the tonal system in music."

[6] The test-note technique was initially developed by Krumhansl and Shepard (1979) to study the hierarchical relationships of similarity and distance between musical notes in perceptual experiments through listeners' evaluations. In this study, they established a major tonal context from the C ascending and descending C scale, and then presented one test note at a time of the chromatic scale for listeners to rate on a seven-point scale how well each one fit into the tonal context (1 = very badly to 7 = very well). The classifications were then interpreted as a measure of the hierarchy, similarity and distance between the test notes and the induced tonalities , obtaining the following classification: first, the notes of the tonic chord, then the other diatonic notes and, finally, the chromatic notes outside the scale. Subsequently, Krumhansl and Kessler (1982) expanded this study by applying the test note technique in various tonal contexts using scales, chords and cadences in major and minor keys. One of the main goals of the authors in this experiment was to obtain a measure of hierarchy, similarity, and distance between the notes of the chromatic scale in relation to the major and minor keys that could be used to classify the distances between the keys.

In the asymmetry view of the structural alignment method**,** in a statement "X is like Y", the Y domain is called the base and the X domain is called the target. Comparisons are made of the target against the base, so having the most systematic and coherent item as a base maximizes the amount of information that can be mapped onto the target. The most informative element is always the base.

In transformation theory, there is asymmetry when representations differ in complexity. The most complex and richest object is the frame of reference. Suppose a subject has reasonable knowledge about China but very little about Korea. Transforming the representation of China into the representation of Korea would require a reasonably short program (which simply deletes large amounts of information about China that is not relevant to Korea), while the program that does the opposite would be more complex, since the slightest information about Korea is of little use to construct the complex representation of China. Thus, transforming the representation of China into the representation of Korea must be more complex than transforming the representation of Korea into the representation of China, i.e., K(China/Korea) must be greater than K(Korea/China).

APPLICATION OF THE CONTRAST, STRUCTURAL ALIGNMENT, AND TRANSFORMATION MODEL TO DIATONIC CHORDS DERIVED FROM THE FOUR TONAL SCALES

The three methods of similarities developed in psychology are applied to diatonic chords in major and minor keys taking into account the four tonal scales: Ionian, Aeolian, Harmonic Minor and Melodic Minor. In this study, the major key is derived exclusively from the Ionian mode and the minor key is derived from the three minor scale forms. According to Forte (1974 [1962], p.12), "these alterations of scale degrees 6 and 7 are so common that we do not consider them as genuine chromatic alterations. They have been assimilated as part of the diatonic minor scale.". For Piston (1987, p.43), "in the period of common harmonic practice, music in the minor mode is rarely limited to one type of minor mode". According to Harrison (1994, p.18),

> Suffice it to say here that, while there are indeed three minor scales, we name pieces not according to these scales but according to the mode of the scales. More simply, we do not speak — as did a hapless former student of mine — of a "symphony in C harmonic minor" or of a "sonata in A melodic minor." The duality of major and minor duality is a higher-order phenomenon than is its manifestation in composition and elementary theory.

The same was said by Ian Guest (2006, vol.1, p.117), for him "it is not common to have music made only in harmonic minor or melodic minor, this classification only fits the scales" and Almir Chediak (1986, vol. 1, p.84) declared that tonality "is a system of sounds based on the major, harmonic minor, melodic minor and natural minor scales". According to Almada (2009, p.167), in the same way, "the minor key can be expressed in three different scales: natural, harmonic and melodic". Motte (1998, p.68) and Gauldin (2004, p.39) stated that the minor mode is a collection of nine notes to be used in any composition in a minor key, and for Roig-Francolí (2011, p.400), and Kostka and Payne (2009, p.59-60), all the notes of the three minor scales are diatonic in the minor key. David Temperley (2018, p.18-23) called the nine-note set derived from the three minor scales *supermode*. In "*Nineteenth-century Harmonic Theory*", Bernstein (2002, p.788-789) mentions that Simon Sechter (1788-1867) merges the three versions of the minor scale into one in his "*Die Grundsatze de Musikalischen Komposition*", from 1853. Later on, Bersntein (2002, p.792) mentions that Mayrberger also considered the three forms of the minor scale in only one. In the same article, Bernstein (2002, p.803) states that "Sechter [1853] and Schoenberg [1978] considered the minor mode in terms of its three forms: harmonic, melodic and natural minor.". Based on these arguments, this study evaluates similarities, distances and groupings between diatonic chords, taking into account the four tonal scales as representatives of major and minor keys.

All the diatonic tetrads of the major and minor keys were compared with their respective tonic tetrads (C7M and Am7). [7] Fig.4 shows the vectors of the contrast, structural alignment and transformation model resulting from the comparison of all diatonic tetrads with the C7M tonic. The black notes connected by black arrows indicate the notes in common, the red notes connected by red arrows indicate the different notes that are over the same interval degree (referring only to the degrees and not to the major, minor, major, and diminished quality), and the red notes within a square without arrows correspond to the different diatonic notes that do not find a match in the other chord.

In **the contrast model** (C), **C7M'**s relationship to itself has the largest vector (4 – 0 – 0 = 4) because all the notes are matched. In the comparison between **C7M and Dm7**, there is 1 note in common, 3 different notes of C7M in relation to Dm7 and 3 different notes of Dm7 in relation to C7M, resulting in the equation **1 - 3 - 3 = -5**. In the comparison between **C7M and Em7**, there are 3 notes in common, 1 different note of C7M in relation to Em7 and 1 different note of Em7 in relation to C7M, resulting in the equation **3 - 1 - 1 = 1**. In the comparison between **C7M and F7M**, there are 2 notes in common, 2 different notes of C7M in relation to F7M and 2 different notes of F7M in relation to C7M, resulting in the equation **2 - 2 - 2 = -2**. In the comparison between **C7M and G7**, there are 2 notes in common, 2 different notes of C7M in relation to G7 and 2 different notes of G7 in relation to C7M, resulting in the equation **2 - 2 - 2 = -2**. In the comparison between **C7M and Am7**, there are 3 notes in common, 1 different note of C7M in relation to Am7 and 1 different note of Am7 in relation to C7M, resulting in the equation **3 - 1 - 1 = 1**. In the comparison between **C7M**

[7] The notes, chords and keys in this article are encrypted using the letters of the alphabet. Notes are referred to as A, B, C, D, E F and G. Major chords and keys: A, B, C etc.; minor chords and keys: Am, Bm, Cm, etc. (7M) means that the chord has a major seventh, (7) means that the chord has a minor seventh, (#5) means that the chord is augmented, (ø) means that the chord is half-diminished and (°) means that the chord is diminished.

and Bø, there is 1 note in common, 3 different notes of C7M in relation to Bø and 3 different notes of Bø in relation to C7M, resulting in the equation **1 - 3 - 3 = -5**.

There are two considerations to be made regarding the method of **structural alignment**. One concerns the order of the four numbers within the vectors: matches in place (MIPS), matches out of place (MOPS), alignable differences, and non-alignable differences. The structural alignment, in fact, does not mention how the comparison vectors are constructed directly, so this choice was based on their indirect reports. The ordering MIPS, MOPS, alignable differences, and non-alignable differences within the four-number vector comes from a hierarchy where, obviously, commonalities come first because they are most important to similarities (Tversky 1977; Krumhansl 1978; Sjoberg 1972) and alignable differences are more important than non-alignable differences because they are more closely related to commonalities (Gentner; Markman 1994, 1997; Markman; Gentner 1993a, 1996, 2000). [8] Therefore, the ordering of the four numbers within the structural alignment vectors in this study is determined by the acronym **IOAN,** which represents, respectively, the sequence: MIPS, MOPS, Alignable Differences and Non-Alignable Differences. The other consideration concerns a constraint or axiom of the structural alignment method called one-to-one mapping (Gentner 1983, 1989). According to one-to-one mapping, each feature, attribute, relationship, or object in one domain or representation can only be matched to a maximum of one feature, attribute, relationship, or object of the other domain or representation. For example, if the top circle of the left setting of fig.2 is matched with the top square of the right setting because both are at the top of the image, the same top left circle cannot be matched with the bottom right circle because an element in one image can only be mapped to a single element of the other image. Many-to-one mapping occurs when two or more elements of a scene are matched to a single element of the other scene. Many-to-one mappings are inconsistent because, in many domains, a coherent interpretation of the relationship between two scenes cannot be formulated if one part of one scene has two competing correspondences in the other scene (Marr; Poggio 1979). In the case of comparisons between musical elements, the restriction of one-to-one mapping means that a note of one chord can only be matched with a single note of the other chord, a note of one scale can only be matched with a single note of the other scale, a note of one key can only be matched with a single note of the other key, and so on in the comparison between any musical elements.

[8] There are several articles that suggest that alignable differences are more important for similarities than non-alignable differences. For example, for Gentner and Markman (1997, p.50), "just as commonalities gain in importance when they are part of a matching system, so too do differences. That is, alignable differences are more salient than non-alignable differences. Intuitively, this focus on alignable differences makes sense, for it leads to a focus on those differences that are relevant to the common causal or goal structure that spans the situations." (see also Kahneman; Tversky 1984; Lindemann; Markman 1996; Markman; Medin 1995; Slovic; Macphillamy 1974).

In the **structural alignment method** (A), the relation of **C7M** to itself has the largest vector (4000) because all the notes are matched. In the comparison between **C7M and Dm7**, there is no match in place (MIP), there is 1 match out of place (MOP), 2 alignable differences, and 2 non-alignable differences, resulting in vector **0122**[9]. In the comparison between **C7M and Em7**, there is no match in place (MIP), there are 3 matches out of place (MOP), no alignable differences, and 2 non-alignable differences, resulting in vector **0302**. In the comparison between **C7M and F7M**, there is no match in place (MIP), there are 2 matches out of place (MOP), no alignable differences, and 4 non-alignable differences, resulting in vector **0204**. In the comparison between **C7M and G7**, there is no match in place (MIP), there are 2 matches out of place (MOP), no alignable differences, and 4 non-alignable differences, resulting in vector **0204**. In the comparison between **C7M and Am7**, there is no match in place (MIP), there are 3 matches out of place (MOP), no alignable differences, and 2 non-alignable differences, resulting in vector **0302**. In the comparison between **C7M and Bø**, there is no match in place (MIP), there is 1 match out of place (MOP), 2 alignable differences, and 2 non-alignable differences, resulting in vector **0122**.

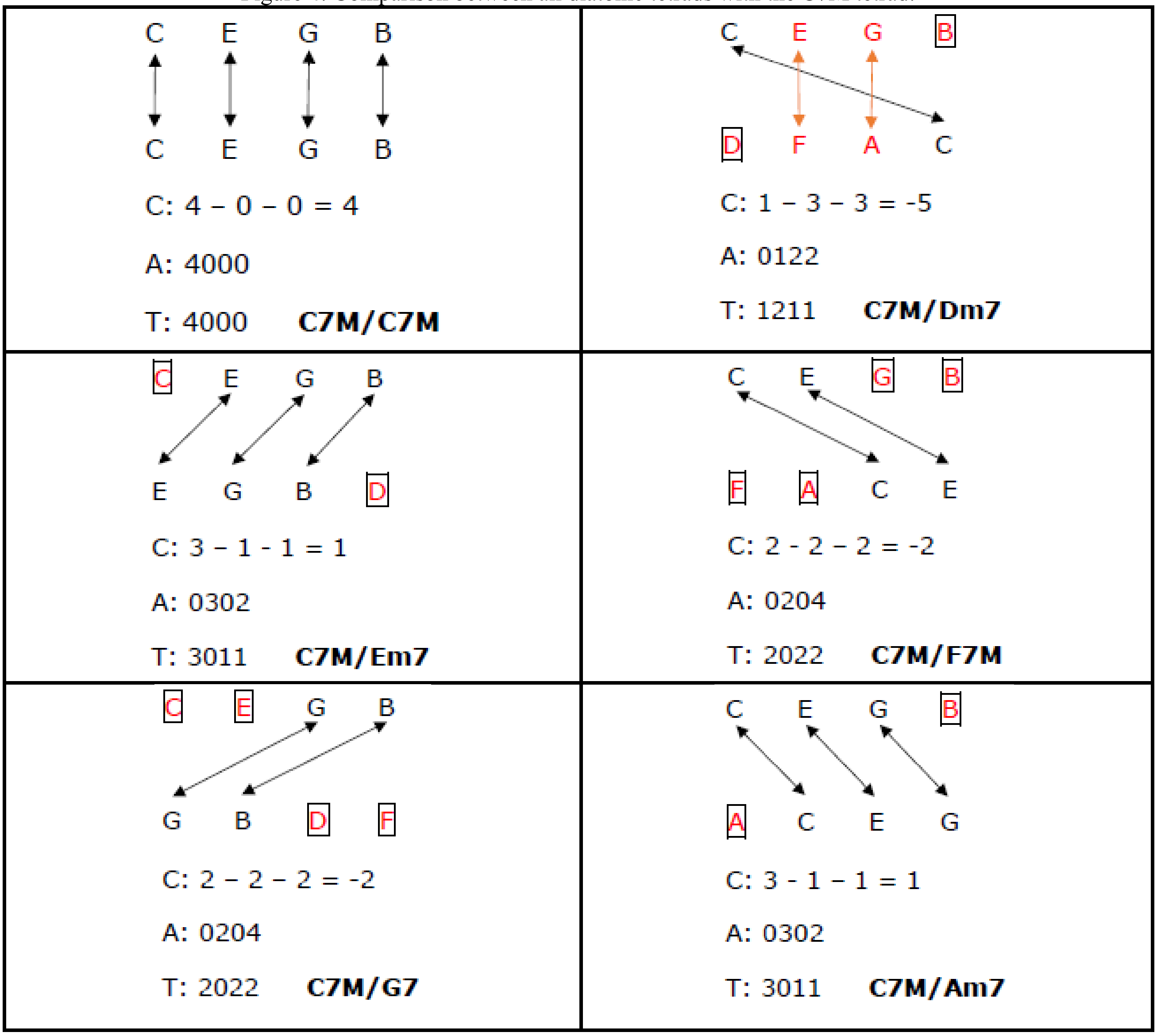
Similarly, the transformation method does not mention how its comparison vectors are constructed directly, so the order of the four-number vectors was also constructed by the authors of this study. In the **transformation** method, the parameter of commonalities comes first because it is the most important in the evaluation of similarities. Secondly, the transformations themselves appear because they are different notes, but they correspond because they are on the same interval degree. In third place is the subtraction parameter that is weighted before addition because, according to Hahn, Richardson and Chater (2001, p.398), when it comes to representations, deletions "tend to be less expensive than insertions, because deletions require only a sufficient specification to identify the component to be deleted, while insertions require a complete specification of the component to be added". Therefore, the ordering of the four numbers within the vectors in the transformation method in this article is determined by the acronym **CTSA,** which represents, respectively, the sequence: **Commonalities, Transformations, Subtractions, and Additions.**

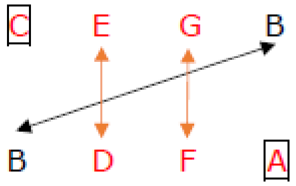
In the **transformation method** (T), the relation of **C7M** to itself has the largest vector (4000) because all the notes are corresponding (fig.4). In the comparison between **C7M and Dm7**, there is 1 note in common, 2 transformations, 1 subtraction, and 1 addition, resulting in the vector **1211**. In the comparison between **C7M and Em7**, there are 3 notes in common, no transformation, 1

[9] In-place and out-of-place correspondences here are considered in terms of interval degree, i.e., the placement of the note within the scale. This means that the note C, which is the fundamental note of the C chord, and the minor seventh of the Dm chord, is an out-of-place match because it is in another interval position (see comparison between C7M and Dm7 in fig.4).

subtraction, and 1 addition, resulting in vector **3011**. In the comparison between **C7M and F7M**, there are 2 notes in common, no transformation, 2 subtractions, and 2 additions, resulting in the vector **2022**. In the comparison between **C7M and G7**, there are 2 notes in common, no transformations, 2 subtractions, and 2 additions, resulting in the vector **2022**. In the comparison between **C7M and Am7**, there are 3 notes in common, no transformation, 1 subtraction, and 1 addition, resulting in the vector **3011**. In the comparison between **C7M and Bø**, there is 1 note in common, 2 transformations, 1 subtraction, and 1 addition, resulting in the vector **1211**.

Figure 4: Comparison between all diatonic tetrads with the C7M tetrad.





C: 1 – 3 - 3 = -5

A: 0122

T: 1211 **C7M/Bø**

Table 1 shows that all methods obtain the same classification order in the evaluations of similarities, distances and groupings of diatonic chords in relation to the C7M tonic. In the contrast, structural alignment, and transformation model, the larger the vector of the commonalities, the more similar and closer are the chords involved in the comparison. Note the descending order of the contrast model vectors: 4, 1, -2, and -5. The number of commonalities decreases, and so do the similarities. In the structural alignment method, the commonalities also decrease, as can be seen by the first two numbers of each vector: 40, 03, 02 and 01, which represent the commonalities in place and out of place, respectively. The other two numbers indicate differences that will be important in other similarity classifications. In the transformation method, the commonalities also decrease, as can be seen by the first number of each vector: 4, 3, 2 and 1, which represent the commonalities. The other three numbers represent the differences that will also be important in other similarity classifications. Therefore, Am7 and Em7 (level 2) are the chords most similar and closest to C7M with the highest vectors (**C**: 1 / **A**: 0302 / **T**: 3011, respectively). Next are the F7M and G7 chords that are equally distant from C7M at level 3 with intermediate vectors (**C**: -2 / **A**: 0204 / **T**: 2022). Dm7 and Bø (level 4) are the farthest chords and therefore most different from C7M with the smallest vectors (**C**: -5 / A: 0122 / **T**: 1211).

Table 1: Comparison between the three similarity models in relation to the C7M chord.

Level	Contrast	Vector	Alignment	Vector	Transformation	Vector
1	**C7M**	**4**	**C7M**	**4000**	**C7M**	**4000**
2	Am7 Em7	1	Am7 Em7	0302	Am7 Em7	3011
3	F7M G7	-2	F7M G7	0204	F7M G7	2022
4	Dm7 Bø7	-5	Dm7 Bø7	0122	Dm7 Bø7	1211

Fig.5 shows the vectors resulting from the contrast, structural alignment and transformation model in the comparison of all tetrads derived from the three minor scales with the stress Am7.

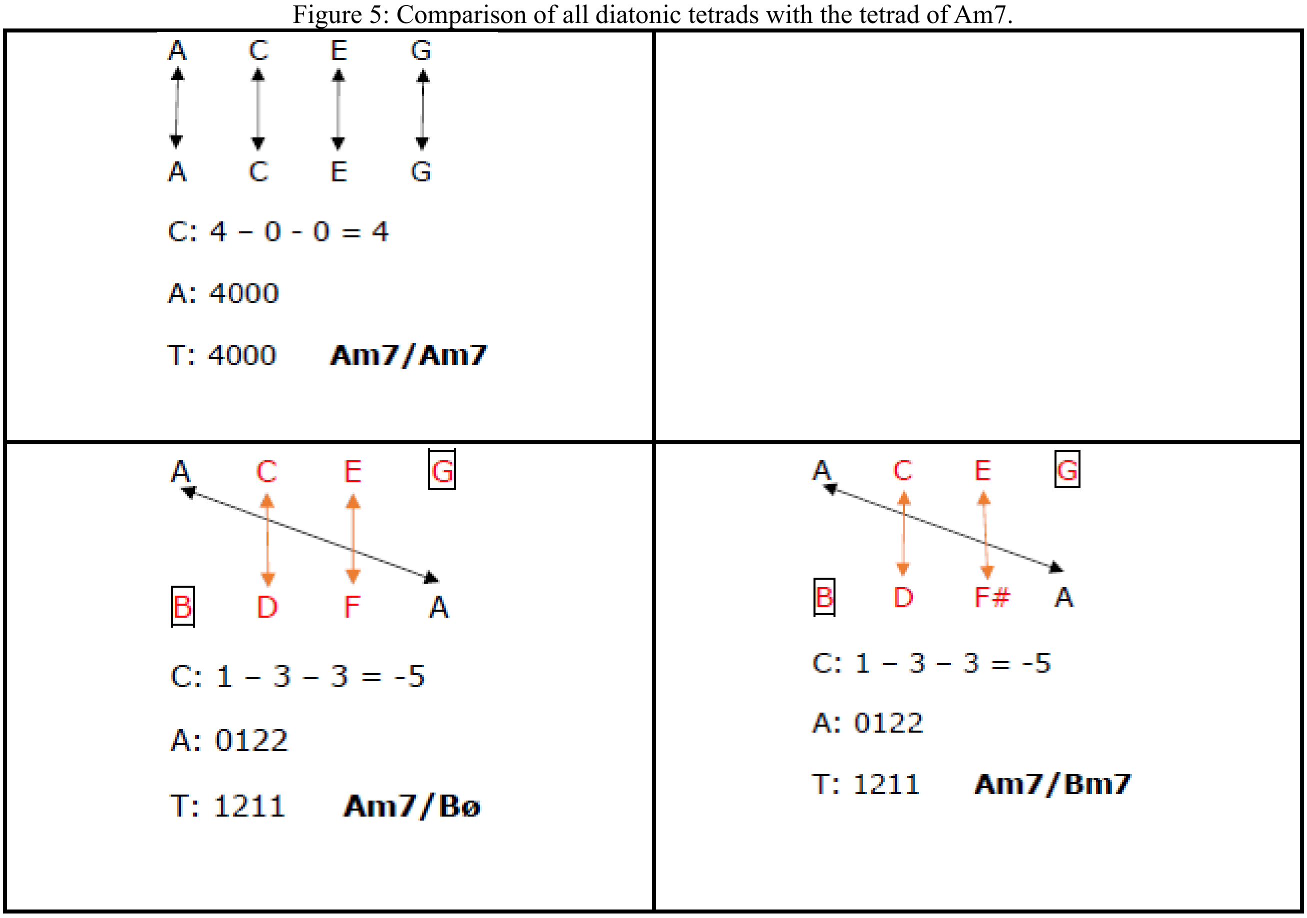
In **the contrast model** (C), the relation of **Am7** to itself has the largest vector **4 - 0 - 0 = 4** because all the notes are corresponding (fig.5). In the comparison between **Am7 and Bø**, there is 1 note in common, 3 different notes of Am7 in relation to Bø and 3 different notes of Bø in relation to

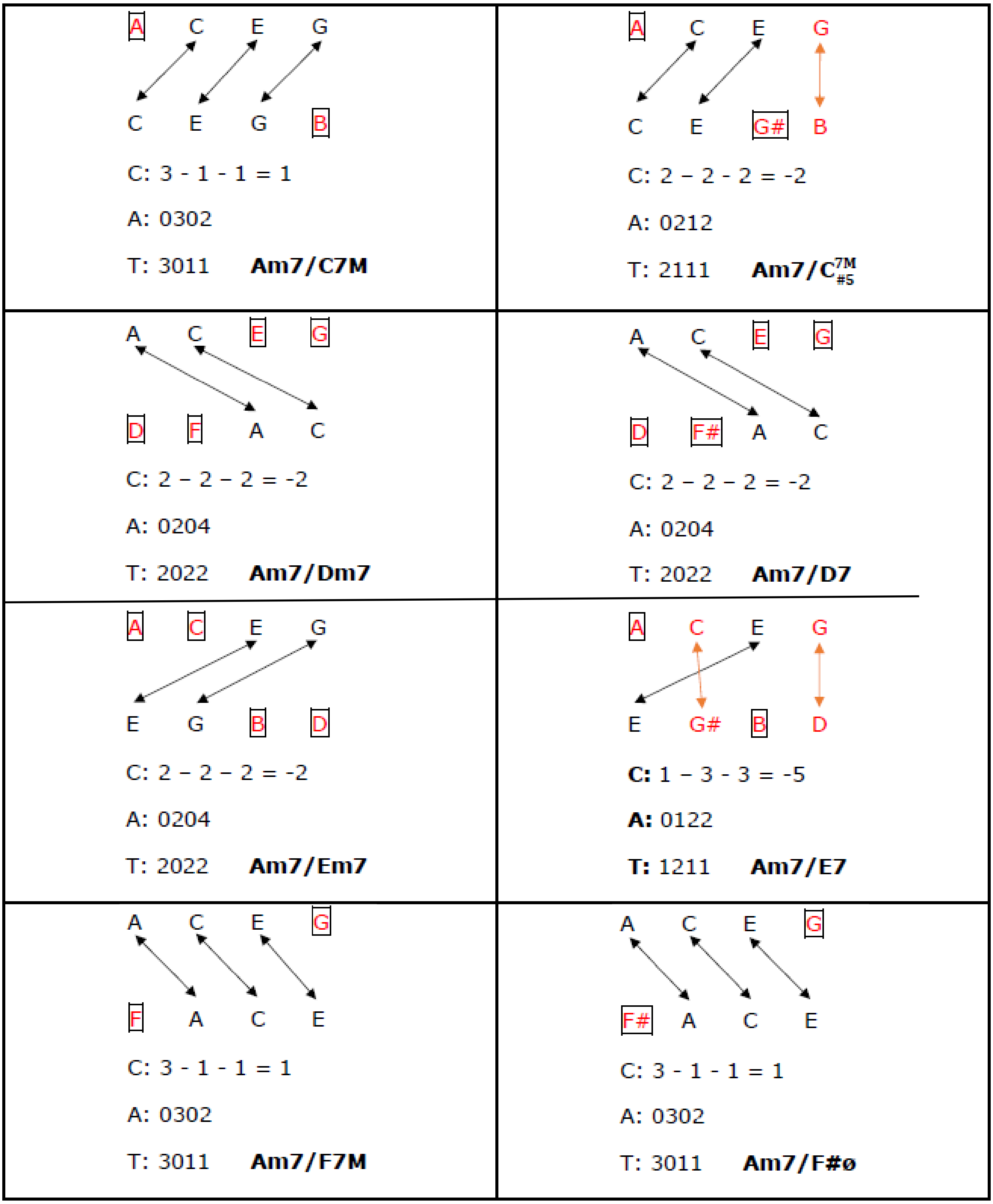
Am7, resulting in the equation **1 - 3 - 3 = -5**. In the comparison between **Am7 and Bm7**, there is 1 note in common, 3 different notes of Am7 in relation to Bm7 and 3 different notes of Bm7 in relation to Am7, resulting in the equation **1 - 3 - 3 = -5**. In the comparison between **Am7 and C7M**, there are 3 notes in common, 1 different note of Am7 in relation to C7M and 1 different note of C7M in relation to Am7, resulting in the equation **3 - 1 - 1 = 1**. In the comparison between **Am7 and $C^{7M}_{\#5}$**, there are 2 notes in common, 2 different notes of Am7 in relation to $C^{7M}_{\#5}$ and 2 different notes of $C^{7M}_{\#5}$ in relation to Am7, resulting in the equation **2 – 2 - 2 =-2**. In the comparison between **Am7 and Dm7**, there are 2 notes in common, 2 different notes of Am7 in relation to Dm7 and 2 different notes of Dm7 in relation to Am7, resulting in the equation **2 - 2 - 2 = -2**. In the comparison between **Am7 and D7**, there are 2 notes in common, 2 different notes of Am7 in relation to D7 and 2 different notes of D7 in relation to Am7, resulting in the equation **2 - 2 - 2 = -2**. In the comparison between **Am7 and Em7**, there are 2 notes in common, 2 different notes of Am7 in relation to Em7 and 2 different notes of Em7 in relation to Am7, resulting in the equation **2 - 2 - 2 = -2**. In the comparison between **Am7 and E7**, there is 1 note1 in common, 3 different notes of Am7 in relation to E7 and 3 different notes of E7 in relation to Am7, resulting in the equation **1 - 3 - 3 = -5**. In the comparison between **Am7 and F7M**, there are 3 notes in common, 1 different note of Am7 in relation to F7M and 1 different note of F7M in relation to Am7, resulting in the equation **3 - 1 - 1 = 1**. In the comparison between **Am7 and F#ø**, there are 3 notes in common, 1 different note of Am7 in relation to F#ø and 1 different note of F#ø in relation to Am7, resulting in the equation **3 - 1 - 1 = 1**. In the comparison between **Am7 and G7**, there is 1 note in common, 3 different notes of Am7 in relation to G7 and 3 different notes of G7 in relation to Am7, resulting in the equation **1 - 3 - 3 = -5**. In the comparison between **Am7 and G#ø**, there is no note in common, there are 4 different notes of Am7 in relation to G#ø and 4 different notes of G#ø in relation to Am7, resulting in the equation **0 - 4 - 4 = -8**. In the comparison between **Am7 and G#°**, there is no note in common, there are 4 different notes of Am7 in relation to G#° and 4 different notes of G#° in relation to Am7, resulting in the equation **0 - 4 - 4 = -8**.

In the **structural alignment method** (A), the relation of **Am7** to itself has the largest vector (4000) because all the notes are matched. In the comparison between **Am7 and Bø**, there is no match in place (MIP), there is 1 match out of place (MOP), 2 alignable differences, and 2 non-alignable differences, resulting in vector **0122**. In the comparison between **Am7 and Bm7**, there is no match in place (MIP), there is 1 match out of place (MOP), 2 alignable differences, and 2 non-alignable differences, resulting in vector **0122**. In the comparison between **Am7 and C7M**, there is no match in place (MIP), there are 3 matches out of place (MOP), no alignable differences, and 2 non-alignable differences, resulting in vector **0302**. In the comparison between **Am7 and** $\mathbf{C^{7M}_{\#5}}$, there is no match in place (MIP), there are 2 matches out of place (MOP), 1 alignable difference, and 2 non-alignable differences, resulting in vector **0212**. In the comparison between **Am7 and Dm7**, there is no match in place (MIP), there are 2 matches out of place (MOP), no alignable differences, and 4 non-alignable differences, resulting in vector **0204**. In the comparison between **Am7 and D7**, there is no match in place (MIP), there are 2 matches out of place (MOP), no alignable differences, and 4 non-alignable differences, resulting in vector **0204**. In the comparison between **Am7 and Em7**, there is no match in place (MIP), there are 2 matches out of place (MOP), no alignable differences, and 4 non-alignable differences, resulting in vector **0204**. In the comparison between **Am7 and E7** there is no match in place (MIP), there is 1 match out of place (MOP), 2 alignable differences and 2 non-alignable differences, resulting in vector **0122**. In the comparison between **Am7 and F7M**, there is no match in place (MIP), there are 3 matches out of place (MOP), no alignable differences, and 2 non-alignable differences, resulting in vector **0302**. In the comparison between **Am7 and F#ø**, there is no match in place (MIP), there are 3 matches out of place (MOP), no alignable differences, and 2 non-alignable differences, resulting in vector **0302**. In the comparison between **Am7 and G7**, there is no match in place (MIP), there is 1 match out of place (MOP), 2 alignable differences, and 2 non-alignable differences, resulting in vector **0122**. In the comparison between **Am7 and G#ø**, there is no match in place (MIP), no match out of place (MOP), there are 4 alignable differences and no non-alignable differences, resulting in vector **0040**. In the comparison between **Am7 and G#°**, there is no match in place (MIP), no match out of place (MOP), there are 4 alignable differences and no non-alignable differences, resulting in vector **0040**.

In the **transformation** method (T), the relation of **Am7** to itself has the largest vector (4000) because all the notes are matched. In the comparison between **Am7 and Bø**, there is 1 note in common, two transformations, 1 subtraction, and 1 addition, resulting in the vector **1211**. In the comparison between **Am7 and Bm7**, there is 1 note in common, two transformations, 1 subtraction, and 1 addition, resulting in the vector **1211**. In the comparison between **Am7 and C7M**, there are 3 notes in common, no transformation, 1 subtraction, and 1 addition, resulting in the vector **3011**. In the comparison between **Am7 and** $\mathbf{C^{7M}_{\#5}}$, there are 2 notes in common, 1 transformation, 1 subtraction, and 1 addition, resulting in the vector **2111**. In the comparison between **Am7 and Dm7**, there are 2 notes in common, no transformations, 2 subtractions, and 2 additions, resulting in the vector **2022**. In the comparison between **Am7 and D7**, there are 2 notes in common, no transformations, 2 subtractions, and 2 additions, resulting in the vector **2022**. In the comparison between **Am7 and Em7**, there are 2 notes in common, no transformations, 2 subtractions, and 2 additions, resulting in the vector **2022**. In the comparison between **Am7 and E7**, there is 1 note in common, 2 transformations, 1 subtraction, and 1 addition, resulting in the vector **1211**. In the comparison between **Am7 and F7M**, there are 3 notes in common, no transformation, 1 subtraction, and 1 addition, resulting in the vector **3011**. In the comparison between **Am7 and F#ø**, there are 3 notes in common, no transformation, 1 subtraction, and 1 addition, resulting in the vector **3011**. In the comparison between **Am7 and G7**, there is 1 note in common, 2 transformations, 1 subtraction, and 1 addition, resulting in the vector **1211**. In the comparison between **Am7 and G#ø**, there is no note in common, there are 4 transformations, no subtraction, and no addition, resulting in the vector **0400**. In the comparison between **Am7 and G#º**, there is no note in common, there are 4 transformations, no subtraction, and no additions, resulting in vector **0400**.

Figure 5: Comparison of all diatonic tetrads with the tetrad of Am7.




A C E G
C E G B
C: 3 - 1 - 1 = 1
A: 0302
T: 3011 Am7/C7M
A C E G
C E G# B
C: 2 – 2 - 2 = -2
A: 0212
T: 2111 Am7/C7M #5
A C E G
D F A C
C: 2 – 2 – 2 = -2
A: 0204
T: 2022 Am7/Dm7
A C E G
D F# A C
C: 2 – 2 – 2 = -2
A: 0204
T: 2022 Am7/D7
A C E G
E G B D
C: 2 – 2 – 2 = -2
A: 0204
T: 2022 Am7/Em7
A C E G
E G# B D
C: 1 – 3 - 3 = -5
A: 0122
T: 1211 Am7/E7
A C E G
F A C E
C: 3 - 1 - 1 = 1
A: 0302
T: 3011 Am7/F7M
A C E G
F# A C E
C: 3 - 1 - 1 = 1
A: 0302
T: 3011 Am7/F#ø

A C E G G B D F C: 1 – 3 – 3 = -5 A: 0122 T: 1211 **Am7/G7**	A C E G G# B D F# C: 0 - 4 – 4 = -8 A: 0040 T: 0400 **Am7/G#ø**
A C E G G# B D F C: 0 - 4 – 4 = -8 A: 0040 T: 0400 **Am7/G#°**	

It can be seen in Table 2 that although it is possible to organize the similarities, distances and groupings of all the diatonic tetrads of the Am tonality in the same order in the three methods, there is a difference in one of the intermediate levels. The chords that appear at level 3 of the contrast model are divided into two groups in structural alignment and transformation. The methods of structural alignment and transformation have the same classificatory result at all levels. It should be noted that the contrast model was taken as an example of classificatory results because its method of assessing similarities is quite clear: $S(A, B) = \theta f(A \cap B) - \alpha f(A\text{-}B) - \beta f(B\text{-}A)$, while the four-number vectors of the other two methods were created by the authors of this study.

For the classification of similarities, the number of commonalities is first evaluated and, if necessary, in case of a tie, the number of differences is evaluated. This occurs at levels 3 and 4 of the structural alignment and transformation methods. It is observed that the chords of level 3 of the contrast model are divided into two groups in the methods of structural alignment and transformation. The four chords that appear highlighted in levels 3 and 4 of the alignment and transformation have the same number of commonalities (02 and 2, respectively), so what differentiates them and classifies them as third and fourth place are the differences. $C^{7M}_{\#5}$ has 3 differences (3#) from the Am7 chord, and D7, Dm7 and Em7 have 4 differences (4#) from Am7.

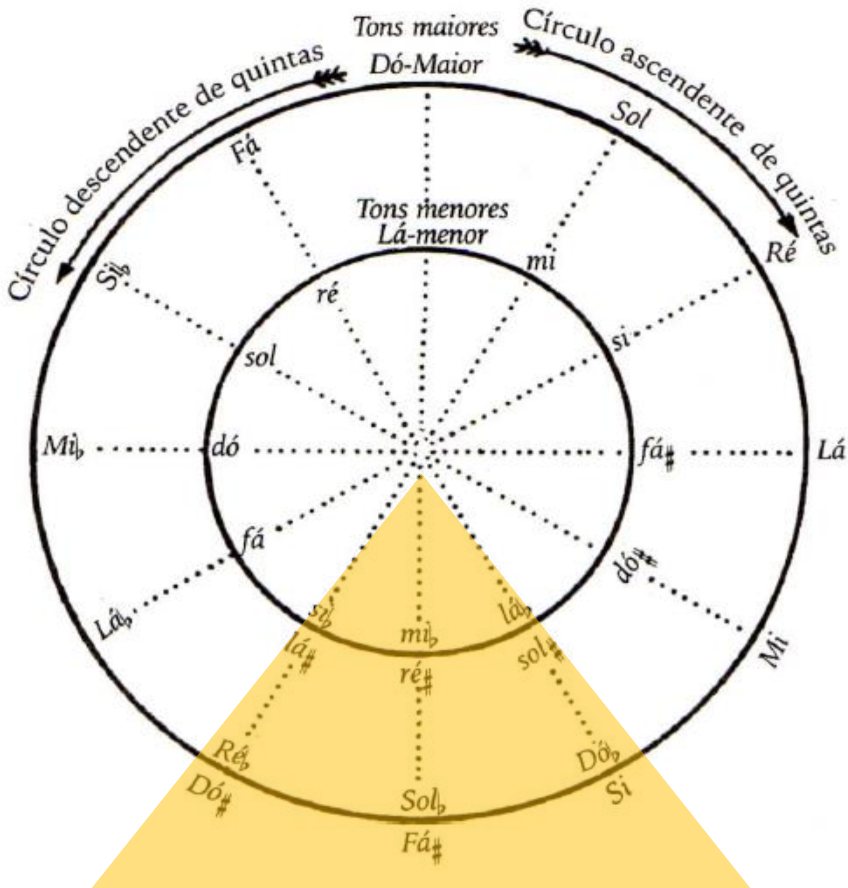
Table 2: comparison between the three similarity models in relation to the Am7 chord.

Level	Contrast	Vector	Level	Alignment	Vector	Level	Transformation	Vector
1	**Am7**	**4**	**1**	**Am7**	**4000**	**1**	**Am7**	**4000**
2	C7M F7M F#ø7	1	**2**	C7M F7M F#ø7	0302	**2**	C7M F7M F#ø7	3011
3	$C^{7M}_{\#5}$ D7 Dm7 Em7	-2	**3**	$C^{7M}_{\#5}$	0212 **(3#)**	**3**	$C^{7M}_{\#5}$	2111 **(3#)**
			4	D7 Dm7 Em7	0204 **(4#)**	**4**	D7 Dm7 Em7	2022 **(4#)**
4	E7 G7 Bm7 Bø7	-5	**5**	E7 G7 Bm7 Bø7	0122	**5**	E7 G7 Bm7 Bø7	1211
5	G#°7 G#ø7	-8	**6**	G#°7 G#ø7	0040	**6**	G#°7 G#ø7	0400

APPLICATION OF THE CONTRAST, STRUCTURAL ALIGNMENT, AND TRANSFORMATION MODEL TO THE MAJOR AND MINOR KEYS DERIVED FROM THE FOUR TONAL SCALES

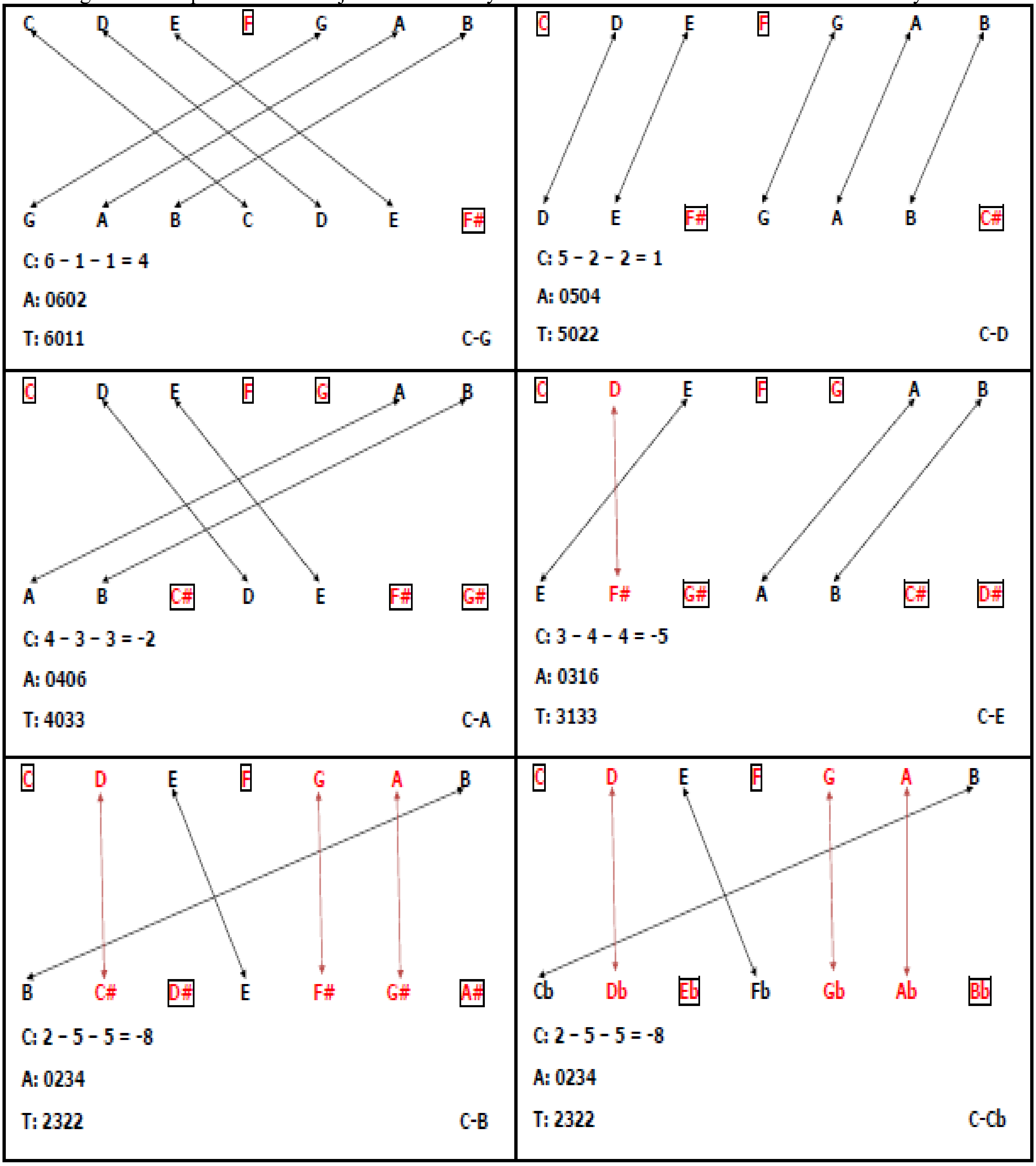
The distances and groupings of the Keys were obtained through the same methods of evaluation of similarities developed in cognitive psychology. The Keys involved in the comparison processes are those that appear in the circle of fifths, as explained by Schoenberg (2001, p.230). It can be seen that the only Keys that appear in an enharmonic way are those highlighted in a yellow triangle in fig.6.

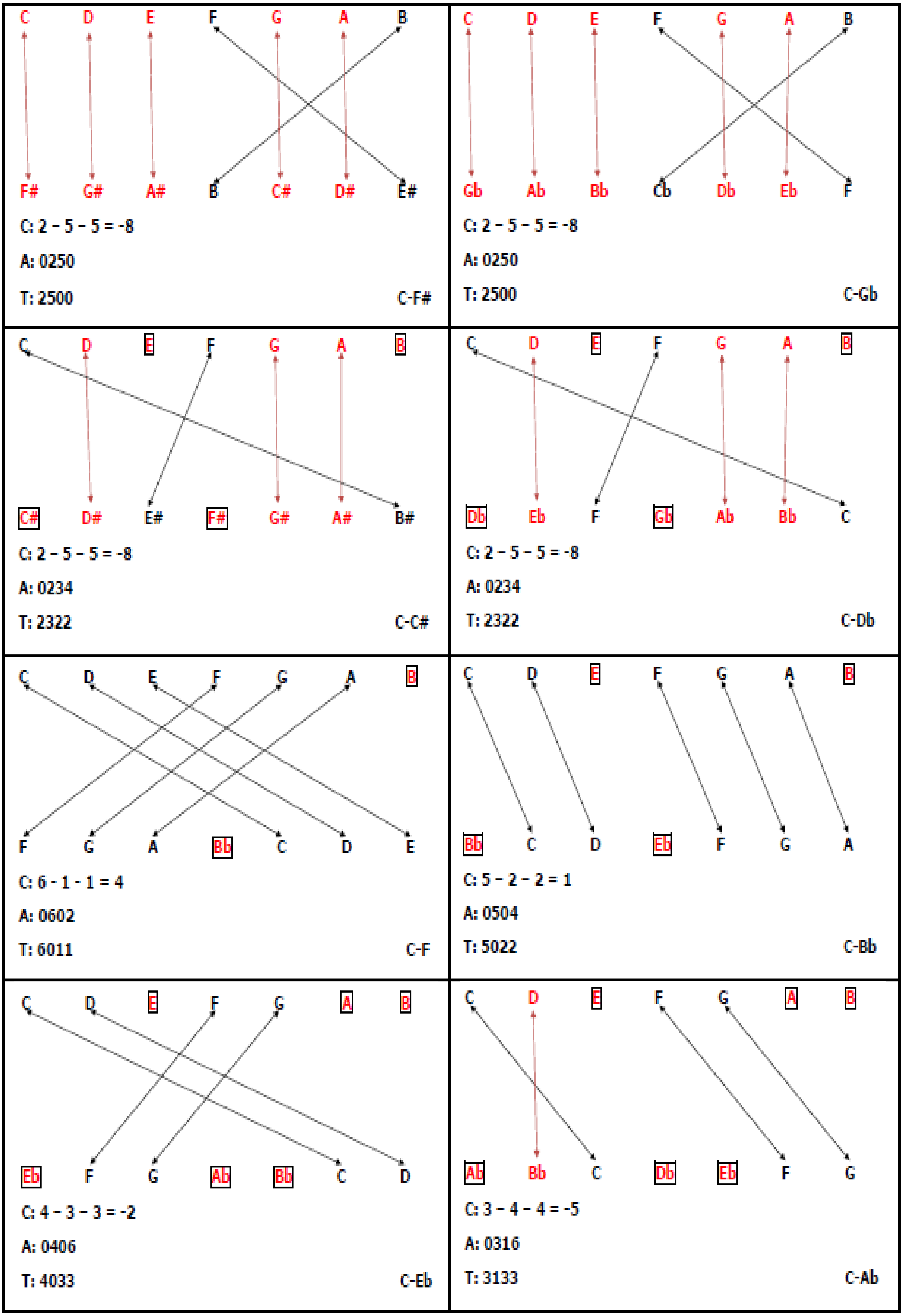
Figure 6: Schoenberg's circle of fifths (2001, p.230).

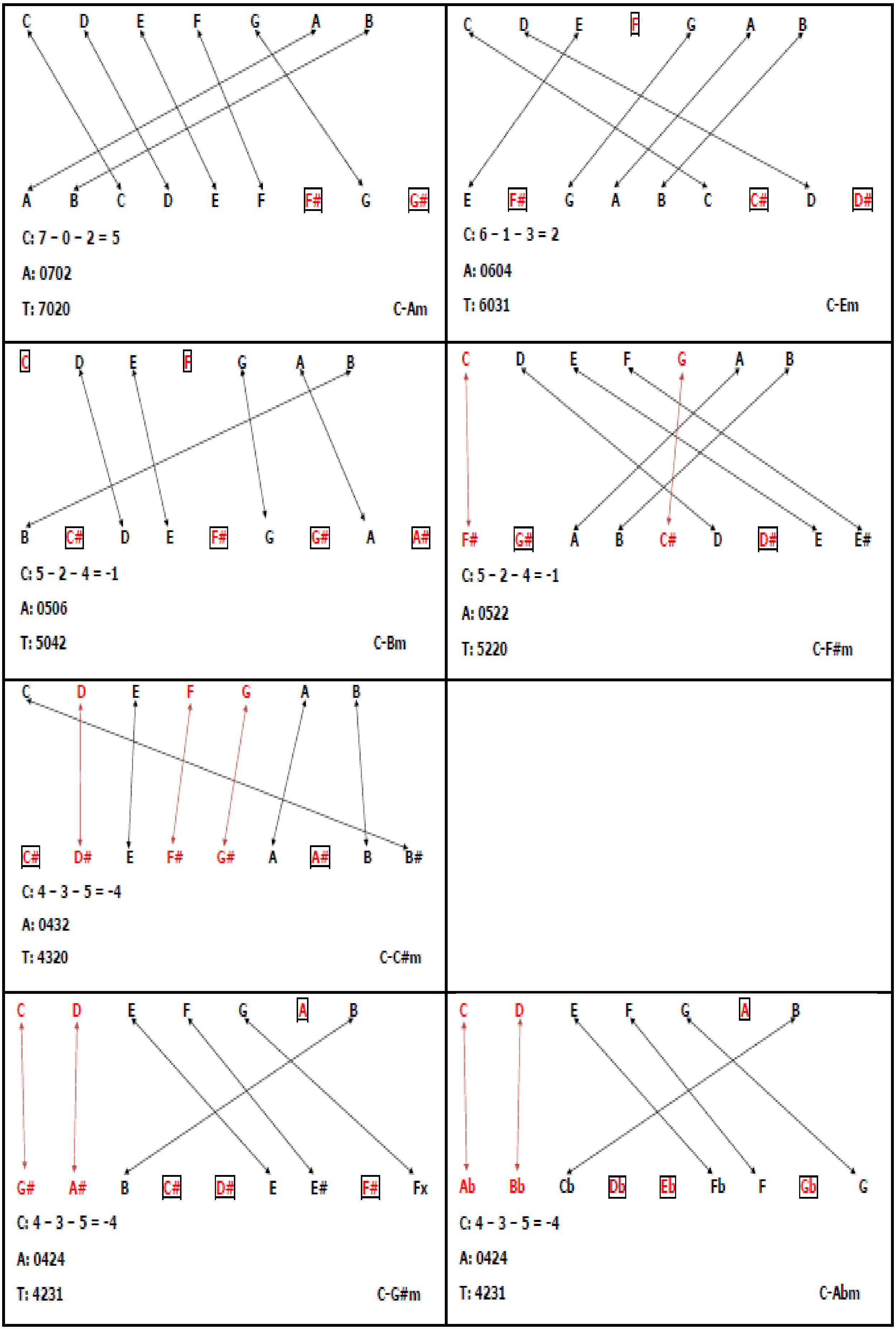


In figs.7 and 8 below, it is possible to see the similarity vectors of the three methods between all major and minor keys in relation to the C and Am keys, taking into account the four tonal scales. The black notes connected by black arrows indicate the notes in common between the keys, the red notes connected by red arrows indicate the different notes that are over the same interval degree (referring only to the degrees and not to the major, minor, augmented and diminished quality) and the red notes within a square without arrows correspond to the different notes that do not find a match in the other key.

Figure 7: Comparison of all major and minor keys derived from the four tonal scales with the key of C.




C D E F G A B
F# G# A# B C# D# E#
C: 2 - 5 - 5 = -8
A: 0250
T: 2500
C-F#
C D E F G A B
Gb Ab Bb Cb Db Eb F
C: 2 - 5 - 5 = -8
A: 0250
T: 2500
C-Gb
C D E F G A B
C# D# E# F# G# A# B#
C: 2 - 5 - 5 = -8
A: 0234
T: 2322
C-C#
C D E F G A B
Db Eb F Gb Ab Bb C
C: 2 - 5 - 5 = -8
A: 0234
T: 2322
C-Db
C D E F G A B
F G A Bb C D E
C: 6 - 1 - 1 = 4
A: 0602
T: 6011
C-F
C D E F G A B
Bb C D Eb F G A
C: 5 - 2 - 2 = 1
A: 0504
T: 5022
C-Bb
C D E F G A B
Eb F G Ab Bb C D
C: 4 - 3 - 3 = -2
A: 0406
T: 4033
C-Eb
C D E F G A B
Ab Bb C Db Eb F G
C: 3 - 4 - 4 = -5
A: 0316
T: 3133
C-Ab


C D E F G A B
A B C D E F F# G G#
C: 7 - 0 - 2 = 5
A: 0702
T: 7020
C-Am
C D E F G A B
E F# G A B C C# D D#
C: 6 - 1 - 3 = 2
A: 0604
T: 6031
C-Em
C D E F G A B
B C# D E F# G G# A A#
C: 5 - 2 - 4 = -1
A: 0506
T: 5042
C-Bm
C D E F G A B
F# G# A B C# D D# E E#
C: 5 - 2 - 4 = -1
A: 0522
T: 5220
C-F#m
C D E F G A B
C# D# E F# G# A A# B B#
C: 4 - 3 - 5 = -4
A: 0432
T: 4320
C-C#m
C D E F G A B
G# A# B C# D# E E# F# Fx
C: 4 - 3 - 5 = -4
A: 0424
T: 4231
C-G#m
C D E F G A B
Ab Bb Cb Db Eb Fb F Gb G
C: 4 - 3 - 5 = -4
A: 0424
T: 4231
C-Abm

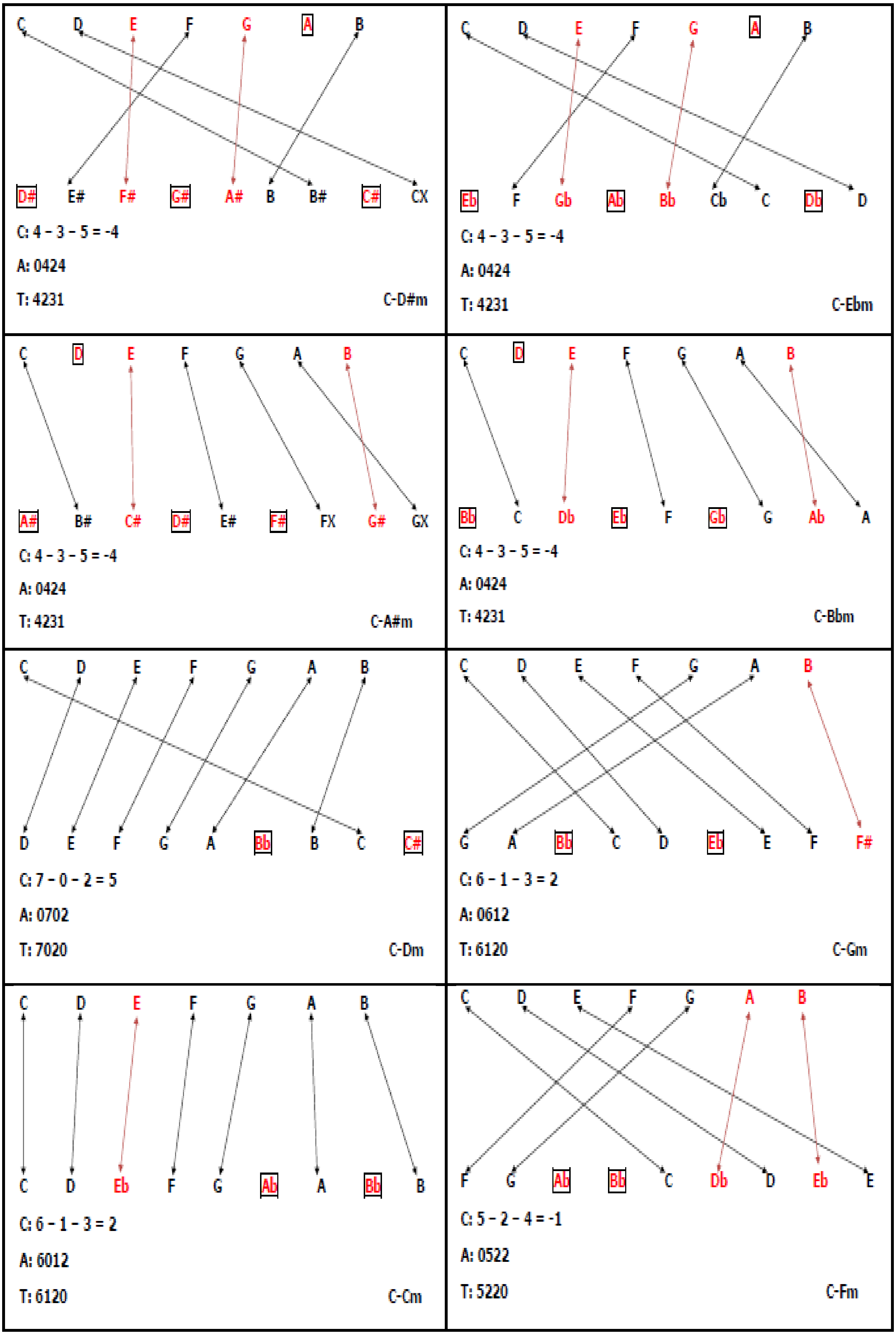
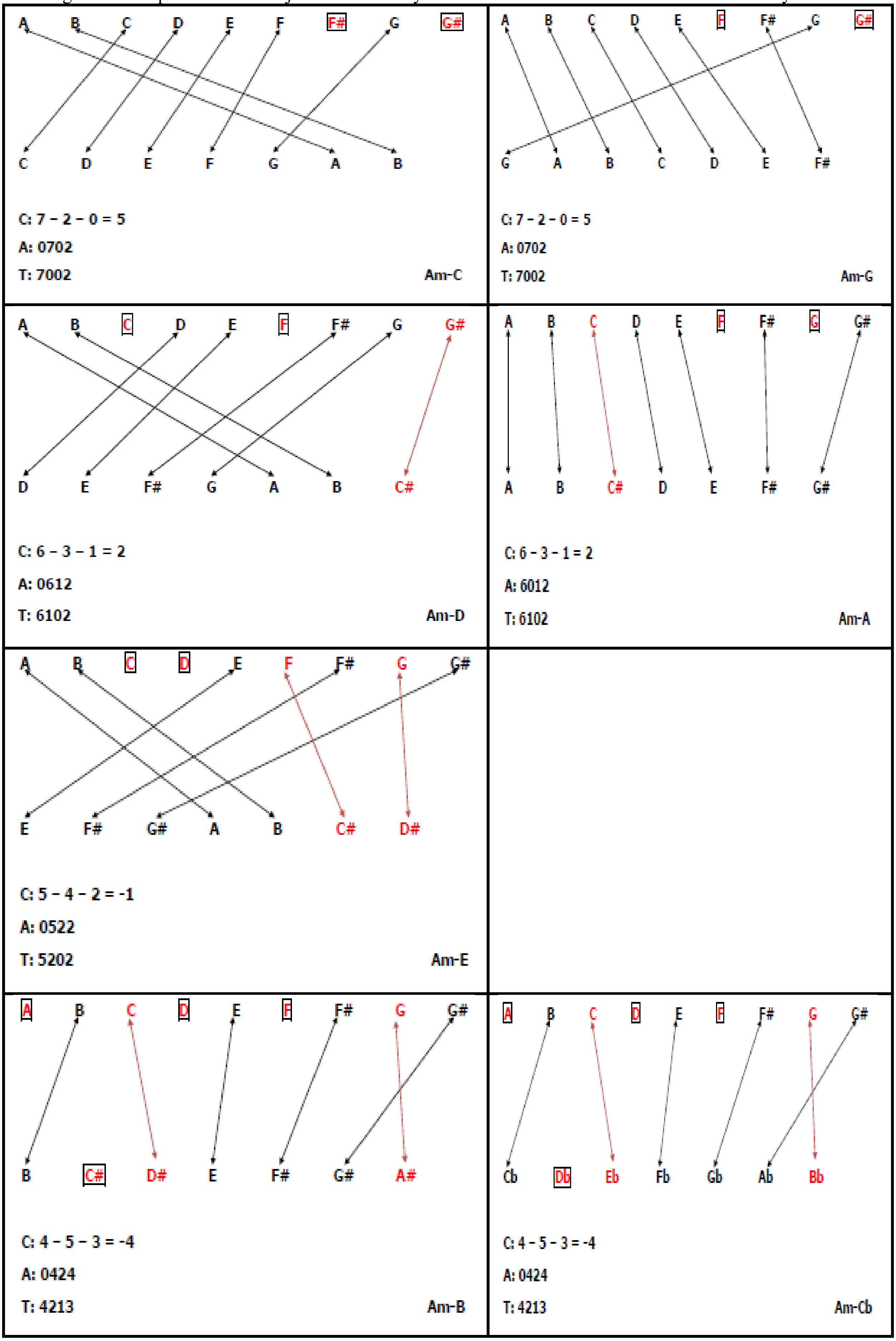
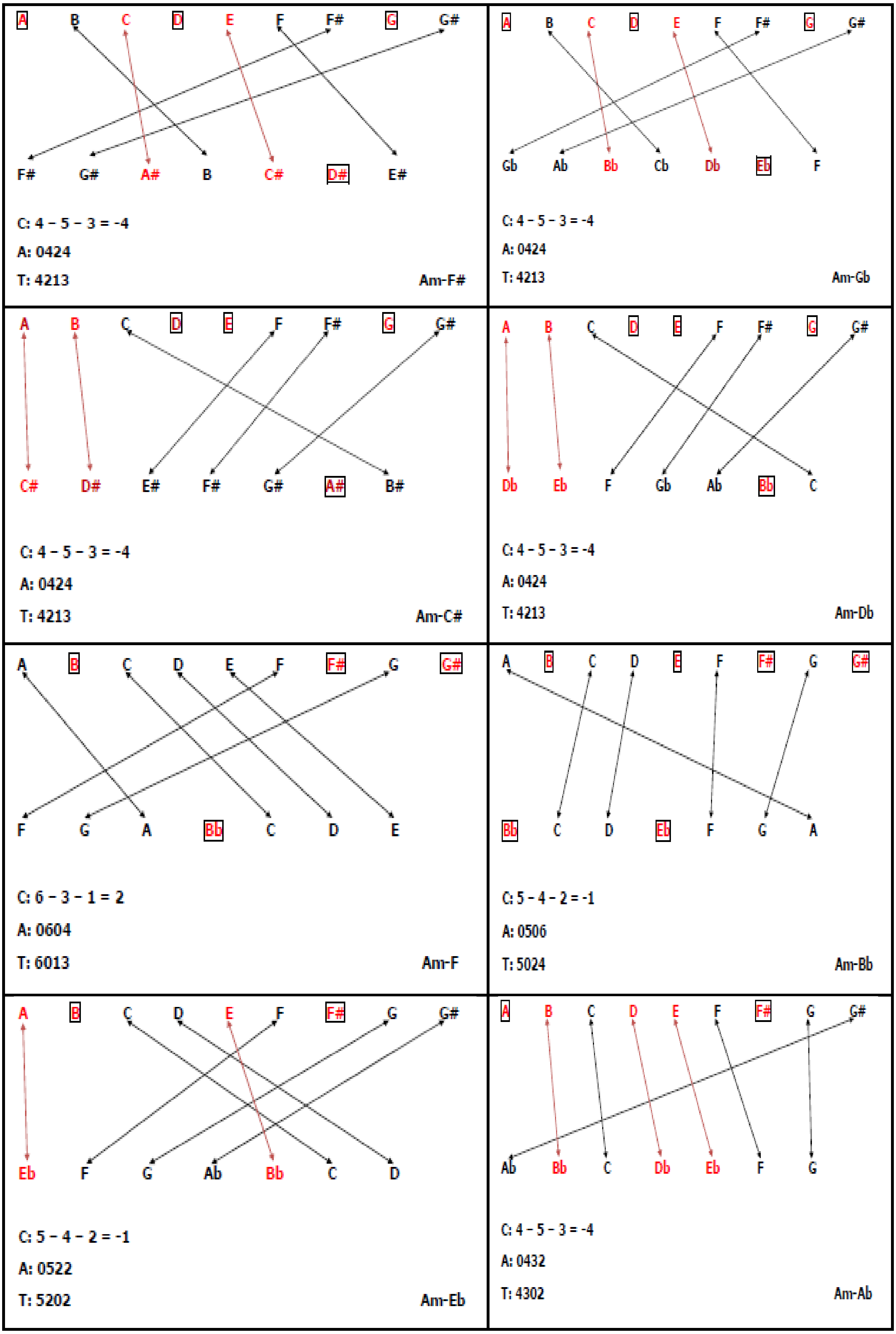
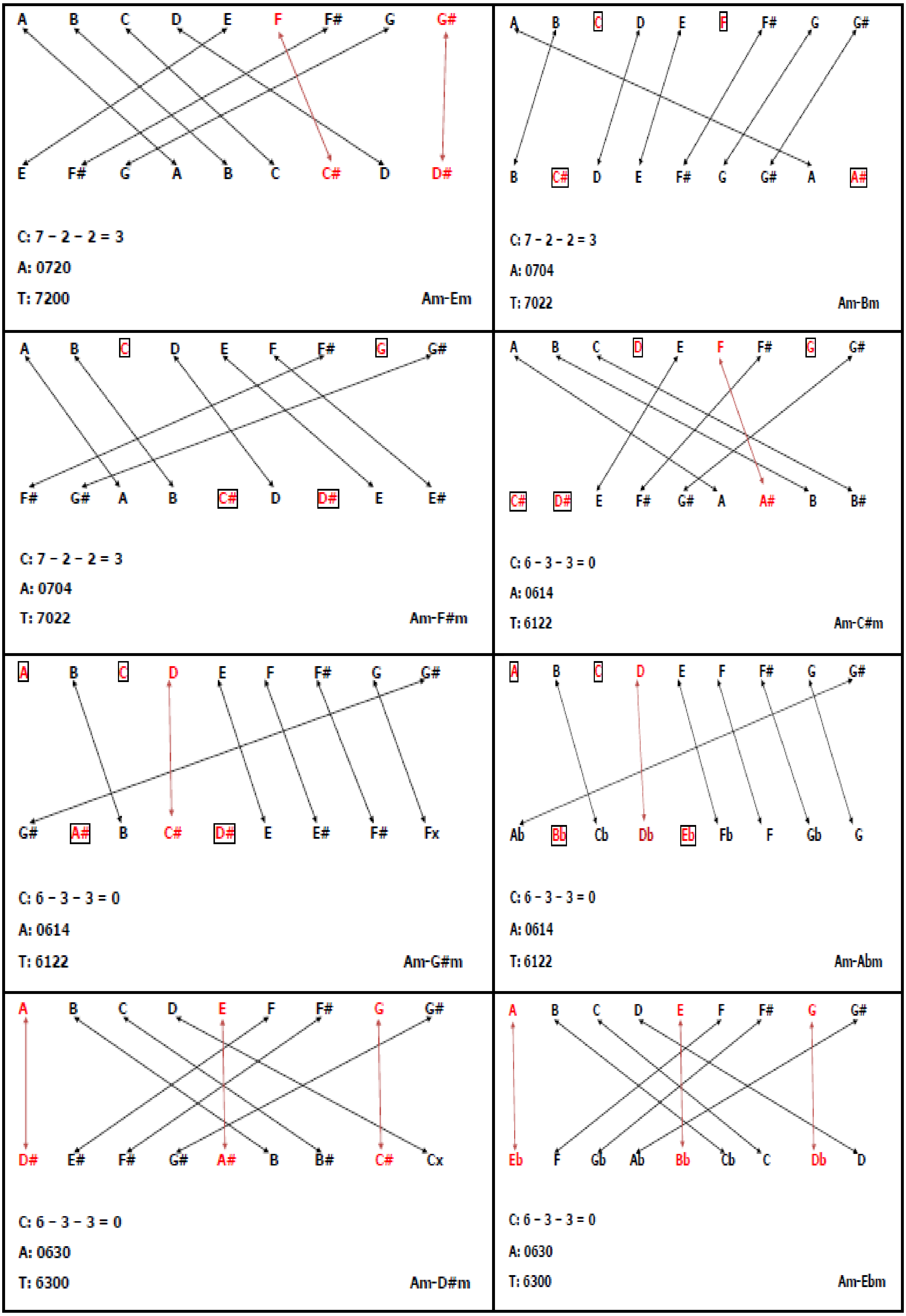

C D E F G A B
D# E# F# G# A# B B# C# CX
C: 4 - 3 - 5 = -4
A: 0424
T: 4231
C-D#m
C D E F G A B
Eb F Gb Ab Bb Cb C Db D
C: 4 - 3 - 5 = -4
A: 0424
T: 4231
C-Ebm
C D E F G A B
A# B# C# D# E# F# FX G# GX
C: 4 - 3 - 5 = -4
A: 0424
T: 4231
C-A#m
C D E F G A B
Bb C Db Eb F Gb G Ab A
C: 4 - 3 - 5 = -4
A: 0424
T: 4231
C-Bbm
C D E F G A B
D E F G A Bb B C C#
C: 7 - 0 - 2 = 5
A: 0702
T: 7020
C-Dm
C D E F G A B
G A Bb C D Eb E F F#
C: 6 - 1 - 3 = 2
A: 0612
T: 6120
C-Gm
C D E F G A B
C D Eb F G Ab A Bb B
C: 6 - 1 - 3 = 2
A: 6012
T: 6120
C-Cm
C D E F G A B
F G Ab Bb C Db D Eb E
C: 5 - 2 - 4 = -1
A: 0522
T: 5220
C-Fm

Figure 8: Comparison of all major and minor keys derived from the four tonal scales with the key of Am.




A B C D E F F# G G#
F# G# A# B C# D# E#
C: 4 - 5 - 3 = -4
A: 0424
T: 4213
Am-F#
A B C D E F F# G G#
Gb Ab Bb Cb Db Eb F
C: 4 - 5 - 3 = -4
A: 0424
T: 4213
Am-Gb
A B C D E F F# G G#
C# D# E# F# G# A# B#
C: 4 - 5 - 3 = -4
A: 0424
T: 4213
Am-C#
A B C D E F F# G G#
Db Eb F Gb Ab Bb C
C: 4 - 5 - 3 = -4
A: 0424
T: 4213
Am-Db
A B C D E F F# G G#
F G A Bb C D E
C: 6 - 3 - 1 = 2
A: 0604
T: 6013
Am-F
A B C D E F F# G G#
Bb C D Eb F G A
C: 5 - 4 - 2 = -1
A: 0506
T: 5024
Am-Bb
A B C D E F F# G G#
Eb F G Ab Bb C D
C: 5 - 4 - 2 = -1
A: 0522
T: 5202
Am-Eb
A B C D E F F# G G#
Ab Bb C Db Eb F G
C: 4 - 5 - 3 = -4
A: 0432
T: 4302
Am-Ab


A B C D E F F# G G#
E F# G A B C C# D D#
C: 7 – 2 – 2 = 3
A: 0720
T: 7200
Am-Em
A B C D E F F# G G#
B C# D E F# G G# A A#
C: 7 - 2 - 2 = 3
A: 0704
T: 7022
Am-Bm
A B C D E F F# G G#
F# G# A B C# D D# E E#
C: 7 – 2 – 2 = 3
A: 0704
T: 7022
Am-F#m
A B C D E F F# G G#
C# D# E F# G# A A# B B#
C: 6 - 3 - 3 = 0
A: 0614
T: 6122
Am-C#m
A B C D E F F# G G#
G# A# B C# D# E E# F# Fx
C: 6 – 3 – 3 = 0
A: 0614
T: 6122
Am-G#m
A B C D E F F# G G#
Ab Bb Cb Db Eb Fb F Gb G
C: 6 - 3 - 3 = 0
A: 0614
T: 6122
Am-Abm
A B C D E F F# G G#
D# E# F# G# A# B B# C# Cx
C: 6 – 3 – 3 = 0
A: 0630
T: 6300
Am-D#m
A B C D E F F# G G#
Eb F Gb Ab Bb Cb C Db D
C: 6 - 3 - 3 = 0
A: 0630
T: 6300
Am-Ebm

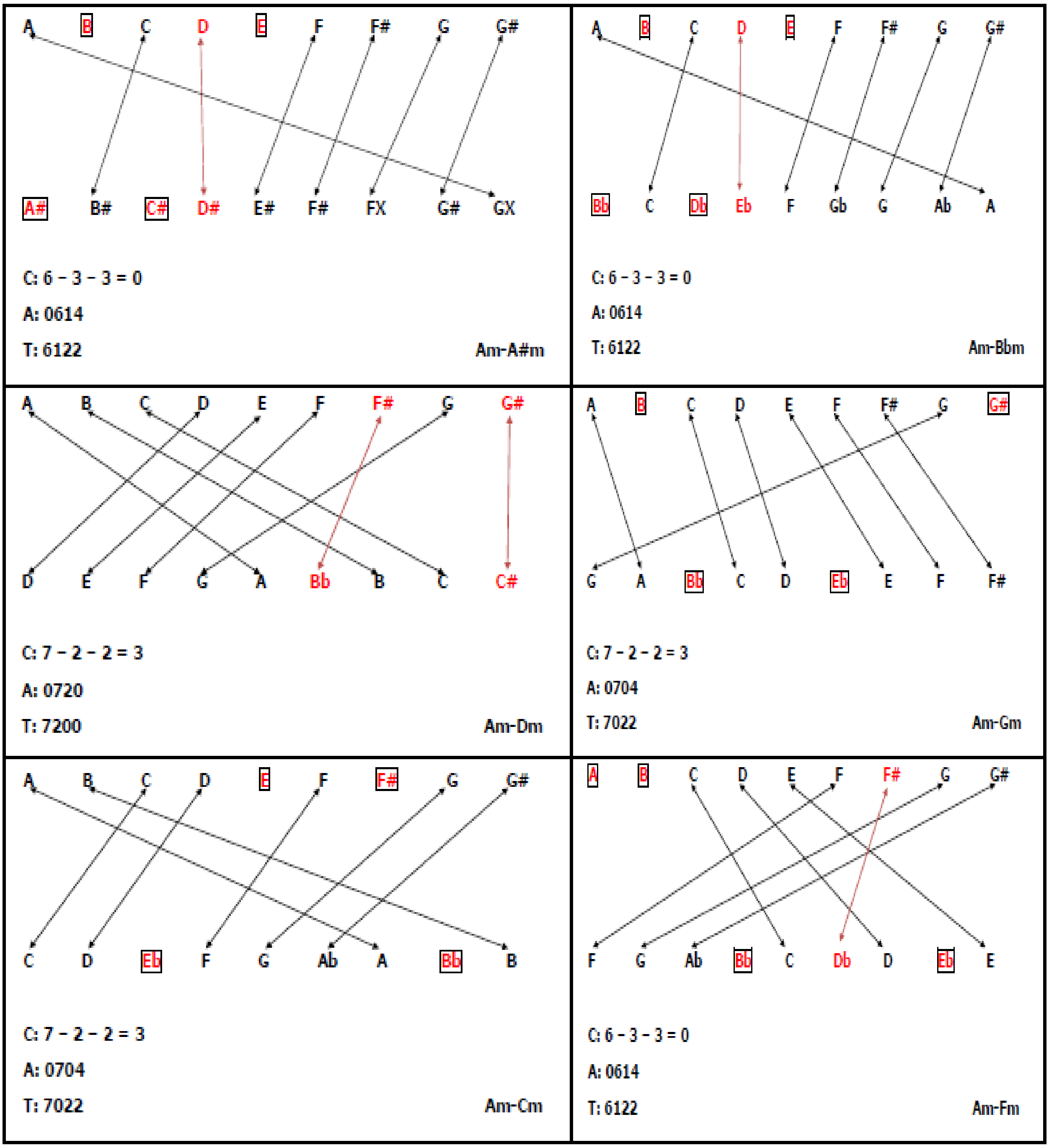


Table 3 shows the classification of similarities, distances and groupings between all major and minor keys in relation to the key of C, taking into account the four tonal scales. The way of classification is the same as that used for chords. First, the commonalities are evaluated, if there is a tie, the differences are observed. For example, in the contrast model, the Keys of Cm, Gm, and Em appear tied for third place with vector (6 – 1 – 3 = 2), but in structural alignment and transformation, they appear divided into two groups, third place (Cm and Gm) and fourth place (Em), respectively. It is observed that the three Keys appear with 6 commonalities in structural alignment and transformation (the first two numbers of the vectors of the structural alignment method represent the commonalities in place and out of place, respectively; and the first number of the vectors of the transformation method represents the commonalities), but Cm and Gm have 3 differences (3#) with respect to C and Em has 4 differences (4#). The differences, in these cases, are the classificatory requirements. What is striking in this case are the commonalities in place of structural alignment, something that does not happen in the contrast model or in the transformation. The key of Cm has its 6 commonalities as correspondences in place (MIPS) and the key of Gm has its 6 commonalities as correspondences out of place (MOPS). In this regard, the key of Cm can take the first position in the ranking list, as no other key has commonalities in its place. In this sense, structural alignment explains why C and Cm are considered the same key in the classical and romantic period, i.e., the change from C to Cm or vice versa was not considered modulation, but only a change of mode. Piston (1987, p.231-232) considered that the parallel major and minor keys (or homonyms) are practically identical and that the classical composition of the eighteenth and nineteenth centuries tended "regarded the two modes as simply two different aspects of one tonality" (also discussed in Krumhansl and Kessler 1982, p.339). For Piston (1987, p.62), "the change of mode from major to minor, or vice versa, does not affect the key [...] And it's one of the most important means of variation." According to Krumhansl and Kessler (1982, p.338-339):

> Additionally, there is a close tie between a major key and the minor key built on the same tonic, which is called the parallel minor. This is true even though their key signatures differ in terms of three sharps or flats. For example, C major (with no sharps or flats) has C minor (with three flats) as its parallel minor. Despite the differences in key signatures, these keys share most of the structurally stable tones—the tonic, fourth, and fifth scale degrees—as well as the seventh, leading tone in the harmonic and ascending melodic forms. Possibly even more important is the fact that they also share the essential dominant, V, chord, which prepares the tonic of either of the parallel keys.

For Schoenberg (2004, p.74; 1999 [1969], p.52), [10] "in classical music, major and minor are often interchanged without much explanation; A passage in minor is followed by another in major, and vice versa, without any harmonic element of connection." According to Schoenberg (2001, p.303-304, p.318; 1978, p.207-208, p.219), what allows the interchange between homonymous tonalities is the dominant one in common, that is, after a dominant one can come either a major or a minor tonic: "a dominant one can introduce a major or minor triad, and it can be the dominant one of a major or minor region. The possibility of permutation between major and minor is anchored in the strength of the dominant one" (Schoenberg 2004, p.73, 1999 [1969], p.51). An idea that was also part of Piston's thought (1987, p.63) when he said that the dominant one proceeds to the major or minor tonic with equal ease (also in Piston 1987, p.233). For Dubovsky *et al.* (2018, p.330), the interaction between the major and minor modes resulted in the enrichment of both modes. In the

[10] The first reference is from the Portuguese edition, Structural Functions of Harmony of 2004. The second reference is from the 1999 edition *of Structural Functions of Harmony* [1969]. The same will apply to the references of other quotations from Schoenberg throughout the text: Portuguese edition and English edition, or vice versa. It is noteworthy that the 1954, 1969, and 1999 editions of *Structural Functions of Harmony* are identical with respect to the page numbers given as reference.

early nineteenth century, composers such as Schubert began to saturate their major pieces with elements of the minor to such an extent that the twenty-four major and minor keys seem to have merged into just twelve major-minor calls. According to Smith (1986, p.112, footnote 23),

> One corollary of this pivotal status is that we can distinguish only twelve, and not twenty-four, keys-since there are only twelve possible leading tones, and hence only twelve possible V chords. Thus major and minor modes must be regarded functionally as different colorings of one key, rather than as different keys-as indeed they are treated by most nineteenth-century composers most of the time.

Without a doubt, the major-minor duality is abundant in the period of common practice of the 18th and early 19th centuries. As described by Schenker (1954, p.86-87):

> Properly speaking, I think any composition moves in a major-minor system. A composition in C, for example, should be understood as in C major-minor (C $\frac{\text{maior}}{\text{menor}}$); for a pure C major, without any C minor ingredient, or, vice versa, a pure C minor, without any C major component, hardly occurs in reality.

This thought was also expressed by Ottman (1992 [1961], p.200) when he said that "a passage of music is sometimes said to be in mixed mode when it includes harmonic structures from both the major and minor modes", and also by Dubovsky et al. (2018, p.330), writing that "all the complexities that characterized the interaction of the greater with the minor formed the major-minor system, with the modes themselves receiving the denomination of major-minor or minor-major, depending on whether they are driven by the major or minor stress."Smith (1981, p.158) also regarded "the modes, especially major and the minor, as different colorings of the same key, rather than as separate tonalities; there are therefore only 12 distinguishable keys." For Ratner (1962, p.249), also, "it is possible to switch modes while retaining the same tonal center. The implications of the interchange of mode for eighteenth- and nineteenth-century music are tremendous. Both harmonic structure and harmonic color are involved.". According to Gauldin (2004, p.502),

> The major and minor modes were clearly delineated in the tonal compositions of the later Baroque period (1670-1750). A movement or an entire piece of music exhibited tonal closureby starting and concluding in the same key and same mode, although interior modulations were possible. Later composers began to experiment with blurring the modes by introducing passages in the parallel major or minor key. This technique, known as modal exchange, was frequently employed by Beethoven and Schubert during the opening decades of the nineteenth century.

Even for Rameau (1971 [1722], p.162-163), about two hundred years before these authors, parallel tonalities meant only a change of mode over the same fundamental one and not a change of tonality:

> Notice that we do not separate the term mode from the term key when a change between major and minor is found on the same tonic note, for we may change the mode from major to

minor or from minor to major without changing the tonic or principal note of the mode. For example, when we pass from a gay theme to a sad one, or from a sad to a gay, as occurs in most Chaconnes or Passacaglias [...] we can say that the key does not change, even though the mode changes. If the note Do is the tonic of the major mode, then it is also the tonic in the minor (Rameau 1971 [1722], p.162-163). [11]

Another differentiation between the classification of the contrast model and the other two, structural alignment and transformation, can be seen in the key of C#m in relation to the prototype or referential key of C. In the contrast model, the key C#m appears in seventh place along with other key as can be seen in Table 4, however, in structural alignment and transformation, it can be placed between the seventh and eighth position as indicated by the arrow, because its vectors 0432 and 4320 in structural alignment and transformation, respectively, which have 4 commonalities and 5 differences, displaces the key of C#m that appears in the ninth position. The Key of C#m was placed in the ninth position because, as previously stated, the contrast model provides the classificatory example for the other two methods.

Table 3: results of the three similarity assessment methods in relation to the tonality of C.

Level	Contrast C	Vector	1	Alignment C IOAN	Vector	Level	Transformation C CTSA	Vector
1	Am Dm	7 – 0 – 2 = 5	2	Am Dm	0702 (2#) 0702 (2#)	2	Am Dm	7020 (2#) 7020 (2#)
2	G F	6 – 1 – 1 = 4	3	G F	0602 (2#) 0602 (2#)	3	G F	6011 (2#) 6011 (2#)
3	Cm Gm Em	6 – 1 – 3 = 2	4	**Cm** Gm	**6012 (3#)** 0612 (3#)	4	Cm Gm	6120 (3#) 6120 (3#)
			5	Em	0604 (4#)	5	Em	6031 (4#)
4	D Bb	5 - 2 - 2 = 1	6	D Bb	0504 (4#) 0504 (4#)	6	D Bb	5022 (4#) 5022 (4#)
5	F#m Fm Bm	5 – 2 – 4 = -1	6	F#m Fm	0522 (4#) 0522 (4#)	6	F#m Fm	5220 (4#) 5220 (4#)
			7	Bm	0506 (6#)	7	Bm	5042 (6#)
6	A Eb	4 – 3 – 3 = -2	8	A Eb	0406 (6#) 0406 (6#)	8	A Eb	4033 (6#) 4033 (6#)
7	C#m G#m (Abm) D#m (Ebm) A#m (Bbm)	4 – 3 – 5 = -4	**9**	**C#m**	**0432 (5#)**	**9**	**C#m**	**4320 (5#)**
			10	G#m (Abm) D#m (Ebm) A#m (Bbm)	0424 (6#) 0424 (6#) 0424 (6#)	10	G#m (Abm) D#m (Ebm) A#m (Bbm)	4231 (6#) 4231 (6#) 4231 (6#)
8	E Ab	3 – 4 – 4 = -5	11	E Ab	0316 (7#) 0316 (7#)	11	E Ab	3133 (7#) 3133 (7#)
9	F# (Gb) B (Cb) C# (Db)	2 – 5 – 5 = -8	12	F# (Gb)	0250 (5#)	12	F# (Gb)	2500 (5#)
			13	B (Cb) C# (Db)	0234 (7#) 0234 (7#)	13	B (Cb) C# (Db)	2322 (7#) 2322 (7#)

Table 4 shows the classification of similarities, distances and groupings between all major and minor keys in relation to the key of Am taking into account the four tonal scales. The same comments as earlier on the parallel or homonymous keys of C and Cm occur here in relation to the

[11] It is noted here that Rameau calls the fundamental note the "tonic note." This study, however, differentiates between these two terms. "Fundamental" refers to the first note of a scale or the fundamental bass of a chord, while "tonic" refers to the first-degree chord of a key.

keys of Am and A. The key of A has its 6 commonalities as correspondences in place (MIPS) and the key of D has its 6 commonalities as correspondences out of place (MOPS). In this regard, the key of A can take the first position in the ranking list, as no other key has commonalities in its place.

As with the key of C#m in relation to the key of C in the contrast model, the key of D#m (Ebm) appears in fourth place along with other keys as can be seen in table 4, however, in the structural alignment and transformation, it can be placed between the fourth and fifth position as indicated by the arrow, because its vectors 0630 in structural alignment and 6300 in transformation, which have 6 commonalities and 3 differences (3#), displaces the key D#m (Ebm) from the sixth position.

Table 4: results of the three similarity assessment methods in relation to the tonality of Am.

Level	Contrast Am	Vector	Level	Alignment Am IOAN	Vector	Level	Transformation Am CTSA	Vector
1	C G	7 – 2 – 0 = 5	1	C G	0702 (2#) 0702 (2#)	1	C G	7002 (2#) 7002 (2#)
2	Em Dm Bm F#m Cm Gm	7 – 2 – 2 = 3	2	Em Dm	0720 (2#) 0720 (2#)	2	Em Dm	7200 (2#) 7200 (2#)
			3	Bm F#m Cm Gm	0704 (4#) 0704 (4#) 0704 (4#) 0704 (4#)	3	Bm F#m Cm Gm	7022 (4#) 7022 (4#) 7022 (4#) 7022 (4#)
3	D A F	6 – 3 – 1 = 2	4	D A	0612 (3#) 6012 (3#)	4	D A	6102 (3#) 6102 (3#)
			5	F	0604 (4#)	5	F	6013 (4#)
4	D#m (Ebm) C#m G#m (Abm) A#m (Bbm) Fm	6 – 3 – 3 = 0	**6**	**D#m (Ebm)**	**0630 (3#)**	**6**	**D#m (Ebm)**	**6300 (3#)**
			7	C#m G#m (Abm) A#m (Bbm) Fm	0614 (5#) 0614 (5#) 0614 (5#) 0614 (5#)	7	C#m G#m (Abm) A#m (Bbm) Fm	6122 (5#) 6122 (5#) 6122 (5#) 6122 (5#)
5	E Eb Bb	5 – 4 – 2 = -1	8	E Eb	0522 (4#) 0522 (4#)	8	E Eb	5202 (4#) 5202 (4#)
			9	Bb	0506 (6#)	9	Bb	5024 (6#)
6	Ab B (Cb) F# (Gb) C# (Db)	4 – 5 – 3 = -4	10	Ab	0432 (5#)	10	Ab	4302 (5#)
			11	B (Cb) F# (Gb) C# (Db)	0424 (6#) 0424 (6#) 0424 (6#)	11	B (Cb) F# (Gb) C# (Db)	4213 (6#) 4213 (6#) 4213 (6#)

APPLICATION OF THE CONTRAST, STRUCTURAL ALIGNMENT, AND TRANSFORMATION MODEL TO THE 48 SERIES OF ALBAN BERG'S VIOLIN CONCERTO

The 48 series of Alban Berg's Violin Concerto (1935) were also classified through the three methods of assessing similarities developed in cognitive psychology. Menezes (2002, p.214) demonstrates that the original series of this concerto is based on triads, containing the four types of basic chords: major, minor, augmented and diminished chords (fig.9). The final part contains a fragment of the wholetone scale.

Figure 9 (Menezes 2002, p. 215, ex.114): series of Alban Berg's Violin Concerto.

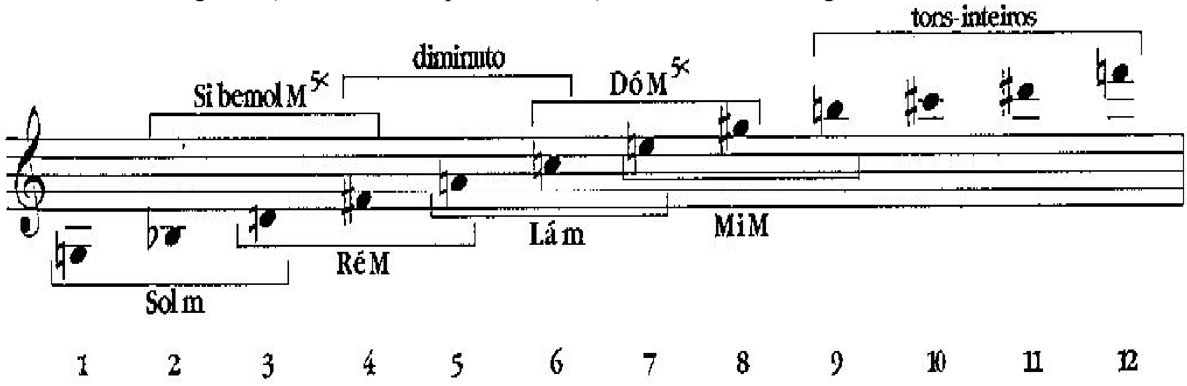


Table 5 shows the 48 series of Alban Berg's Violin Concerto, i.e. the 12 transpositions of the original series, the 12 transpositions of the inversion, the retrograde and the retrograde of the inversion. The transpositions of the 4 main series are numbered from 0 to 11.

Table 5: 48 series of Alban Berg's Violin Concerto.

	I0	I3	I7	I11	I2	I5	I9	I1	I4	I6	I8	I10	
O0	G	Bb	D	F#	A	C	E	G#	B	C#	D#	F	R0
O9	E	G	B	D#	F#	A	C#	F	G#	Bb	C	D	R9
O5	C	Eb	G	B	D	F	A	Db	E	F#	Ab	Bb	R5
O1	Ab	B	Eb	G	Bb	Db	F	A	C	D	E	F#	R1
O10	F	G#	C	E	G	Bb	D	F#	A	B	C#	D#	R10
O7	D	F	A	C#	E	G	B	D#	F#	G#	Bb	C	R7
O3	Bb	Db	F	A	C	Eb	G	B	D	E	F#	Ab	R3
O11	F#	A	Db	F	Ab	B	Eb	G	Bb	C	D	E	R11
O8	Eb	F#	Bb	D	F	G#	C	E	G	A	B	C#	R8
O6	C#	E	Ab	C	Eb	F#	Bb	D	F	G	A	B	R6
O4	B	D	F#	Bb	Db	E	Ab	C	Eb	F	G	A	R4
O2	A	C	E	Ab	B	D	F#	Bb	Db	Eb	F	G	R2
	RI0	RI3	RI7	RI11	RI2	RI5	RI9	RI1	RI4	RI6	RI8	RI10	

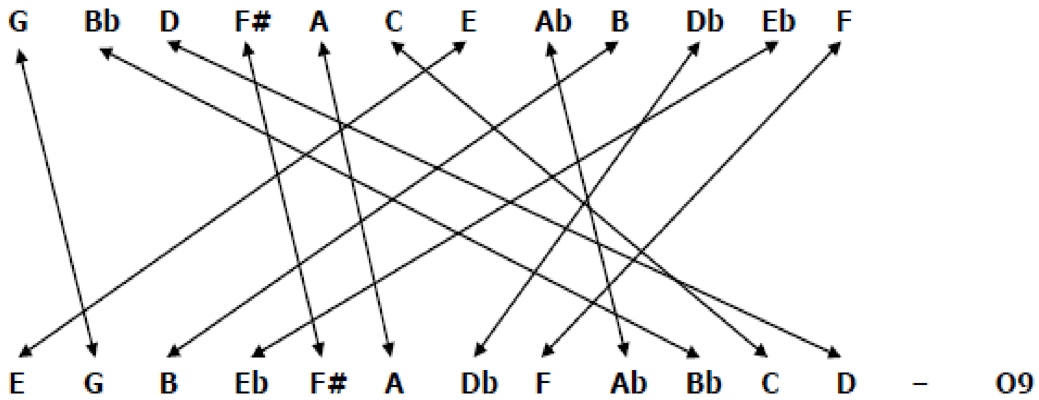
Fig.10 shows the comparisons with the highest similarity assessment in relation to the original series (O0). The black notes connected by black arrows indicate the common notes in different positions, the red notes connected by red arrows indicate the common notes in the same position. As can be seen in Table 6, all series were classified into only four groups, so only one example of each group will be described in the following paragraphs.

According to the **contrast model** (C), in the comparison between **O0 and O9**, there are 12 notes in common, no notes other than O0 in relation to O9 and no notes other than O9 in relation to O0, resulting in the equation **12 - 0 - 0 = 12**. In the comparison between **O0 and RI3**, there are 12 notes in common, no notes other than O0 in relation to RI3, and no notes other than RI3 in relation to O0, resulting in the equation **12 - 0 - 0 = 12**. In the comparison between **O0 and I0**, there are 12 notes in common, no notes other than O0 with respect to I0, and no notes other than I0 with respect to O0, resulting in the equation **12 - 0 - 0 = 12**. In the comparison between **O0 and R2**, there are 12 notes in common, no notes other than O0 with respect to R2, and no notes other than R2 with respect to O0, resulting in the equation **12 - 0 - 0 = 12**. Figure 11 shows that the contrast model obtains the same result in all comparisons. This is true for all transpositions of the original series, all regressions, reversals, and retrogrades of the inversion.

In the **structural alignment method** (A), in the comparison between **O0 and O9** there is no match in place (MIP), there are 12 matches out of place (MOP), no alignable difference and no non-alignable difference, resulting in vector **0/12/00**. In the comparison between **O0 and RI3** there are 4 matches in place (MIP), 8 matches out of place (MOP), no alignable differences, and no non-alignable differences, resulting in the **4/8/00 vector**. In the comparison between **O0 and I0** there are 2 matches in place (MIP), 10 matches out of place (MOP), no alignable differences, and no non-alignable differences, resulting in the **2/10/00 vector**. In the comparison between **O0 and R2** there is 1 match in place (MIP), 11 matches out of place (MOP), no alignable difference, and no non-alignable difference, resulting in the vector **1/11/00**.

In the **transformation method** (T), in the comparison between **O0 and O9** there are 12 commonalities, no transformation, no subtraction, and no addition, resulting in the vector **12/000**. In the comparison between **O0 and RI3** there are 12 commonalities, no transformation, no subtraction, and no addition, resulting in the vector **12/000**. In the comparison between **O0 and I0** there are 12 commonalities, no transformation, no subtraction, and no addition, resulting in the vector **12/000**. In the comparison between **O0 and R2** there are 12 commonalities, no transformation, no subtraction, and no addition, resulting in the vector **12/000**. Figure 11 shows that the transformation method also obtains the same result in all similarity comparisons (12/000), as does the contrast model.

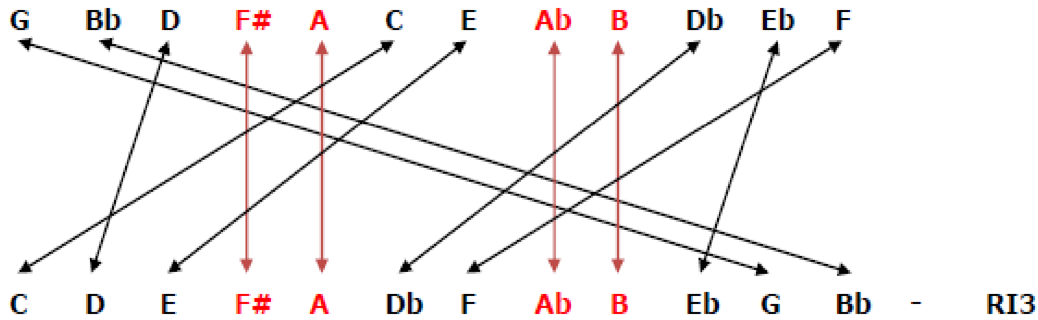
Figure 10: Similarities between the series shown in Table 6.



C: 12 – 0 – 0 = 12

A: 0/12/00

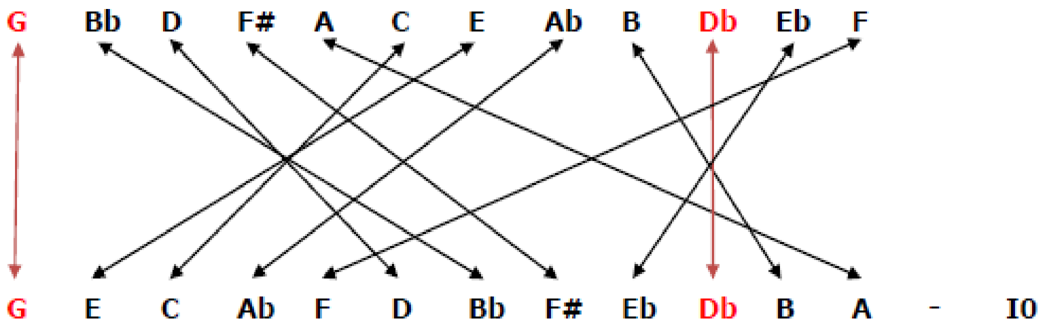
T: 12/000



C: 12 – 0 – 0 = 12

A: 4/8/00

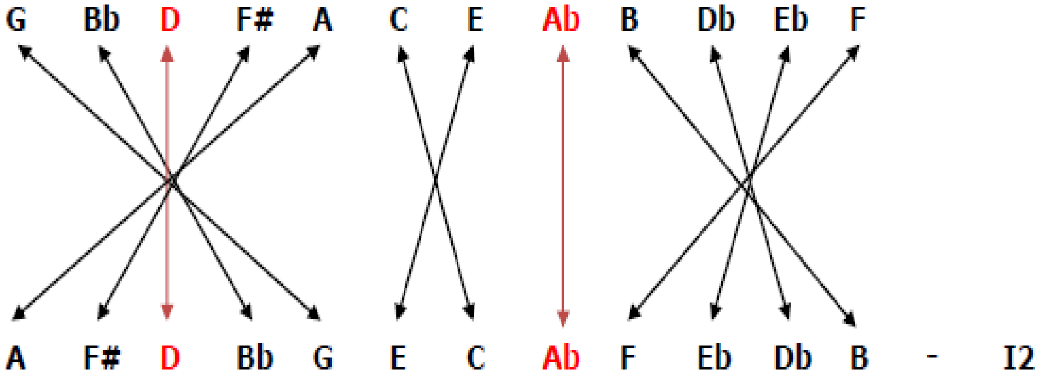
T: 12/000



C: 12 – 0 – 0 = 12

A: 2/10/00

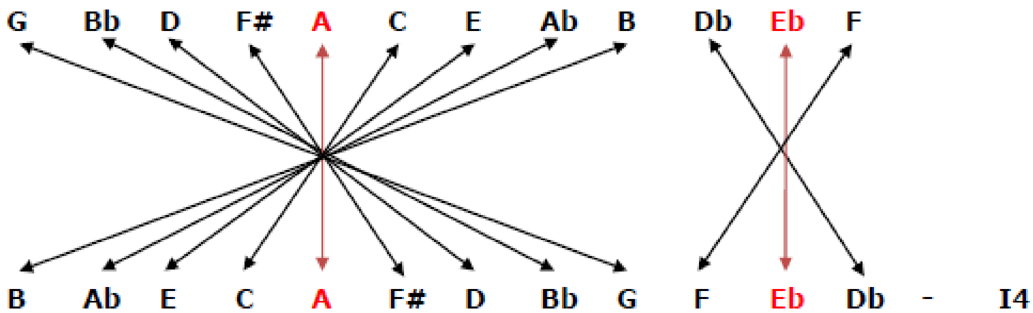
T: 12/000



C: 12 – 0 – 0 = 12

A: 2/10/00

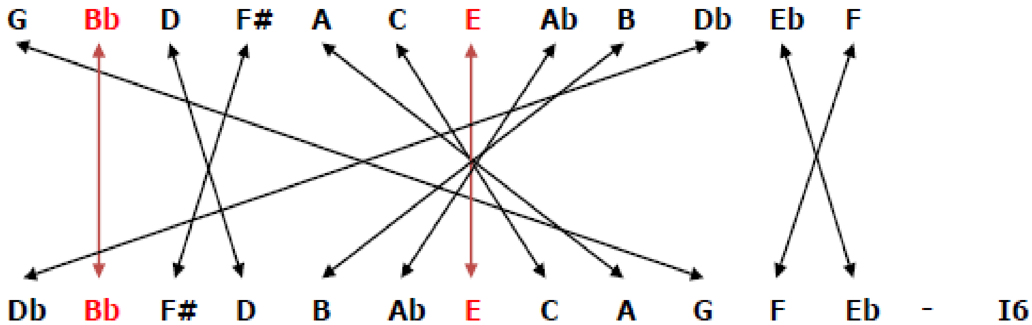
T: 12/000



C: 12 – 0 – 0 = 12

A: 2/10/00

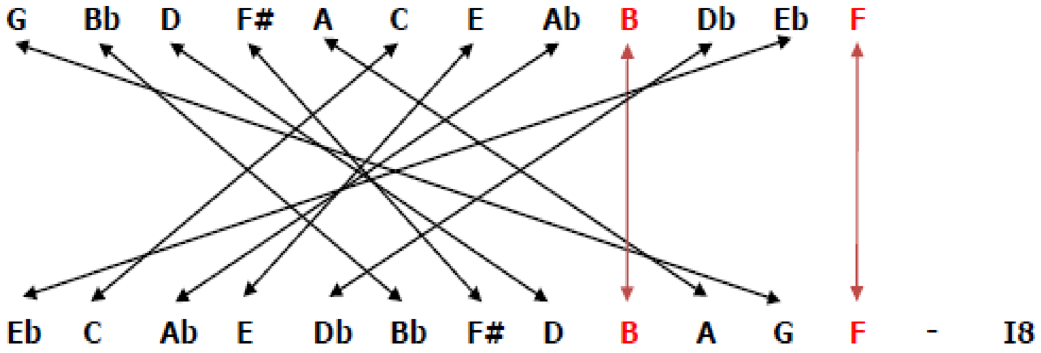
T: 12/000



C: 12 – 0 – 0 = 12

A: 2/10/00

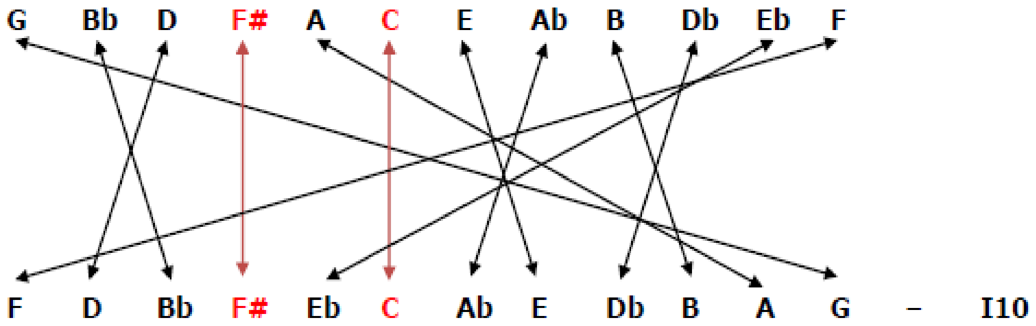
T: 12/000



C: 12 – 0 – 0 = 12

A: 2/10/00

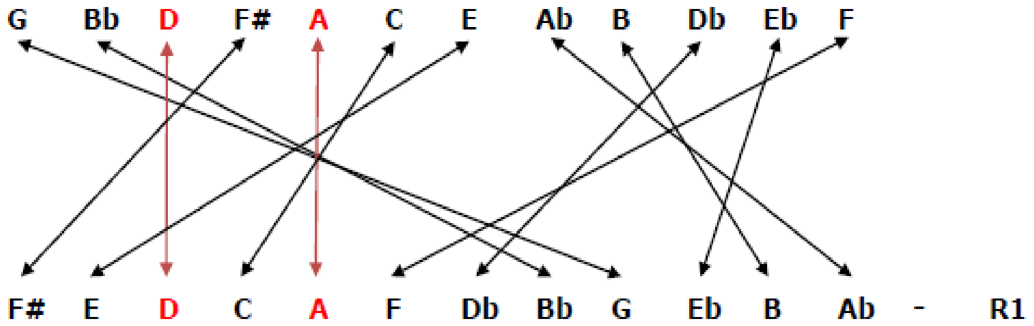
T: 12/000



C: 12 – 0 – 0 = 12

A: 2/10/00

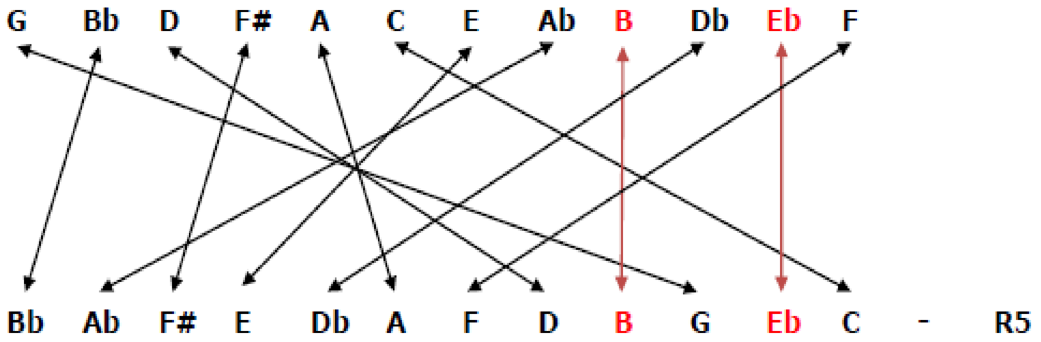
T: 12/000



C: 12 – 0 – 0 = 12

A: 2/10/00

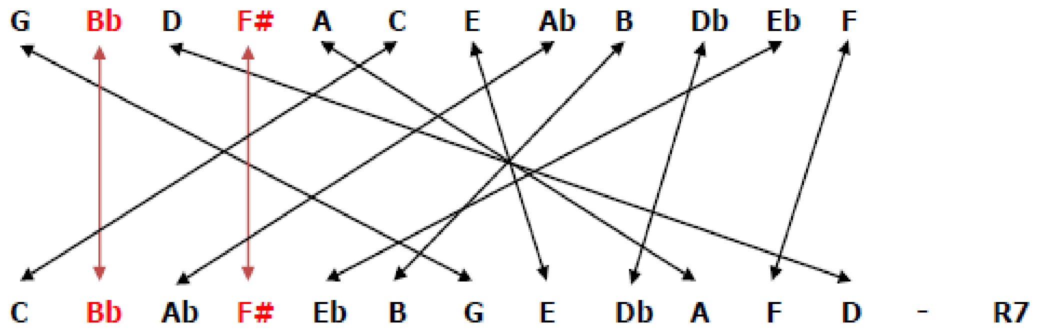
T: 12/000



C: 12 – 0 – 0 = 12

A: 2/10/00

T: 12/000



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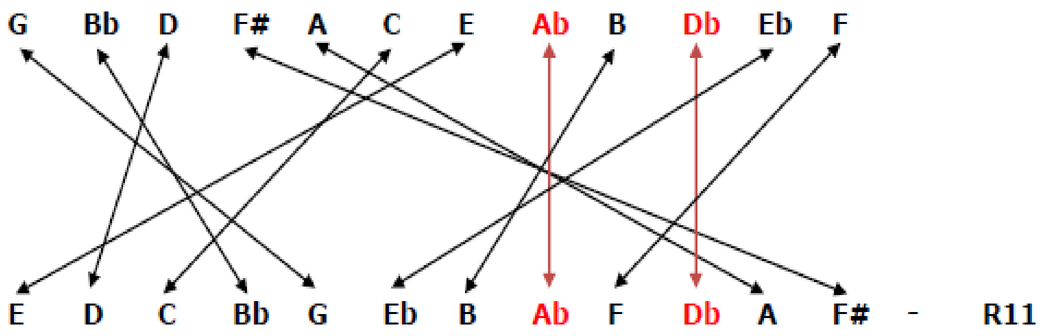

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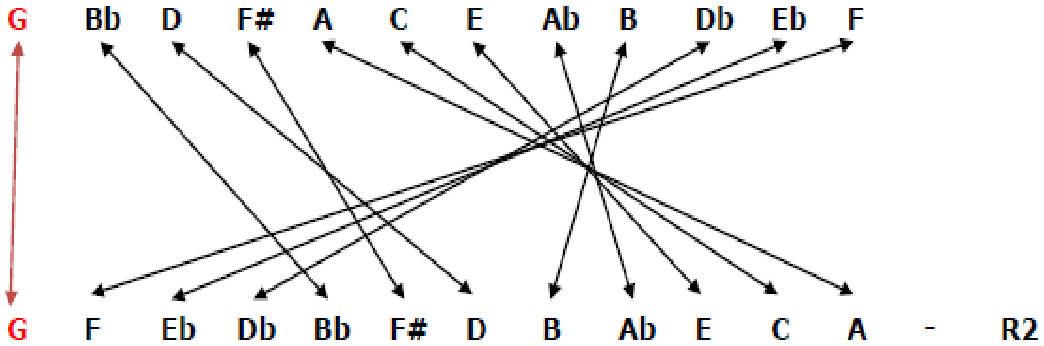

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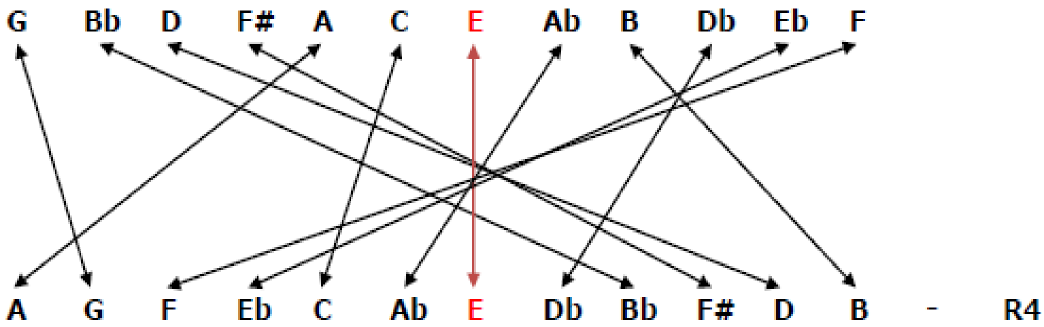

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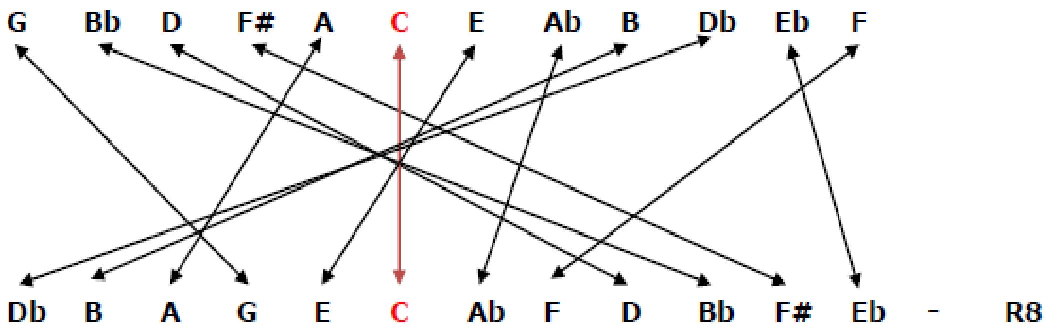

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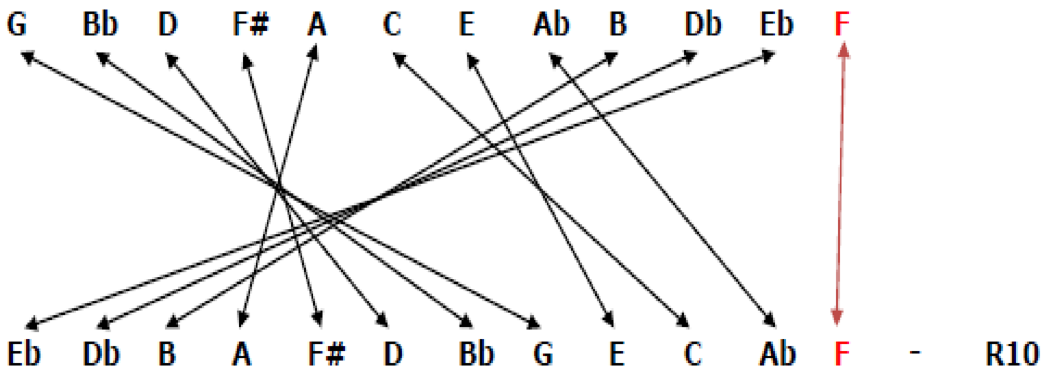

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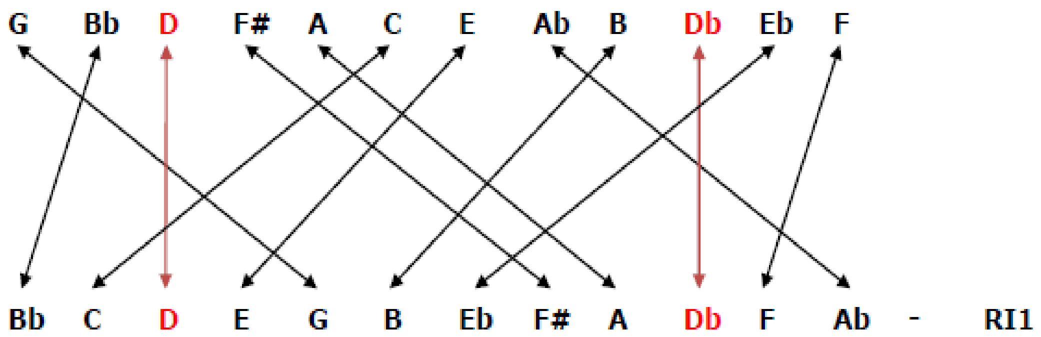

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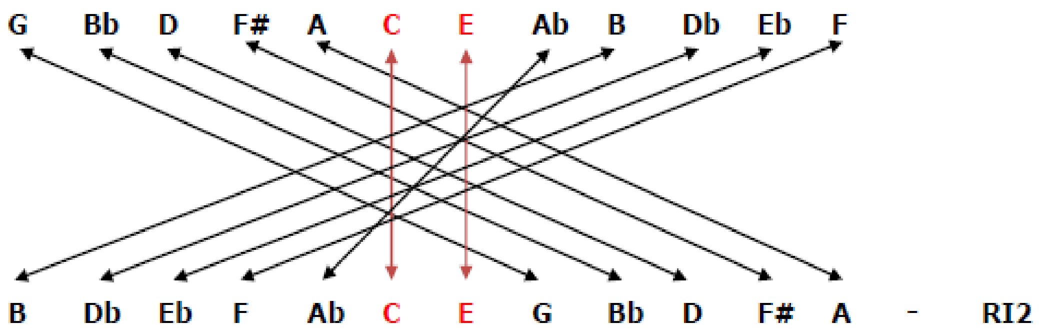

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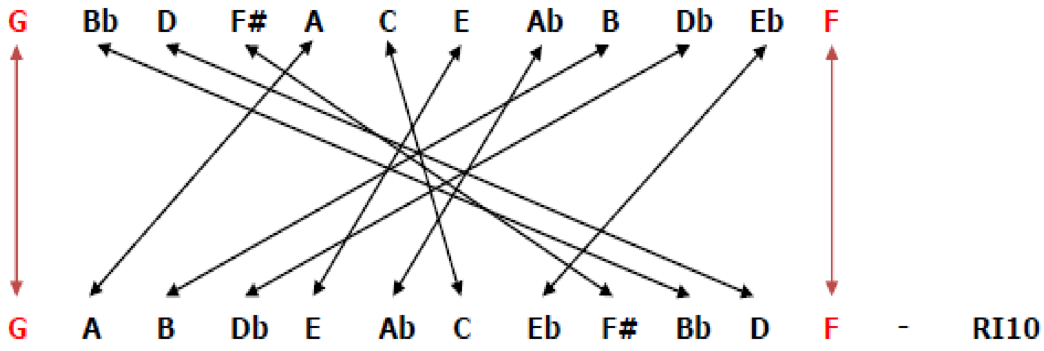

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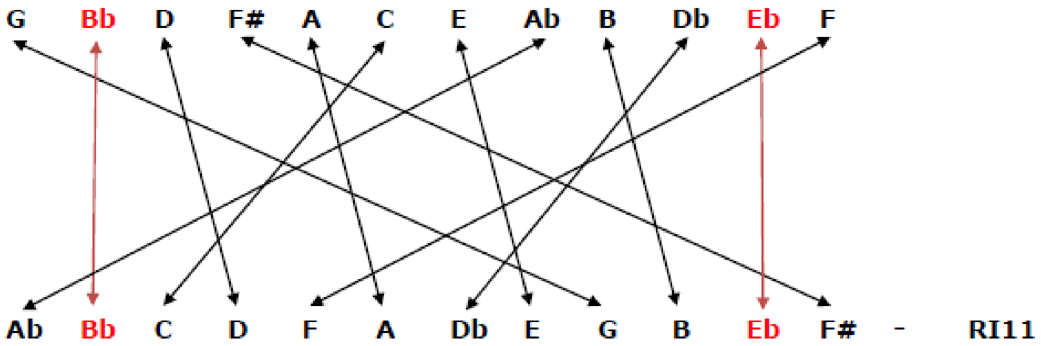


C: 12 – 0 – 0 = 12

A: 2/10/00

T: 12/000

It is observed that the only method that obtains differentiated results and, therefore, is useful in the classification of similarities, distances and groupings of the twelve-tone series is structural alignment (Table 6). The contrast model and the transformation do not obtain differentiated and significant results that allow a classification of similarities, distances and groupings of the series because they contain exactly the same vectors in their respective methods. The coincidences of placements of some notes in the series make sense only for structural alignment, as the parameters matches in place and out of place (MIPS and MOPS) make all the difference.

Based on the way used in the methods of classification of similarities in which the commonalities are the first parameters to be considered and then, if there is a tie, the parameters of the differences are considered, it is observed in table 6 that RI3 is the most similar series and closest to the original (O0) with the vector 4/8/00. In second place are the series with vector 2/10/00. In third place are the series with vector 1/11/00. The most dissimilar series and, therefore, the furthest from the original (O0) are those with the vector 0/12/00. This includes all transpositions of the original series and the remaining series that did not obtain differentiated and significant values of similarities.

Table 6: classification of similarities, distances and groupings of all 48 series of Alban Berg's Violin Concerto in comparison with the original series.

1	2	3	4
4/8/00	**2/10/00**	**1/11/00**	**0/12/00**
RI3	**I0**		All original series
	I2		All other series
	I4		
	I6		
	I8		
	I10		
	R1	**R2**	
	R5	**R4**	
	R7	**R8**	
	R11	**R10**	
	RI1		
	RI2		
	RI10		
	RI11		

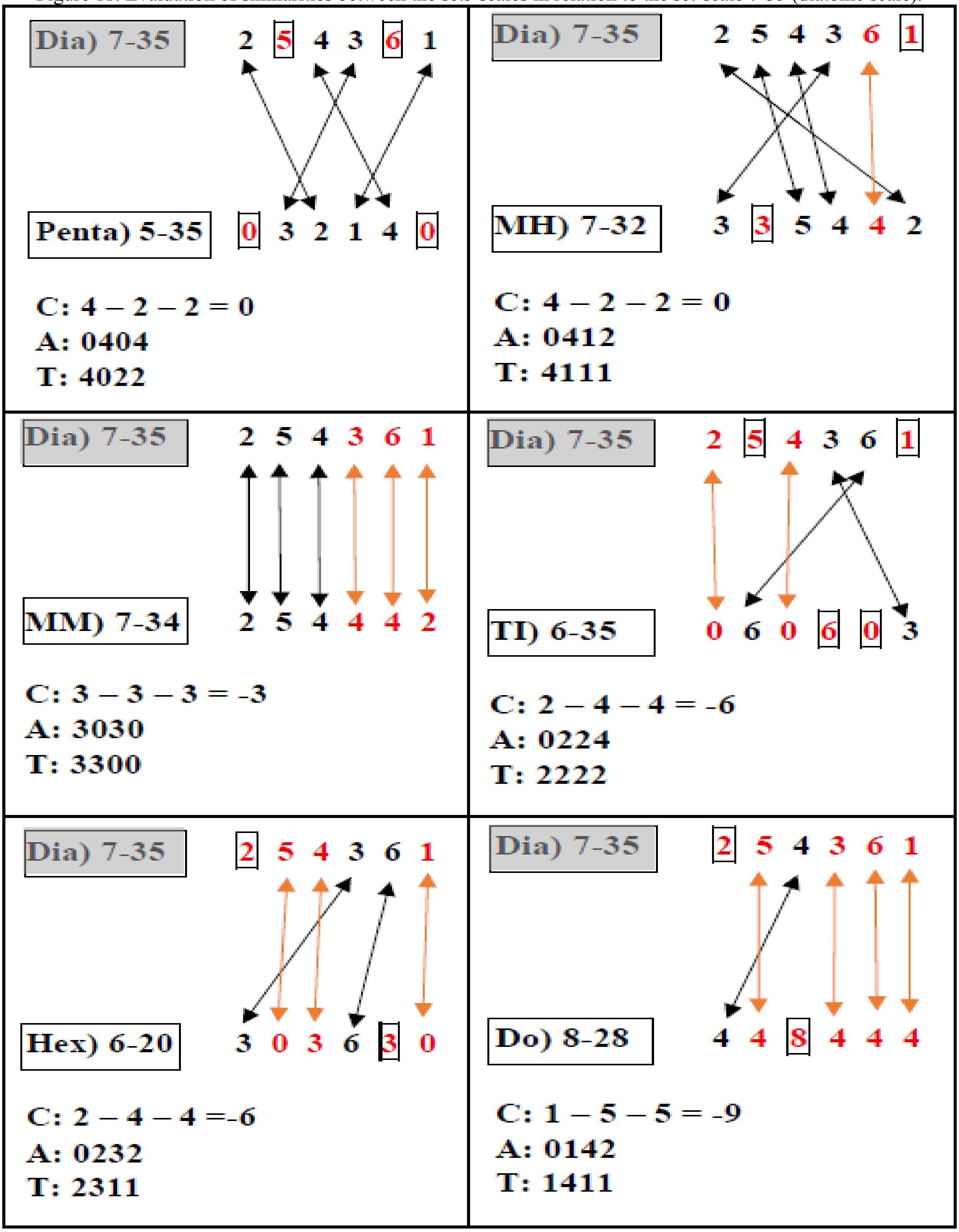
APPLYING THE CONTRAST, STRUCTURAL ALIGNMENT, AND TRANSFORMATION MODEL TO SOME REPRESENTATIVE SETS OF SCALES

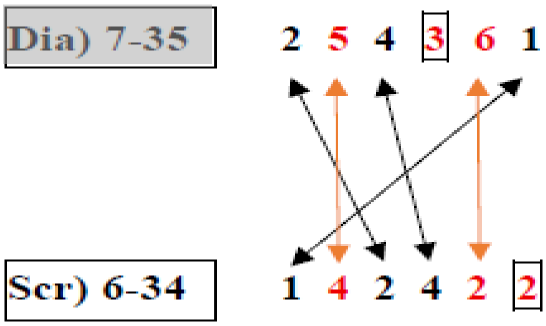
The sets chosen for comparison are those that refer to the diatonic (7-35), pentatonic (5-35), harmonic minor (7-32), melodic minor (7-34), whole tones (6-35), hexaphonic (6-20), diminished dominant (8-28) and Scriabin (6-34) scales. The prototype or referential set-scale is the diatonic set (7-35) from which all other set-scales are compared (tab.7). As in Forte (1973, p.46-60), similarities were classified according to the interval vectors of the *set-classes* rather than based on sets or notes. The black notes connected by black arrows indicate the interval vectors in common, whether they are in the same position or not. The red notes connected by red arrows indicate different vectors, but they are under the same positioning relationship within the vectors. The red notes within a square indicate different vectors that do not find a match in the other set. The vectors of the contrast model (C), structural alignment (A) and transformation are shown in fig.11.

Table 7: scale sets under consideration.

Set-classes **(sets)**	**Grades**	**Vectors**
Diatonic 7-35 (013568A)	**C C# D# F F# G# A#**	**254361**
Pentatonic 5-35 (02479)	C D E G A	032140
Harmonic minor 7-32 (0134689)	C C# D# E F# G# A	335442
Melodic minor 7-34 (013468A)	C C# D# E F# G# A#	254442
Whole tones 6-35 (02468A)	C D E F# G# A#	060603
Hexaphonic 6-20 (014589)	C C# E F G# A	303630
Domdim 8-28 (0134679A)	C C# D# E F# G A A#	448444
Scriabin 6-34 (013579)	C C# D# F G A	142422

Figure 11: Evaluation of similarities between the sets-scales in relation to the set-scale 7-35 (diatonic scale).





C: 3 – 3 – 3 = -3
A: 0322
T: 3211

Based on the comparisons of the interval vectors, it can be seen in Table 8 that the **contrast model** separates the sets representing scales into four groups. The sets of scales most similar to the 7-35 (diatonic) set scales are 7-32 (harmonic minor) and 5-35 (pentatonic) with vector (4 - 2 - 2 = 0). In second place are the scale-sets 7-34 (melodic minor) and 6-34 (Scriabin) with vector (3 - 3 - 3 = -3). In third place are the sets of scales 6-20 (hexaphonic) and 6-35 (whole tones) with vector (2 - 4 - 4 = -6) and in fourth and last place is set scale 8-28 (dominant, diminished) with vector (1 - 5 - 5 = -9).

In the **structural alignment** method, you can align your results with the same ordering as the contrast model. To do this, it is sufficient to consider first the number of commonalities and then the number of differences, as has been done since the beginning of this study in all methods and comparisons. Thus, the scales 7-32 and 5-35 appear in first and second place, respectively, because there are 4 commonalities, but what differentiates them is the number of differences. The 7-32 set-scale has 3 differences (3#) and the 5-35 set-scale has 4 differences (4#). The same is true of the 7-34 and 6-34 scale sets, which appear in third and fourth place, respectively. Both have 3 commonalities, but it is the number of differences that classifies them. The 7-34 set-scale has 3 differences (3#) and the 6-34 set-scale has 4 differences (4#). The 6-20 and 6-35 scale sets appear in fifth and sixth place, respectively, because they both have 2 commonalities, but the 6-20 scale set has 5 differences (5#) and the 6-35 scale set has 6 differences (6#). The last place belongs to the 8-28 set-scale with 1 commonality and 6 differences (6#).

In the **transformation** method it is possible to equate the same ordering of the contrast model with some displacements, as happened with the structural alignment method. The scale-sets 7-32 and 5-35 that appear tied in first place in the contrast model appear divided into two levels in the transformation method, levels 1 and 2, respectively. The scales 7-34 and 6-34 that appear tied for second place in the contrast model appear divided into two levels in the transformation method, levels 3 and 4, respectively. The scales 6-20 and 6-35 that appear tied for third place in the contrast model appear divided into two levels in the transformation method, levels 5 and 6, respectively. The 8-28 scale set that appears fourth and last in the contrast model also appears last in the transformation method, however, in seventh place.

In the **transformation** method, the classification of similarities of the scale-sets is also done first from the number of commonalities and then from the number of differences. For example, the scale sets 7-32 and 5-35, which appear in first and second place, respectively, have 4 interval vectors in common, but are differentiated by the number of differences. The 7-32 set-scale has 3 differences (3#) and the 5-35 set-scale has 4 differences (4#). This ranks them first and second, respectively. The scale sets 7-34 (melodic minor) and 6-34 (Scriabin) have 3 interval vectors in common, but they differ by the number of differences, the scale set 7-34 has 3 differences (3#) and the scale set 6-34 has 4 differences (4#). This ranks them third and fourth, respectively. The 6-20 (hexaphonic) and 6-35 (whole tones) scale sets have 2 interval vectors in common, but they differ by the number of differences, the 6-20 scale set has 5 differences (5#) and the 6-35 scale set has 6 differences (6#). This ranks them fifth and sixth, respectively. The scale set 9-28 (domdim) appears in seventh and last place because it has only 1 interval vector in common and 6 differences (6#).

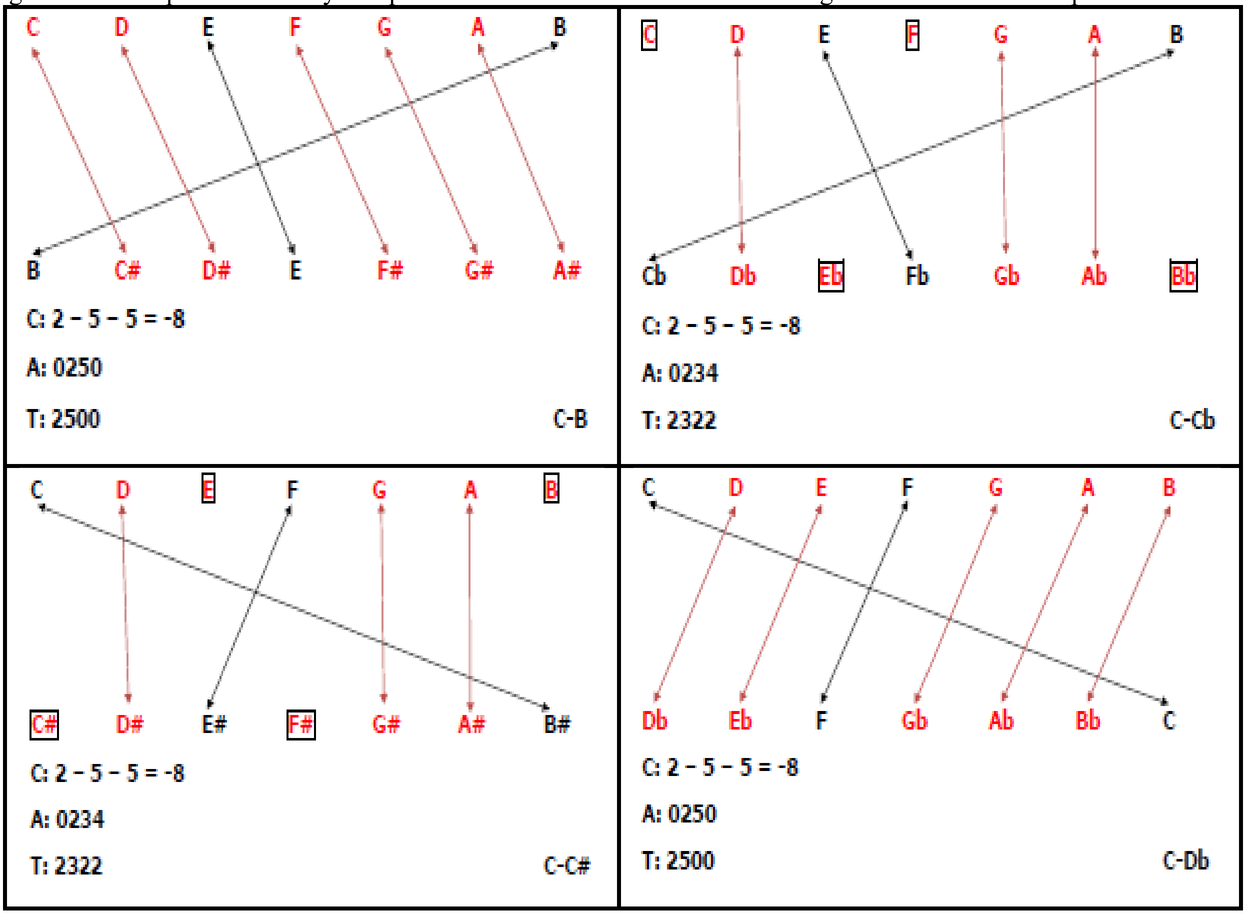
Table 8: similarities between interval vectors of scale sets.

Level	Sets	Contrast 7-35 (day)	Level	Alignment 7-35 (day)	Level	Transformation 7-35 (day)
1	**7-32 (MH)**	4 – 2 – 2 = 0	**1**	0412 (3#)	**1**	4111 (3#)
	5-35 (Penta)		**2**	0404 (4#)	**2**	4022 (4#)
2	**7-34 (MM)**	3 – 3 – 3 = -3	**3**	**3030 (3#)**	**3**	3300 (3#)
	6-34 (Scr)		**4**	0322 (4#)	**4**	3211 (4#)
3	**6-20 (Hex)**	2 – 4 – 4 = -6	**5**	0232 (5#)	**5**	2311 (5#)
	6-35 (TI)		**6**	0224 (6#)	**6**	2222 (6#)
4	**8-28 (domdim)**	1 – 5 – 5 = -9	**7**	0142 (6#)	**7**	1411 (6#)

CONCLUSION

The three methods of assessing similarities developed in cognitive psychology proved to be very useful in classifying the distances and groupings of the various musical elements. The relationships of similarities and proximities between the diatonic chords derived from the four tonal scales and the tonic chord C7M only needed the evaluation of the commonalities to define their classificatory order. However, the similarities and proximity relationships between the diatonic chords derived from the four tonal scales and the tonic chord Am7 had to evaluate first the number of commonalities and then the number of differences in order to define their classificatory order. The same occurred in the evaluation of similarities, distances and groupings of major and minor tonalities in relation to the prototype or referential tones of C and Am. By defining the contrast model as an example to be followed by the other methods, we confronted some disparities in the cases of the key of C#m in relation to the key of C and D#m (Ebm) in relation to the key of Am. In all other cases, it was possible to adjust the similarities, distances and groupings of all keys with the contrast model. As for the series of Alban Berg's Violin Concerto, it is noted that it was possible to divide them into four different groups. The only useful psychological method in the classification of similarities, distances and groupings of the twelve-tone series was structural alignment because there are no differentiated and significant results in the methods of the contrast and transformation model. The in-place and out-of-place matches (MIPS and MOPS) parameters made all the difference in the ranking. The sets representing scales also submitted the results of structural alignment and transformation to the results obtained in the contrast model, however, without any type of confrontation or disparity between them, except for the division of the groups into two, as happened with the previous categories: chords and tonalities. It is worth remembering that the vectors of four numbers and their ordering were elaborated by the authors of this article, and it is possible, obviously, to create other vectors and other ordering of them, including new methods of connection between musical elements. For example, instead of connecting the alignable differences of the structural alignment method based on the same interval degree, they could be connected based on the conduction of parsimonious voices by semitone, however, this was tried as an option for this study, but the results were not suitable for the equal temperament, i.e., the chords and enharmonic keys obtained differentiated classificatory results. It can be seen in fig.12 that the enharmonic keys C-B/C-Cb and C-C#/C-Db obtain different results in the methods of structural alignment and transformation when the different notes are connected by the shortest semitone path. This is one of the reasons why the contrast model was taken as an example for the classification of the other methods, it does not obtain differentiated results when enharmonic elements are compared.

Figure 12: Example of similarity comparisons between enharmonic tones using the shortest semitone path connections.



It is now suggested that new research applies the methods of evaluation of similarities developed in cognitive psychology in the classification of distances and groupings of other musical elements, such as other chords, other scales, other twelve-tone series, other sets and even sound objects, provided that comparable characteristics can be attributed to them. Finally, the main objective of this research was to suggest the methods of evaluation of similarities developed in cognitive psychology in the theoretical and practical formations of music, which, in a way, is one of the main objectives of interdisciplinarity, that is, to seek interactions and solutions in different areas of knowledge.

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