The manipulable trigonometric cycle: A tool for teaching trigonometric functions

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ABSTRACT

The present research aims to analyze the use of the Manipulable Trigonometric Cycle as a teaching resource to facilitate the teaching-learning process of trigonometry. It is justified by the reason that some teachers do not teach trigonometry in basic education, as it is considered Mathematics content that is difficult to teach and very difficult for students to understand. Therefore, it was decided to use the manipulable object as an auxiliary tool for teaching trigonometry, believing that it could be a very effective methodology for education. This investigation had a quantitative-qualitative approach, as the quantified data was obtained through the application of the Manipulable Trigonometric Cycle, and they were subsequently analyzed, interpreting the variations in the results and Questionnaires were used to obtain the data. The research sample were students from two classes of the second year of high school integrated into the technical course in agriculture at the Federal Institute of the North of Minas Gerais. Based on the assumption that Manipulable Materials help in the development of learning, stimulates creativity, imagination and autonomy, exercises concentration, promotes socialization, contributes to the construction of students' personality and logical reasoning. It is concluded that the teaching material can become a great ally for the teacher, helping him in the teaching-learning process of the students, thus contributing to an improved construction of knowledge, as long as it is explored properly, so that the student begins to build their own concepts.

Keywords: Mathematics, Trigonometry, Manipulable Trigonometric Cycle.

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INTRODUCTION

Trigonometry is a very important content of Mathematics, especially in the applicability of its concepts, so its teaching in basic education must be well worked so that students have a good learning. But this is not what the students think, because according to Thomaz (1999), the students find Mathematics very complicated, incomprehensible, distant and cold, being a cluster of exercises that they do for the sake of doing and do not know why or for what purpose of this discipline.

This students' perception of Mathematics is reflected in the results of international assessments, such as PISA (Programme for International Student Assessment), which according to INEP (National Institute of Educational Studies and Research Anísio Teixeira), has the initiative to evaluate students in the age group of 15, the age at which students are immersed in the teaching of trigonometry. This evaluation is done every three years and the last one took place in 2015. According to PISA, Brazil lost five positions compared to its previous edition, in 2012, falling from 58th to 63rd place, with only 70 countries participating. This position is due to three areas evaluated: Math, Reading, and Science.

It can be seen that the teaching of Mathematics in Brazil is regressing, and this may be due to the rejection that students have for the subject. As Reis (2005) has already pointed out, some of the causes are, the lack of motivation of the teacher when teaching and the lack of motivation of the students to learn, the idealization accepted by the students that Mathematics is difficult, the rigor of Mathematics, the negative experiences that the students had with this subject, the absence of relationship between the Mathematics taught in schools and the daily life of the students, the relationship that the teacher establishes with the students, the way of teaching and the way of evaluating.

Given these circumstances, this research presents a resource that will facilitate the teaching of trigonometry, in a playful and motivating way so that students, using the manipulable trigonometric cycle, promote a better understanding, visualizing and manipulating the material. This research is justified by the fact that some teachers do not teach trigonometry in basic education, including one of the authors, because it is considered a very difficult mathematics content to teach and very difficult for students to understand. Therefore, it was decided to use the manipulable object as an auxiliary tool for the teaching of trigonometry, believing that it can be a very effective methodology for education.

Thus, this research investigated how the use of the manipulable trigonometric cycle contributes to the teaching-learning process of trigonometry, facilitating the teacher's work in teaching and motivating students to learn.

To try to present an alternative for such investigations, this research has a general objective, to analyze the use of the Manipulable Trigonometric Cycle as a facilitating didactic resource in the teaching-learning process of trigonometry. The Specifics are:

- ✓ Construct a Manipulable Trigonometric Cycle, student version, at low cost, so that it can be reproduced in public schools, especially in state schools, where resources are reduced;
- ✓ Construct a Manipulable Trigonometric Cycle, teacher version, so that it can be reproduced by teachers, in order to present the content collectively;
- ✓ Visualize the values of trigonometric functions in a manipulative way in the cycle;
- ✓ Identify the advantages and disadvantages of using the Manipulable Trigonometric Cycle.

THEORETICAL FRAMEWORK

TRIGONOMETRY TEACHING

Based on the historical context, the importance of trigonometry can be seen. Taking this content of Mathematics to teaching, it is observed that trigonometry has been evolving due to the needs of civilizations, its transmission in teaching is also gradual, since,

> Trigonometry, as we can see, is a very important content, which was developed in order to calculate distances using the measurement of angles as a basis. This content is initially introduced to the student in Elementary School II, with the trigonometric relations sine, cosine and tangent, in addition to the Pythagorean Theorem. And later in high school with the law of sines, law of cosines, area of any triangle, trigonometric circumference, among others, further deepening the concept of Trigonometry and its applicability. (Júnior, 2017, p. 32).

But, in order for all this trigonometry content to be taught in such a way that the student feels attracted and understands all the importance of the subject, Lins (2016) argues that our school system today needs to adapt, promoting knowledge and skills that enable us to share in the globalization process.

This problem happens more frequently in high school, where students are almost ending their school life and eagerly preparing for their entry into the job market, but for the National Curriculum Parameters for High School (PCNEM) learning is much more important, as the text points out:

> In high school, the final stage of basic schooling, Mathematics should be understood as a part of human knowledge essential for the formation of all young people, which contributes to the construction of a worldview, to read and interpret reality and to develop skills that will be required of them throughout their social and professional lives. (Brasil, 2000, p.111).

Taking all this consideration into the teaching of trigonometry today, the (PCNEM) also point out that it is traditionally "presented disconnected from applications, investing a lot of time in the algebraic calculation of identities and equations to the detriment of the important aspects of trigonometric functions and the analysis of their graphs". (BRASIL, 2000, p. 121-122). This may be one of the many indications of students' distaste for the teaching of Mathematics, where they are only presented with calculations and numbers, without any approach to the why and why of so many calculations.

One of the problems in the teaching of mathematics is the lack of interest of most students in school activities. Prediger, Berwanger and Mörs (2009) conducted research with teachers of the final grades, and they pointed out that there are still students who are interested, study and perform challenging activities in schools, but there are those who are not interested in anything, especially in activities that require logical thinking, such as the contents of Mathematics.

The causes of the problems that are present in education today are several, and one of them is pointed out by Conceição, Mendes and Borges (2015), where the teacher is one of the causes of the classes being boring, causing this lack of interest. In the authors' research, students suggest that math classes should be more interactive and dynamic.

The traditional way of teaching does not arouse interest in students, because students with access to so many means of communication and technologies are disinterested and unmotivated to study in an outdated way. It is not enough for teachers to just master the content of specific mathematics, they also need to be up to date with current educational means, interacting with some technological means and new teaching methods in their classes. (Conceição, Mendes and Borges, 2015).

In order for students to truly learn, it is necessary to know their interests, give them the opportunity to question, disagree, create hypotheses, make assumptions about the level of knowledge they have already learned, participate in classes and in the entire educational process, committing themselves to the school and to their own social education (Bitencourt and Batista, 2011).

It is notorious that the problems encountered in the teaching of Mathematics do not come from just one cause, but from several, and they can be noticed by teachers in their practice in the classrooms.

In the teaching of Mathematics, especially trigonometry, which requires more reasoning and analysis, is where the greatest losses in the quality of teaching occur. The students start to create a certain accommodation, so that, even if the teacher has innovative and motivational methodologies, the students are unmotivated and have an attitude of rejection of the discipline.

In the final grades is where more problems occur in teaching, since there is a lack of motivation and lack of interest of students in studying, there are few practical activities for some contents, a lot of difficulty in interpreting students, lack of a support network for teaching in the classroom, such as reinforcement classes for students who are having difficulties, lack of conditions for better planning and preparation of classes, such as teaching material and resources. (Prediger, Berwanger and Mörs, 2009).

Nowadays, there is a lack of communication between the mathematical language and the Portuguese language, in which students claim that they are either good in one discipline or in the other, creating a distance between the two disciplines. The lack of Portuguese language in the subject of Mathematics means that students do not interpret and understand mathematical concepts, only memorizing and executing them in a mechanized way. (Conceição, Mendes and Borges, 2015).

Therefore, if young people continue to have this perception that learning mathematics is just memorizing their formulas, without understanding all its fundamentals and without being aware that it is significant knowledge, students will continue to think that it is just an important subject to pass the year and in the future have a good job and not important in itself.

MANIPULABLE TEACHING MATERIALS

After a brief approach to some of the causes of students' lack of interest in studying Mathematics and, primarily, in the content of Trigonometry, one should look for some resource to improve the teaching of Trigonometry, as it is a very important content both in everyday life and in the school context. Many difficulties have been observed in the students' learning in the laws of Trigonometry, difficulties caused by the vision developed by the teacher in the classroom, distant from the student's reality, preferring the technique to the detriment of meaning and applicability. (Souza, 2018).

Therefore, in order for the teacher to achieve all this interest and curiosity of the students to the content of Mathematics, it is suggested by this research the Manipulable Didactic Materials, which are "real objects that the student is able to touch, feel, manipulate and move, objects that represent an idea. For many, a well-conducted activity must go through manipulation, representation and symbolization, which would be the springboard to reach abstractions." (Scolaro, 2018, p. 11).

Manipulable materials were introduced into education because of Pestalozzi and Froebel in the nineteenth century, and in the early twentieth century it was due to Montessori and Decroly (Fiorentini; Miorim, 1990). Pestalozzi argued that education should begin with the perception of concrete objects, carrying out concrete actions and experiments. In Brazil, these didactic resources appeared around 1920, a time marked by the emergence of the empiricist-activist, a trend in the teaching of Mathematics in this period (Nacarato, 2005). So it was from there that the idea of manipulation and visualization of concrete objects, exercising practical activities such as measurements, counts and comparisons, resulting in the assimilation of students from theory to practice.

Many teachers use only the textbook as a work tool, but the illustrations of the manipulable materials in the books do not replace the materials themselves, since, with them, in a Mathematics laboratory, it would be better to visualize the proposed problems (Santos, 2011). Therefore, it is important to use concrete materials, but there are some caveats, because,

> Generally, Mathematics teachers use manipulable material as support for the student's learning difficulties in a given subject, which evidences the strong influence of the New School movement, which involved the use of this type of material with the objective that the student can learn by doing. (Santos, 2011, p. 17).

The New School movement was a mobilization of European and North American educators, around the nineteenth century, which suggested a new understanding of the needs of childhood, aiming at a renewal of the mentality of educators and pedagogical practices, in addition to having the intention of placing the student at the center of the educational process. Educators defended this renewal, due to the accommodation in which children and young people were forced to follow traditional methods. The New School arrived in Brazil in the 1920s with the reforms in education, in several Brazilian states, it sought modernization, democratization, industrialization and urbanization of society. (Menezes, 2001).

Therefore, this resource is important because they are "playful materials, pedagogically structured for the learning of the various mathematical contents, since, through their use, they provide better interaction and socialization among students, contributing to a greater exchange and sharing of ideas among them". (Camacho, 2012, p.27). In addition to the benefits that the teacher will achieve, the use of Didactic Materials (DM) in the teaching of Mathematics, the students, also, with a good use of the classes, make that,

> The use of DM in the classroom expresses a great importance in relation to the construction of knowledge, because each student has his or her own way of learning, and using alternative means this teaching-learning process completes in a significant way, makes it for the student not to learn mechanically, nor to learn non-playfully, but leads him to value action, manipulation and experimentation. (Oliveira; Freire; Santos, 2011, p.07).

In this way, for a better development of the classes, when the teacher is going to teach them, two types of manipulable materials can be developed, namely:

> Static manipulable material: a concrete material that does not allow transformation by continuity, i.e. alteration of its physical structure from its manipulation [...]. The dynamic manipulable material: concrete material that allows transformation by continuity, that is, the physical structure of the material changes as it undergoes transformations, through operations imposed by the subject who manipulates it [...]. (RODRIGUES; Gazire, 2012, p. 190).

Thus, according to Rodrigues and Gazire (2012), dynamic manipulable material is more advantageous than static manipulable material, since it facilitates a better perception of properties, as well as the realization of rediscoveries that can guarantee a more significant learning in the study of trigonometry.

In this form of interaction of the students, solving the activities, reflection and improvement of cognitive schemes arise during the exploration of the materials, such as knowing how to do, knowing how to question, knowing how to say, knowing how to argue, such as knowing how to live together and work collectively. This knowledge contributes to the construction of autonomy to make choices and decisions, developing citizenship actions (Santos, 2011).

These manipulable handmade resources provide, at any level of education, the full development of mathematical content, but this is relative to the subject and not as a form of recreation (Souza, 2018). Therefore, teaching can often begin with the concrete, that is, revealing the potential of sight and touch, which theory often does not reach, not that they are not important, but they are not sufficient for the student's learning (Santos, 2011).

When choosing this teaching methodology, the educator should first, according to Oliveira, Freire and Santos (2011), answer four questions, namely: Why use manipulable teaching materials? What materials will you use? When and how should it be used? And what concepts will be explored? Using these questions as a reflection will make it much easier to plan lessons when using a manipulative object. In this way, the classes, in addition to being well planned, will be diversified, because,

> The transformation of the classroom into a teaching and learning laboratory will generate a new school environment. The insertion of the student as an active subject in the school process will enable a new reality in the school. Thus, it is necessary to develop a Methodology for the Teaching of Mathematics that aggregates technological development to motor development, associating research in the classroom with electronic resources and handmade didactic materials built by students, in order to better understand the curricular contents. (Souza, 2018, p.16).

"Manipulable materials can be a great ally in Mathematics classes, but not to replace the teacher, but to integrate into their classes; in addition, there is a need for teachers to find theoretical support to support their planning" (Santos, 2011, p.10). Therefore, it cannot be sure that a manipulable object, as a didactic resource, is the solution to the problems of teaching trigonometry, but a very viable attempt due to its importance that can "provide students with greater interaction in the classroom, autonomy and security, creativity, responsibility, motivation, understanding of mathematical concepts, assimilation of content and construction of their own knowledge". (Oliveira; Freire; Santos, 2011, P.08).

METHODOLOGY

The present research had a quantitative-qualitative/qualitative-quantitative approach, since the quantified data were obtained in the construction and application of the manipulable object, and later an analysis of them, tending to interpret the variations of the results.

This type of research has its importance, since it has four methodological definitions, namely: *triangulation*, which seeks to compare and contrast statistical data with qualitative data obtained simultaneously; *embedded*, in which a set of quantitative data supports the other qualitative data or vice versa, both also obtained simultaneously; *explanatory*, where qualitative data are used to clarify quantitative results or vice versa and the *exploratory*, of which qualitative results contribute to the development of the consecutive quantitative method.(CRESWELL; CLARK apud SOUZA; KERBAUY, 2017).

The nature of the research was applied, as she "It aims to generate knowledge for practical application, aimed at solving specific problems. It involves local truths and interests." (Scott; Gerhardt, 2009, p. 35).

The objectives of the research were exploratory, since it provided a "general, approximate view of a given fact. This type of research is carried out especially when the chosen topic is little explored and it becomes difficult to formulate precise and operationalized hypotheses." (Gil, 2008, p. 27). It was also descriptive, since its primary purpose is to describe the characteristics of a given population, phenomenon or the relationship between variables. In addition, explanatory research was used, since it is the one that most investigates the study of reality, since it explains reality and the why of things. (Gil, 2008).

The research design was also based on bibliographic research, as it "is developed from material already elaborated, consisting mainly of books and scientific articles". (Gil, 2008, p. 50). This was the case of this research, as the idea originated from Oliveira's (2015) research based on its construction. The difference in the construction of the material was in the way the rulers are handled to obtain the trigonometric ratios, where in this research it is not necessary to change rulers, with just one the student finds the ratios of the sine, cosine and tangent. But with Oliveira's (2015) construction, students can also obtain the ratios of the cosecant, secant, cotangent.

The research was carried out at the Federal Institute of Northern Minas Gerais (IFNMG) Salinas Campus, located at Fazenda Varginha, km 02 of the MG-404 Highway, CEP no. 39560-000, Salinas/MG. The target audience of this research were students from two classes of the second year of high school integrated into the technical course in agriculture.

The data were obtained during three 50-min classes of a diagnostic questionnaire, to obtain the students' previous knowledge, followed by a workshop with an activity using the manipulable materials, which is the phase of the application of the research, and at the end an evaluative questionnaire was given to the students, which aimed to evaluate the performance, success, interest, difficulties and importance in the use of the object in trigonometry classes.

ANALYSIS AND DISCUSSION

In this chapter, the analysis and the results obtained in the application of the research will be presented, in the 3 stages: the Diagnostic Questionnaire, the Workshop and the Evaluative Questionnaire, Appendix D.

ANALYSIS OF THE DIAGNOSTIC QUESTIONNAIRE

The Diagnostic Questionnaire was developed with the intention of analyzing the profile of students and verifying their interest in studying Mathematics and their willingness to use manipulable materials in classrooms. Below, I present the quantitative graphs, in view of the students' answers. A total of 51 students participated in this questionnaire. It was found that most of the classes are composed of male students, and the age of the students is between 15 and 17 years old.

Students were asked if they like to study Mathematics and 52% of students answered that they do not like it and 48% answered that they do**.** It can be seen that most of them do not like to study this discipline

Another question was whether students have difficulties in studying mathematics, and most students answered yes**.** This result can be analyzed with some of the students' answers in the Diagnostic Questionnaire, according to some speech extracts; *"I can't interpret mathematical expressions,"* another student justifies, *"I have a big problem interpreting mathematical problems."* This answer is for the reason, according to Conceição, Mendes and Borges (2015), there is a division between the mathematical language and the Portuguese language, because students who like one subject do not like the other, harming the interdisciplinarity of the contents and harming mathematics in the interpretation of the problems.

Most of the students answered that they also have many difficulties in the contents of Trigonometry, in which most find this content complicated, especially in the conversion of angles into radians or vice versa and in the complex formulas of Trigonometry.

Another question was whether the students know any type of manipulable material, and 51% of the students answered yes and 49% answered that they do not**.** It turns out that a little more than half of the students know some kind of manipulable material. The students who answered that they already knew it, also answered what the materials were, in which most were the Rubik's cube, golden material and geometric shapes. They said they used these materials in elementary school, where students reported that they consider it very important as it makes it easier to understand the content.

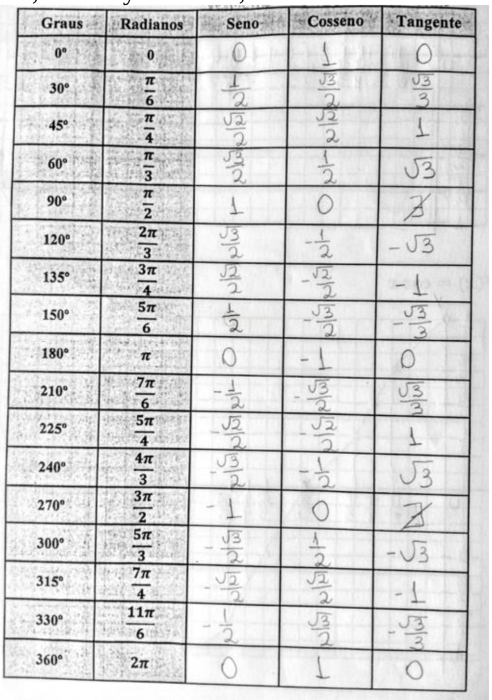
The last question of the Diagnostic Questionnaire was about the importance of using different methodologies in mathematics classes such as software, applications, video lessons, among others. The vast majority of students, 98%, think it is important to use different methodologies in math classes**.** It is observed that the majority thinks it is important, in which analyzing this percentage with some of the students' answers in the Diagnostic Questionnaire, we have justifications such as: *"changing the way of teaching in some classes helps in learning"; "The class is more enjoyable, dynamic and less tiring and also helps to increase interest in the content."* This answer is discussed by Rodrigues and Gazire (2012), where they address that different methodologies in classrooms make Mathematics more dynamic and understandable, bringing theory closer to the practice of content.

ANALYSIS OF THE ACTIVITY CARRIED OUT IN THE WORKSHOP

The workshop was held at IFNMG – Salinas Campus. In the first class of the workshop, the Trigonometric Manipulable Cycle was presented in the teacher version, explaining how it is used and what it is used for. In the second class, the first part of the Activity/workshop was applied, asking the students to fill in the trigonometric table with the values of the notable arcs of the sine, cosine and tangent, using the Manipulable Trigonometric Cycle, student version. In the third class, the students did the second part of the Activity, Appendix C, in which they used the values of the notable arcs identified in the Trigonometric Manipulable Cycle and filled in the table of the first part of the activity, to construct the graphs of the functions $f(x) = sen\, x,\ f(x) = cos\, x$and .$f(x) = tan\, x$

During the first stage of the Activity, the difficulty that the students had was in the manipulation of the Cycle, and in identifying which ruler they should use to find the values of sine, cosine and tangent, since it was something new for them, but to solve this it was necessary to reinforce the explanation, individually, of how it is to be used. Another difficulty was to recognize which were the axes to find the values of the trigonometric ratios, but it was also a problem solved individually. In Figure 5 we can see a table duly filled in by one of the students, in the first part of the workshop activity. During the application of the workshop, it was noticed that the colors of the Trigonometric Cycle helped the students a lot, as it differentiated the trigonometric axes, making it easier to fill in the table.

Figure 5: Trigonometric table, made by a student, filled with the values of sine, cosine and tangent

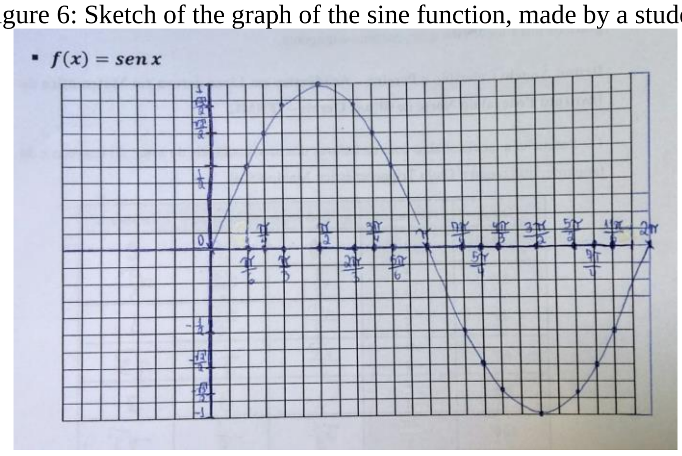


Graus	Radianos	Seno	Cosseno	Tangente
0°	0	0	1	0
30°	$\frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$
45°	$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1
60°	$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$
90°	$\frac{\pi}{2}$	1	0	$\nexists$
120°	$\frac{2\pi}{3}$	$\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$	$-\sqrt{3}$
135°	$\frac{3\pi}{4}$	$\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{2}}{2}$	-1
150°	$\frac{5\pi}{6}$	$\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{3}}{3}$
180°	π	0	-1	0
210°	$\frac{7\pi}{6}$	$-\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$
225°	$\frac{5\pi}{4}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{2}}{2}$	1
240°	$\frac{4\pi}{3}$	$-\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$	$\sqrt{3}$
270°	$\frac{3\pi}{2}$	-1	0	$\nexists$
300°	$\frac{5\pi}{3}$	$-\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$-\sqrt{3}$
315°	$\frac{7\pi}{4}$	$-\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	-1
330°	$\frac{11\pi}{6}$	$-\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{3}}{3}$
360°	2π	0	1	0

Source: Workshop Activity

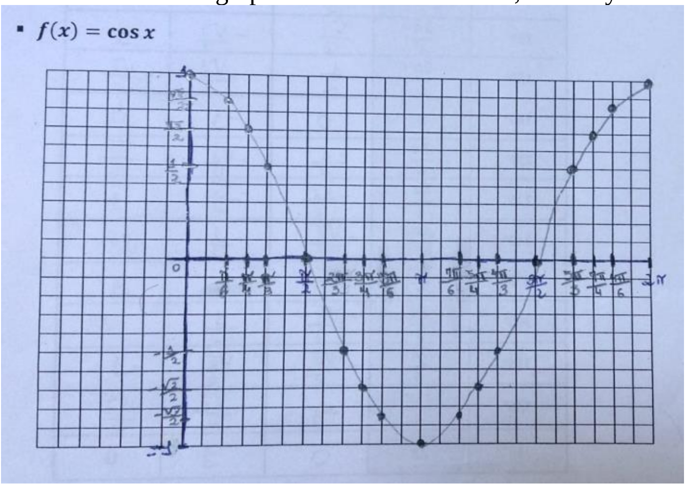
During the second part of the workshop, some difficulties arose from the students in the construction of the graphs, especially in the $f(x) = tan\,x$, since it was the first time that the students visualized and built the trigonometric graphs alone without the teacher having previously presented them on the board. The first difficulty was to assemble the Cartesian planes in the meshes, because the values are very different from what they were used to in other functions, another problem arose when they went to indicate the points of the function in the plane, since some students filled in the table wrong, causing the function to be incorrect. These small problems were solved with the students individually, during the workshop, and this made it possible for the students to elaborate the entire proposed activity. In Figure 6, Figure 7, and Figure 8, we see correctly plotted graphs of some students.

Figure 6: Sketch of the graph of the sine function, made by a student



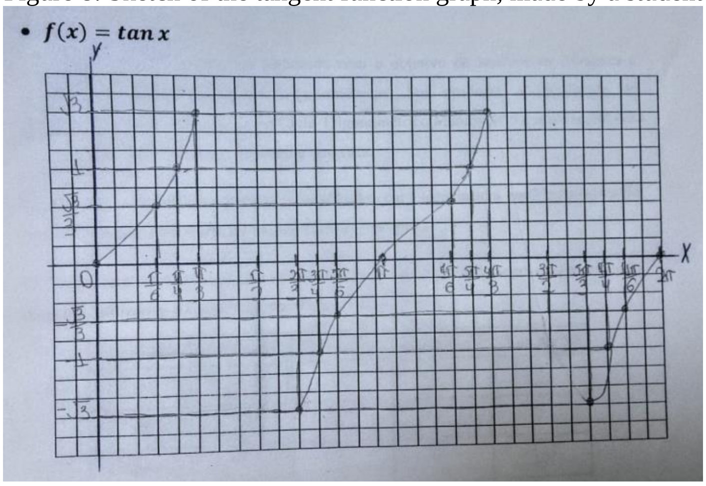
Source: Workshop Activity

Figure 7: Sketch of the graph of the cosine function, made by a student



Source: Workshop Activity

Figure 8: Sketch of the tangent function graph, made by a student



Source: Workshop Activity

Despite some problems in the execution of the Activity in the workshop, the results were very good, as the students were able to achieve the intended objectives. And in order to analyze more precisely the importance of this methodology, we will analyze the Evaluative Questionnaire below.

ANALYSIS OF THE EVALUATIVE QUESTIONNAIRE

The Evaluative Questionnaire was elaborated with the intention of evaluating the use of the Manipulable Trigonometric Cycle, after the completion of the workshop activities. A total of 55 students participated in this questionnaire.

In the Questionnaire, students were asked if they had any difficulty in using the Manipulable Trigonometric Cycle, and 73% of the students answered that they did not have difficulty and 27% answered yes**.** It can be seen that most of them did not present problems in the manipulation of the Cycle. The students who presented difficulties justified why, and most reported that they were having problems at the beginning of the handling of the Trigonometric Manipulable Cycle, because they did not know that type of material, but as soon as they learned they were able to do the activities. The students also reported that they had a bit of trouble plotting the tangent chart.

Another question was whether the students aroused interest and curiosity in the content covered, based on the method applied in the activity done during the workshop, and 76% of the students answered yes and 24% answered no. It can be seen that most of them aroused interest, in which this percentage can be analyzed with some answers of the students in the Evaluative Questionnaire, according to some excerpts of speech: *"I had a different view of what I thought about the sine, cosine and tangent",* another student commented that: *"It encouraged me to want to learn more about the content",* another student said that: *"I was impressed with the elaboration of the material and how easy it was to do the functions."*

It was observed that from these speech extracts that according to Camacho (2012), this methodology aims to facilitate the understanding of mathematical contents, since it awakens motivation and encourages learning, also awakens students' curiosity, concentration and creativity.

In the questionnaire, the students answered if the use of the Manipulable Trigonometric Cycle helped in the teaching of trigonometric functions and, unanimously, 100% of the students answered yes**.** It can be seen that all students think that the Cycle helps in teaching, in which this percentage can be analyzed with some answers from the students in the Evaluative Questionnaire, where some of the students' comments were that the classes are more diverse, interactive and not tiring, another comment was that it is better than just listening to the teachers.

The students were also asked if they would like to know and use Manipulable Materials more often in the study of Mathematics, and 96% of the students answered yes and 4% answered no**.** It is observed that most of them would like to know and use other manipulable materials, in which we can analyze this percentage based on some extracts of the students' speech: *"Mathematics always needs to seek different methods",* another student commented that: *"he would like to know because he has many difficulties in the subject".*

It was also asked to the students that from the use of the Manipulable Trigonometric Cycle, they think that the use of different methodologies in mathematics classes such as software, applications, video classes, among others, contribute to the teaching-learning process, and again, unanimously, 100% of the students answered yes.

It can be seen that all students think that these differentiated methodologies contribute to the teaching of Mathematics, because according to Santos (2011), the purpose of these methodologies is to assist in the teaching and learning process, showing that the resolution of trigonometric problems can be worked in an attractive, constructive, interesting and motivating way. And so were the students' responses, as we can see in an excerpt of speech: *"I am interested in different things, because it makes the classes more attractive and not repetitive."*

CONCLUSION

This research sought to show a different way of learning to obtain the necessary data for the construction of sine, cosine and tangent graphs, through the Manipulable Trigonometric Cycle, since the students' perception of Mathematics was not very good. The handling of the Manipulable Material allowed the students to make several observations, enabling them to solve the activities more quickly, reducing the time of application of the content, favoring the teacher in the execution of all the curricular content.

Thus, we can say that we were able to answer the question of this research, because several contributions of the Trigonometric Manipulable Cycle were found in the teaching of the Sine, Cosine and Tangent Functions. As for the objectives of this research, it was possible to construct the Cycles both the version for the teacher and the version for the students. And from it it was noticed that when developing the activities with Manipulable Trigonometric Cycle, it was observed that it is a facilitating means in the teaching-learning process, seen as something interesting and motivating to the students, mainly because it promotes a better visualization of the Functions, which was one of the objectives.

With the realization of this proposal, we realized the importance, as teachers, of one of the main tools in the teaching-learning process, planning. Taking the time to think about each stage of the class to be taught, selecting the didactic tools to be used, and choosing the topics to be worked on, enabled moments of pleasure and satisfaction, added to the students' participation.

This research showed an experience that presented significant results for the classes, with the application of a playful activity in the teaching of mathematics in a dynamic and creative way, as proven by the numbers in this sample. Other works can be carried out with proposals or suggestions that inspire teachers to expand new ideas of activities to facilitate the learning of their students. Thus, this research leads us to the constant search for strategies that, like these, can meet the desires of students in search of quality in the teaching-learning process.

Finally, it is important to emphasize that the use of new methodologies is not the act of solving the problems involved in the teaching of Mathematics. And to achieve the main objective of student learning, schools and teachers must always reflect and seek new strategies to be used in the classroom.

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