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Negative binomial distribution and multiplicities in proton collisions

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ABSTRACT
The production of particles in proton collision experiments provides fundamental information about the mechanisms of conversion of the initial energy of protons into a given number of secondary particles by measuring their Multiplicity Distribution in strong nuclear interactions. In this context, the Negative Binomial Model has been extensively used in theoretical studies and in parameterization of experimental information, making it important, therefore, to know the hypotheses involved in the elaboration of the model, in order to provide adequate application and interpretation of results. Thus, we make a discussion of the Negative Binomial Model based on the performance of a classical random experiment, which leads and assists in obtaining the analytical expression of the Probability Function. We also express the Probability Function in terms of the Gamma Function, the multiplicity variable and the mean multiplicity adopting a mathematical procedure not found in the specific literature. Implications of the application of the model in the study of Multiplicity Distributions in collisions between protons are discussed.

Keywords: Multiplicity Distributions, Negative binomial distribution, Multiple Particle Production, Proton Collisions.

1 INTRODUCTION

Protons are particles that make up the atomic nucleus and have a complex internal structure composed of other particles called quarks and gluons, which are some of the elementary particles that make up matter [1,2]. One of the purposes of understanding the structure of matter and its interactions, protons are accelerated into beams and collided at collision energies of the order of 109 - 1012 electron volts [2]. In this order of collision energy, quarks and gluons interact predominantly through the strong nuclear force [1,2] and one of the results of collisions between protons, and the consequent interactions between quarks and gluons, is the creation of a set of new particles, a process called multiple particle production [3]. In general, we denote the number of particles produced by () and denote their probability of production. In this article we discuss the set of particles produced in collisions between protons, which is a fundamental experimental physical quantity for understanding the mechanisms of strong nuclear interactions of quarks and gluons [3]. Experimental data on MD were made available through the CERN ISR experiments (1984), CERN UA5 Collaboration (1987-1989) and the E735 experiment performed at Tevatron (1998) [4,5,6,7]. Currently, the Large Hadron Collider $"N"N = 2,4,6,\ldots n...$"$P(N)$"$\{N,P(N)\}$ (LHC), which is the largest particle accelerator in history [2], has made new experimental information about DM available. Details about the experiments at the **LHC** can be found in the Refs. [2,8]. In order to understand the mechanisms of particle production and with experimental data on DM in a wide range of collision energy, mathematical models and/or Probability Distributions are necessary to carry out studies and investigations [9,10,11,12,13,14,15,16]. In this context, the Negative Binomial Probability Distribution (DBN), also referred to as Pascal's Distribution, has been extensively used to study and parameterize DM's observed in proton-antiproton collision experiments [15,16,17]. The wide use of the Negative Binomial Model in several analyses motivated the development of this work, which has two main purposes. One of them is to present a didactic discussion of the Negative Binomial Model and, secondly, to encourage new research on multiplicities due to the relevance of the theme for the understanding of the dynamics of particle production. Thus, due to the didactic proposal of the work, in the next section we present necessary and directed concepts to define Probability Distribution, reviewing the Bernoulli Distribution with the purpose of defining the notation to be used and also discussing aspects about independence of events. In Section 3 we present and discuss our proposal to obtain the DBN, exploring a classic random experiment. We also express the analytical formula of DBN in terms of the Gamma Function and the variable, as often used in applications to describe Multiplicity Distributions. In Section 4, the final considerations are presented."n"

2 ON THE DEFINITION OF PROBABILITY DISTRIBUTION

We assume that readers have knowledge of probability concepts such as random experiment, events, sample space, and classical definition for calculating probabilities. In probabilities work, we define the events and then calculate the probability of these events occurring using the classical definition for probability calculation [18,19]. Based on the example discussed in Ref. [18], consider the random experiment*:* "toss of two honest coins*", which has as its sample space the set {*cc, cr, rc, rr}, where "c" represents heads and " r" tails. Calculating the probabilities of events:

$A =$ Occurrence of no face;

$B =$ Occurrence of a face;

$C =$ Occurrence of two faces;

Get:

$$P(A) = \frac{1}{4}, \ , \ . \qquad (1) P(B) = \frac{1}{2} P(C) = \frac{1}{4}$$

A formal mathematical procedure can be adopted for the calculation of these probabilities. This procedure consists of representing events by numbers instead of words. This can be done by introducing the concept of *Random Variable* (VA) and defined, in this specific example, as:X

X: Number of faces. (2)

Since we are now interested in the representation of events by numbers, and no longer by words, we then observe that , can take the values = {0,1,2} to represent the events , and , and with the following correspondences:XXA BC

$X = 0$ → Occurrence of no face (Event);A

$X = 1$ → Occurrence of a face (Event);B

$X = 2$ → Two-Faced Occurrence (Event). C

In general notation we write:

$$X = \{x_1, x_2, \ldots, x, \ldots \ldots\} \qquad (3)$$

to indicate the possible EV values. Schematically:X

Table 1 - Possible values for the random variable, defined in (2), and respective probability values. X

X	$P(X)$	Corresponding event
0	1/4	No face ()A
1	1/2	One face ()B
2	1/4	Two-faced ()C

In the two numerical columns of Table 1, we identified the set , which is called the Probability Distribution of VA [20], which is easy to represent graphically. We observed that the treatment of AV is naturally more comprehensive [18,20]. We have here a discussion aimed at introducing the concept of the Probability Function.$\{X, P(X)\}X$

2.1 PROBABILITY DISTRIBUTION - BERNOULLI MODEL

The repetition of Bernoulli's successive essays is the source of several interesting theoretical problems [21], giving rise to other models such as the Binomial, Geometric and even the Negative Binomial. Thus, we address some relevant aspects that lead to obtain the analytical expression of the Probability Distribution Function in this model and also to define the notation to be used. Every random experiment in which we distinguish only two possible, and mutually exclusive, outcomes is called an experiment or Bernoulli assay. Mutually exclusive events are events that cannot occur together, . An example is the performance of the random $A \cap B = \emptyset$*experiment "flipping a coin",* of course, there are only two possible outcomes that are mutually exclusive, i.e., the occurrence of heads excludes the possibility of the occurrence of tails and vice versa. Now consider a single performance of any experiment classifiable as a Bernoulli experiment and in which we define, by convention, one of the results as success (S) and the other as failure (F). We indicate the value of the probability of success by the digit and the value of the probability of failure by pq**.** Because they are mutually exclusive events, it follows that . As an example, we use the aforementioned experiment, $p + q = 1$ *"flip a coin",* we define the occurrence of heads as success (with probability) and the occurrence of tails as failure (). Note that the definition of success is arbitrary. As mentioned, the VA should numerically express the possible outcomes and can be defined in the Bernoulli model [18] as:$p = 1/2$ $q = 1/2$

$$X\text{: Number of successes in a single performance of a Bernoulli experiment.} \quad (4)$$

Since the experiment is performed only once, the VA assumes the values equal to 1 and 0, where it indicates the occurrence of the successful result and indicates the occurrence of failure. Thus we can write$XX = 1X = 0X = \{0,1\}$*: P(S)=)= and* $P(X = 1$P(F)= p= $P(X = 0)$, q *in tabular form:*

Table 2 – Tabular representation of the results of a Bernoulli experiment.

X	$P(X)$
0	q
1	p

The analytic expression representing the Probability Function in the Bernoulli model is then written in the form:

$$P(X = x) = p^x q^{1-x}, \quad (5)$$

naturally producing the values indicated in Table 2. Note that the Probability Function is defined as the function that assigns to each value assumed by the AV the probability of the corresponding event [18].

2.2 INDEPENDENT EVENTS AND MULTIPLICATION OF PROBABILITIES

The multiplication of the values of the probabilities, and , which can result from the independence between several realizations of the same random experiment, is a necessary condition for the deduction of the analytical expression of the Probability Distribution Function in the Negative Binomial Model, from a generalization of the Bernoulli Model. In general, discussions involving independence between events are addressed along with discussions about conditional probability [18,19]. Thus, consider two events and e that belong to the same sample space. A conditional probability of the event occurring is defined by knowing that the event $pqABAhas\ occurred$, denoted by , in the form [19]: $BP(A/B)$

$$P(A/B) = \frac{P(A\cap B)}{P(B)}, \qquad (6)$$

where denotes the probability of intersection between the events and [19], i.e., denotes the probability of the event and event occurring. $P(A\cap B)ABA$ and denote probabilities of occurrence of the events and , such that in (6). $BP(A)P(B)ABP(B) \neq 0$

As an example of applying conditional probability, consider the random experiment *"roll a dice",* where we define the events:

$A =$ Exit number 4; and

$B =$ Exit even number.

The calculation of the conditional probability results $P(A/B) = 1/3$, meaning the probability of rolling the number 4 knowing that an even number occurred on the roll of the dice. We now note that an important consequence of the definition of conditional probability is obtained by writing the expression (6) in the form [19]:

$$P(A\cap B) = P(A/B).P(B). \qquad (7)$$

The expression (7) is also referred to as the "Probability Multiplication Theorem" [19]. Regarding independence, two events A *and* B *are independent if the information on the occurrence of* B *does not change the probability attributed to event* A *[19,20], as follows:*

$$P(A/B) = P(A). \qquad (8)$$

Substituting (8) into (7) formally defines [18,19] that events A and B are independent if, and only if,

$$P(A\cap B) = P(A).P(B). \qquad (9)$$

The last expression means that the probability of an event *A and* an event *B,* being independent, is calculated by the product of the probabilities of the occurrence of these events. A Eq. (9) It is generalized to the case where we have the occurrence of several independent events. Therefore, if the events *A1, A2,......,Am,* are independent [18] then:

$$P(\bigcap_{i=1}^{m} A_i) = P(A_1).P(A_2)........P(A_m). \quad (10)$$

It is worth noting that if two events A *and* B are mutually exclusive, then they are dependent events, for if A occurs, event *B* does not occur. That is, the occurrence of one event conditions the non-occurrence of the other event [18].

3 PROBABILITY FUNCTION - NEGATIVE BINOMIAL MODEL AND APPLICATION

In this Section, we obtain the Probability Function in the Negative Binomial model, supported by the elements presented in the last Section. We used the analogy with performing a simple random experiment to exemplify some stages of the model's elaboration.

3.1 COIN FLIPS AND THE PROBABILITY FUNCTION

Consider several repetitions of the *"flip a fair coin" experiment*. As discussed in 2.1, each release is a Bernoulli experiment and there are, of course, two possible and mutually exclusive outcomes. In the previous section, we defined success as the occurrence of heads with probability value, which remains constant in each repetition of the experiment, and failure, the occurrence of tails with probability. Now consider that among the various possible repetitions of the experiment we wish to obtain two successes and we have adopted the letter for this indication. This means that we want to get 2 successes (2 heads) in several repetitions of the random $pqkk = 2$*experiment "flip an honest coin"*. If we attribute to AV the meaning: X*"number of times we must repeat the experiment until we obtain 2 successes", we* have to assume the numerical values: X

$$X = \{2,3,4,5,......\}. \quad (11)$$

Of course, if we want to obtain successes the experiment will have to be repeated at least 2 times, thus implying that the first possible value for the VA will be the number of desired successes, as indicated in (11). $k = 2$*The exemplified situation is generalized in the context of the Negative Binomial Model*, where AV is formally stated as [18,19]:

X: *Number of repetitions of a Bernoulli experiment until a number of successes are obtained.* k (12)

Then you can take $X$the values:

$$X = \{k, k+1, k+2, k+3, ...\}. \quad (13)$$

We emphasize that the definition of AV in (12) implies that the result of the last repetition of the experiment will always be success, since we must repeat the experiment *until* the number of successes is obtained. To guide the achievement of the Probability Function, we constructed the representation indicated in Table $3k$, referring again to the random experiment *"flipping a coin"*. We also fixed the occurrence of two successes, i.e., two faces, which are represented in Table 3 by their probability of success. $k = 2p$In the first horizontal line, we indicate the number of repetitions, considering up to five repetitions of the experiment and this being an arbitrary choice. In the first column on the left, we indicate some possible values of the AV that represent the total number of repetitions of the random experiment. In the last column on the right, the proportionality between the Probability Function, , and the values of the probabilities of success, , and failure are indicated. As we must perform successive repetitions of the experiment until we obtain the two intended successes (), we present the following interpretation of Table 3:$P(X = x)pqk = 2$

$x = 2$: means to obtain successes in repetitions of the random experiment, with the probability being constant in each repetition.$k = 2x = 2p$

$x = 3$: means to achieve successes in repetitions of the experiment, and so on. Note that from this point on we have combinations of results and use bold to highlight the successes in the possible combinations. Specifically, in order to achieve successes in repetitions of the experiment, there are two possible combinations of results indicated in the 5th and 6th lines. They are success, failure, success and represented by (in the adopted notation. The other possibility is failure, success, success, or simply. As mentioned, due to the definition of VA in this model, the last roll always results in success, as we stop the repetitions of the experiment when the number of successes is reached. Continuing the interpretation of Table 3, we note that in order to obtain successes in repetitions of the experiment, there are 3 possible combinations of results indicated in the 8th, 9th and 10th lines. They are success, failure, failure, success, failure, success, failure, success, and failure, failure, success, success. The possible combinations to obtain successes in repetitions of the experiment are indicated from the 12th to the 15th lines. It is essential to note that the various repetitions of the experiment and their respective results, of success or failure, are independent of each other and imply that we must effect the product of the probabilities of success and failure to obtain the analytic expression of the Probability Function. Specifically, the realization and outcome of the 1st coin toss does not interfere with the outcome of the 2nd toss. In turn, the results of the 1st and 2nd launches do not interfere with the result of the 3rd launch, implying that the various repetitions of the experiment and their respective results are independent of each other, as discussed in Subsection 2.2. Thus, in order to obtain the analytic expression of the Probability Function, we use Eq. (10) and we produce the values of the probabilities, or , associated with the results of success or failure obtained in each repetition of the experiment. Based on Table 3, the proportionalities between the Probability Function and the values of the probabilities are verified, namely:$k = 2x = 3k = 2x = 3\boldsymbol{pqp})q\boldsymbol{pp}kk = 2x = 4(\boldsymbol{pqqp})(q\boldsymbol{p}q\boldsymbol{p})(qq\boldsymbol{pp})k = 2x = 5pqP(X = x)pq$

$x = 2 \Rightarrow P(X = 2) \propto p^2$, signifying successes in repetitions of the randomized experiment.$k = 2x = 2$

$x = 3 \Rightarrow P(X = 3) \propto pqp$, signifying successes in independent repetitions of the experiment. This case implies that one of the outcomes is failure, with probability $k = 2x = 3q$. We must then also consider the other possible configuration, i.e.,

$P(X = 3) \propto qpp$or. $P(X = 3) \propto qp^2$

It is important to note, in this case, that the probability of obtaining *k=2* successes in *x=3* repetitions of the experiment is calculated by:

$$P(X = 3) = 2qp^2, \qquad (14)$$

where the factor 2 expresses the number of the two possible configurations. Continuing with our interpretation, for *x=4* there are the settings:

$P(X = 4) \propto pqqp \Rightarrow P(X = 4) \propto q^2p^2$,

$P(X = 4) \propto qpqp \Rightarrow P(X = 4) \propto q^2p^2$,

$P(X = 4) \propto \mathrm{qqpp} \Rightarrow P(X = 4) \propto q^2p^2$.

Since there are 3 possible configurations, referring to the achievement of successes in repetitions of the experiment, we write: $k = 2x = 4$

$$P(X = 4) = 3q^{4-2}p^2, \qquad (15)$$

We emphasize that factor 3 expresses the number of possible configurations in this case. Before proceeding further, it is convenient to express the Probability Function in terms of the variables *x* and *k*. Thus Eq. (15) is even partially rewritten in the form:

$$P(X = x) = 3q^{x-k}p^k. \qquad (16)$$

In the Eqs. (14) and (15) factors 2 and 3 represent, respectively, the number of possible configurations and are groupings called *Combinations* [19]. We exemplify the calculation of the number of possible combinations in this model using again the example of Table 3, of obtaining successes in repetitions of the experiment. Since the result of the last repetition is always success, we have that the other success (2-1)=1, or, can occur in any of the other (4-1)=3, or , repetitions of the experiment. Factor 1 represents the exclusion of success that occurs in the last release. In other words, since the last repetition always results in success, the other success can occur on the 1st, 2nd, or 3rd rolls. Therefore, the number of possible combinations is calculated by [18,19]:$k = 2x = 4\ (k - 1)(x - 1)$

$$\binom{4-1}{2-1} = \frac{3!}{1!2!} = 3. \tag{17}$$

In general:

$$\binom{x-1}{k-1} = \frac{(x-1)!}{(k-1)!(x-k)!}\ . \tag{18}$$

Thus, by reason of the Eqs. (16), (17), and (18), the analytic expression of the Probability Function, Eq. (16), is written in the form:

$$P(X = x) = \binom{x-1}{k-1} q^{x-k} p^{k} \tag{19}$$

or even:

$$P(X = x) = \binom{x-1}{k-1} (1-p)^{x-k} p^{k}, \tag{20}$$

$x \geq k$tag. A Eq. (20) expresses the fact that the probability function, , is proportional to the probability of success raised to the number of desired successes, and proportional to the probability of failure raised to the number of failures and which is calculated by . A Eq. (20) then allows the calculation of the probability of obtaining successes in repetitions of any experiment classifiable as a Bernoulli experiment, with the probability constant in each repetition of the experiment of the occurrence of the event defined as success. As an alternative interpretation, we can state that Eq. (20) provides the probability that before the number of intended successes occurs, the other successes can be allocated in any order in the remaining positions, which represent the results of the other repetitions of the randomized experiment. The e parameters characterize this distribution, which can be represented by the notation $P(X = x)pk, q(x - k)kxpk(k - 1)(x - 1)kpX{:}BN,\ (k, p)$meaning that the VA follows the Negative Binomial Probability Distribution and depends on the parameters and . Xkp

3.2 MULTIPLICITY DISTRIBUTION

In order to provide the connection between the concepts of DBN and the physical quantity Multiplicity Distribution, we make some comments in order to illustrate this physical quantity in a simple and qualitative way. As mentioned, protons are particles with a complex internal structure composed of quarks and gluons. The Multiplicity Distribution is sensitive to the number of collisions between quarks and gluons contained in colliding protons and, in general, to the fundamental mechanisms of particle production [22]. Specifically, in collisions between two protons there can be the creation of 2 particles, or 4, or 6 or, in general, there can be the production of "" particles. We note that "" is an even number due to the conservation of electric charge in the process of producing new particles [1]. To address the problem, we define the AV:nn

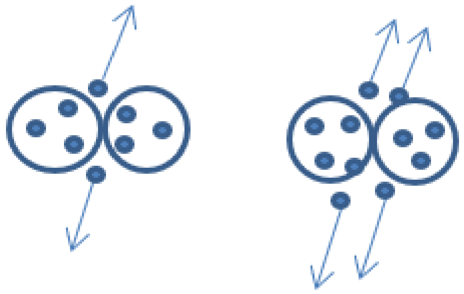
$$N\text{: } \textit{Number of particles produced in the collision.} \tag{21}$$

Such that:

$$N = \{2,4,6,\ldots,n,..\}. \tag{22}$$

This set is called the *"Multiplicity Set of electrically charged particles*", or simply "*Multiplicity*". Each element "" is associated with its corresponding probability of production, that is, the value of the multiplicity corresponds to the value of the probability , meaning the probability that in the collision between protons two particles will be produced. In general, it is the value of the probability that "particles" will be produced in the collision. In this way, the "$nP(N = n)n = 2P(2)$ $P(N = n)n$*Multiplicity Distribution"* *set is constituted*$\{N, P(N)\}$, which is one of the most basic characteristics of proton collisions at high energies and which has been the object of experimental and theoretical studies in order to enable the understanding of the strong nuclear interaction. For pedagogical purposes only, we present in Fig. 1 a non-realistic illustration of the process of production of particles in collision between protons.

Figure 1 - Illustration of particle production in the collision between two protons, represented by the larger circles. Smaller circles, inside the larger ones, represent the constituents of protons. The particles produced in the collision are illustrated by the small circles with arrows.



{2, P(2)} {4, P(4)}

3.3 NEGATIVE BINOMIAL APPLIED TO THE STUDY OF MULTIPLICITY DISTRIBUTION

The use of DBN [10], the superposition of two DBN [11,23] or mathematical models of particle production that use DBN or its limit cases as an element of the model [9,24,25,26] has provided adequate parameterizations of the Multiplicity Distributions. Thus, we performed the mathematical procedures necessary for the application of DBN in analyses involving DM, in which the Negative Binomial Probability Function is usually expressed in terms of the Gamma Function and also eliminating the variable x and the probability value p from the expression (19). In our treatment we saw that (x-k), in Eq. (20), represents the number of failures in x repetitions of the experiment in which we expect to obtain k successes. We introduce the variable "n" by writing:

$$n = x - k \Rightarrow x = n + k. \qquad (23)$$

Substituting (23) for (19) results that:

$$P_{k,p}(n) = \binom{n+k-1}{k-1} q^n p^k. \qquad (24)$$

To facilitate the use of the complete interface [20] following:

$$\binom{n+l}{l} = \binom{n+l}{n}, \qquad (25)$$

We change the variable: in the expression (24) obtaining$l = k - 1$

$$P_{k,p}(n) = \binom{n+l}{l} q^n p^k \,. \qquad (26)$$

Using the relation (25) and noting that Eq. (26) is rewritten in the form:$p + q = 1$,

$$P_{k,p}(n) = \binom{n+k-1}{n}(1-p)^n p^k, \quad (27)$$

or equivalently

$$P_{k,p}(n) = \frac{(n+k-1)!}{n!(k-1)!}(1-p)^n p^k. \quad (28)$$

Due to the introduction of the variable and its meaning, Eq. (23), the expression (28) gives the probability of failures and () successes occurring in any order, before a number of successes occur in a Bernoulli experiment with a probability of success [3]. Calculations involving factorials are, in general, algebraically laborious, so it is convenient to express Eq. (28) in terms of the Gamma Function is defined [27] as:"n"$nk-1kp$

$$\Gamma(n) = \int_0^\infty e^{-x}x^{n-1}\,dx, \quad (29)$$

that converges if . If it is a positive integer, we resort to the identity [27]$n > 0n$

$$n! = \Gamma(n+1). \quad (30)$$

Thus, the factorials present in Eq. (28) are expressed in terms of the Gamma Function, namely:

$$(k-1)! = \Gamma(k). \quad (31)$$
$$(n+k-1)! = \Gamma(n+k). \quad (32)$$

Replacing (30), (31) and (32) in Eq. (28) It results in:

$$P_{k,p}(n) = \frac{\Gamma(n+k)}{\Gamma(n+1)\Gamma(k)}(1-p)^n p^k. \quad (33)$$

It also follows that in practical applications, the probability is often not known, however the average value of a sample of experimental data can be obtained [28]. Thus, since the average multiplicity of the set of particles produced in proton collisions, it is related to the probability of success by equation [3]:$p < n > Np$

$$p^{-1} = 1 + \frac{<n>}{k} \Rightarrow p = \frac{k}{k+<n>}. \qquad (34)$$

Substituting (34) for (33) results that:

$$P(n,k,<n>) = \frac{\Gamma(n+k)}{\Gamma(n+1)\Gamma(k)}\left[1-\frac{k}{<n>+k}\right]^n\left[\frac{k}{<n>+k}\right]^k. \qquad (35)$$

After doing some algebraic work on Eq. (35) We obtain the Probability Function in the form often used in multiplicity investigations [3]:

$$P(n,k,<n>) = \frac{\Gamma(n+k)}{\Gamma(n+1)\Gamma(k)}\left[\frac{<n>/k}{1+<n>/k}\right]^n\frac{1}{[1+<n>/k]^k}. \qquad (36)$$

The analytical form of the DBN now depends on the values of the parameters and , which can be determined in Eq adjustments. (36) to the corresponding experimental data. The expression (36) also allows for versatile graphing and consequent comparisons with experimental data, when compared to Eq. (28). Drawing on Eq. (36) Figure 2 shows graphs of the DBN. In the left pane of the figure, the value of is fixed, while the mean values, , are varied. It is noted that as it decreases, the distribution narrows. On the right panel it is kept constant and as it decreases the opening of the distribution is checked.$< n > k k < n >< n >< n > k$

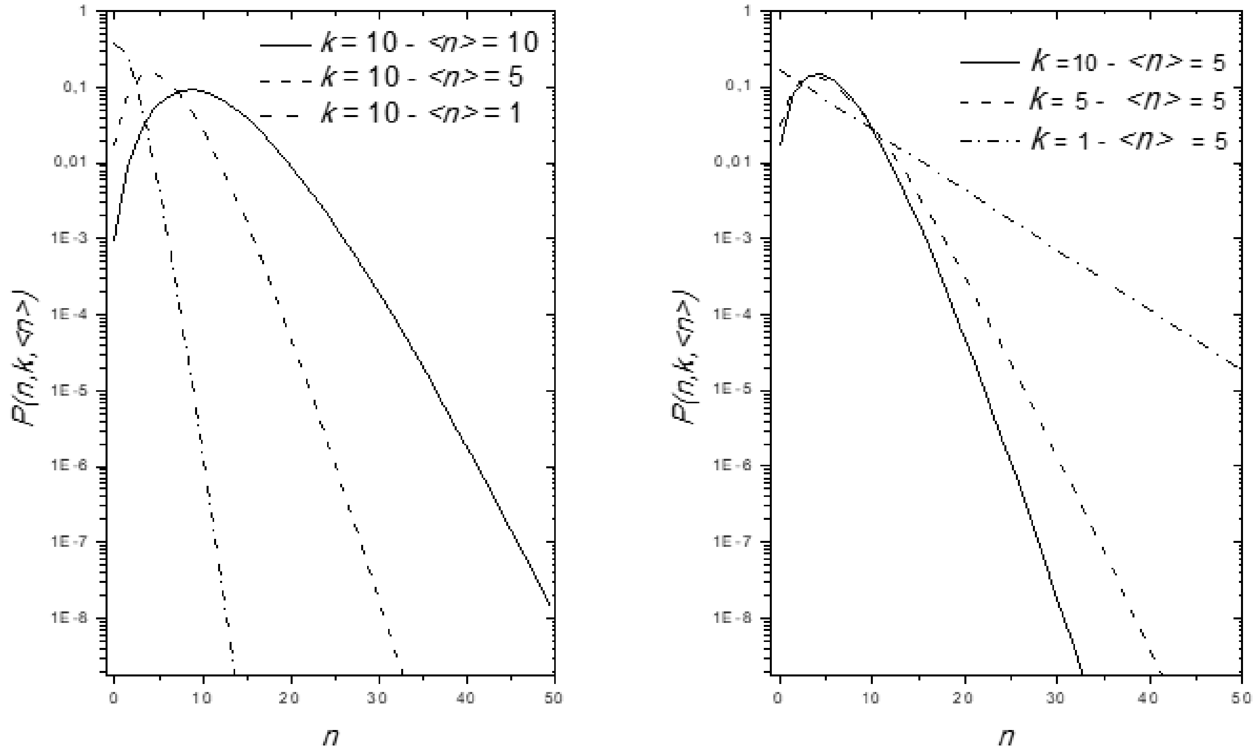
4 DISCUSSION AND FINAL CONSIDERATIONS

Since the first application of DBN by the UA5 Collaboration in 1985 [29], it has been frequent and widely used in investigations involving Multiplicity Distributions in colliding systems such as proton-proton, proton-antiproton, electron-positron and muon-proton [3]. The fact that experimental results of DM in many different experiments and in a wide range of collision energy can be parameterized by DBN, by the superposition of two of them or even limit cases of this distribution has not been considered as an accidental fact and, as pointed out by Giovannini and Ugoccioni [11], the impression is that there may be an approximate universal regularity in this fact. Thus, due to the start of the **LHC's activities,** making available new experimental information on collisions between protons, we have a propitious moment to carry out tests, improvements and development of mathematical models and/or calculation structures for investigations of the mechanisms of multiple particle production in these collisions. With this motivation we discuss the Negative Binomial Model obtaining its Probability Function using a simple scenario, proposing a didactic approach to the elements and hypotheses that characterize this distribution. The purpose is to facilitate and provide adequate application and interpretation of results, as well as to disseminate the research theme. Since we are dealing with interactions between protons, it is natural to expect that the mechanisms of interactions between quarks and gluons that produce particles can be understood in terms analogous to the elements that make up the scenario of the Negative Binomial Model. However, such an understanding has not yet been possible [3]. Although this understanding does not yet exist, and because it does not seem to be an accidental fact, attempts to theoretically generate DBN, based on general principles of particle production involving interactions between quarks and gluons, have been made over the years. A phenomenological approach used for this purpose is referred to as the "*Clan Model*" introduced in High Energy Physics at the XVII International Symposium on Multiparticle Dynamics by L. Van Hove and A. Giovannini [30]. However, any discussion on the subject is outside the scope of this work. We conclude by emphasizing that the discussion presented in this work, i n order to obtain the DBN, can be motivating and help studies and applications to other diverse systems. There is also the possibility of being adapted to facilitate and enable the teaching of other models of probability distributions, such as Binomial and Geometric.

Table 3 – Possible representation to guide the obtaining of the Probability Function in the Negative Binomial Model considering the random experiment *"flip an honest coin"*. The occurrence of success in a repeat of the experiment is represented by and the occurrence of failure by . The repetitions of the random experiment and its results are independent of each other, implying the multiplication of the probabilities and . We use bold to highlight the offset of the probability associated with the success event in the possible combinations. In the last column on the right, the proportionality between the Probability Function and the probabilities and .*pqpqpq*

	First	2nd	Third	4th	5th	
						Proportionalities between the Probability Function and the probabilities and . $P(X = x)pq$
$x = 2$	$\boldsymbol{p}$	$\boldsymbol{p}$				$P(X = 2) \propto pp$ or $P(X = 2) \propto p^2$
$x = 3$	$\boldsymbol{p}$	q	$\boldsymbol{p}$			$P(X = 3) \propto qp^2$
	q	$\boldsymbol{p}$	$\boldsymbol{p}$			$P(X = 3) \propto qp^2$
$x = 4$	$\boldsymbol{p}$	q	q	$\boldsymbol{p}$		$P(X = 4) \propto q^2p^2$
	q	$\boldsymbol{p}$	q	$\boldsymbol{p}$		$P(X = 4) \propto q^2p^2$
	q	q	$\boldsymbol{p}$	$\boldsymbol{p}$		$P(X = 4) \propto q^2p^2$
$x = 5$	$\boldsymbol{p}$	q	q	q	$\boldsymbol{p}$	$P(X = 5) \propto q^3p^2$
	q	$\boldsymbol{p}$	q	q	$\boldsymbol{p}$	$P(X = 5) \propto q^3p^2$
	q	q	$\boldsymbol{p}$	q	$\boldsymbol{p}$	$P(X = 5) \propto q^3p^2$
	q	q	q	$\boldsymbol{p}$	$\boldsymbol{p}$	$P(X = 5) \propto q^3p^2$

Figure 2 - Graphs of the Negative Binomial Distribution, Eq. (36). In the left panel, the parameter value is kept constant by varying the values of the mean value. In the right pane it is constant and the values are changed.$k < n >< n > k$



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