CHAPTER 117

Numerical solution for finding time constants in Harriot's method for identifying overdampened second-order systems

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ABSTRACT
The identification of systems seeks to use techniques that, from input and output signals, can find a dynamic model that describes the system. Among the simplest identification techniques are those based on step response. In this work, the Harriot method is approached, a graphical method that identifies over dampened second-order models. This study presents a numerical solution to find the time constants of the model, allowing its application in a computational and automated way. To validate the proposed approach, it was applied in a motor-cogenerator didactic module and compared with other deterministic identification techniques, such as Ziegler-Nichols, Hägglund, and Sundaresan, to compare the results. The results showed that the numerically resolved Harriot method performed equivalently or even better compared to the other deterministic identification techniques. The R^2 and Akaike Information Criterion (AIC) were used as metrics to quantify the performance of each model. It was concluded that the numerical solution proposed for the Harriot method allows its computational application and presents a promising performance. Future work may explore the use of this technique as part of an auto-tuning scheme for PID controllers.

Keywords: Identification of Dynamical Systems, Harriot's Method, Overdampened Second Order Systems.

1 INTRODUCTION

The identification of dynamical systems is a discipline closely linked to the automatic control of systems. To the extent that the most widespread techniques of linear controller design commonly start from a model of the system to be controlled, obtaining such models is necessary for the design of controllers (Cobos, 2021).

Although it is possible to obtain models from a phenomenological study of the systems, in general, such models are difficult to obtain, because they depend both on a high knowledge of the physical, chemical, and even biological laws that govern the system, as well as on parameters that are often difficult to measure. That said, the identification of systems, in turn, seeks to use techniques that, from the knowledge of the input and output signals of the system, can find a dynamic model that describes it (Ljung, 2009).

Among the simplest identification techniques are those that are classified as deterministic based on step response. Such techniques have weaknesses because they ignore the presence of stochastic components in the collected signals and, in general, start from the assumption of a known structure for the model, which, in practice, is not always true. Such techniques are still widely used, especially in an industrial environment, for their ease of use and the practicality of the tests that, usually, only involve taking the mesh in question to the manual mode, at a point of operation, and giving an increase in the manipulated variable, thus generating a step. The reaction curve to this step stimulus is collected and, from it, a model is obtained (Coelho, 2016).

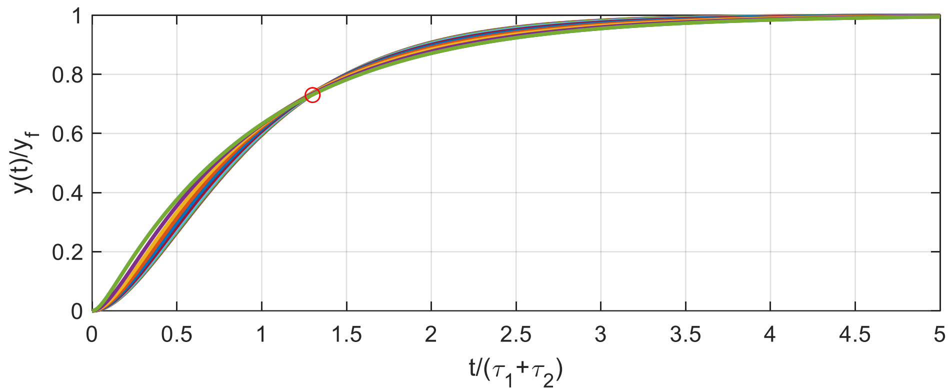
Among these classical techniques, this work will address Harriot's method, which is a graphical method that identifies over dampened second-order models:

$$G_m(s) = \frac{K}{(\tau_1 s + 1)(\tau_2 s + 1)}$$

where K is the static gain of the system, $\tau_1 \tau_2$ and are the time constants and L is the dead time.

Harriot (2007) showed that for different combinations of time constants for the overdamped second-order model when plotted with time normalized by the sum of the time constants, $t/(\tau_1 + \tau_2)$ the reaction curves always intersect approximately at the point $y(t)/y_f = 0{,}73$, which corresponds to $t/(\tau_1 + \tau_2) = 1{,}3$, as exemplified in Figure 1, where y(t) is the process variable and y_f is its regime value in response to a step input

Figure 1: Normalized response for over dampened second-order systems.



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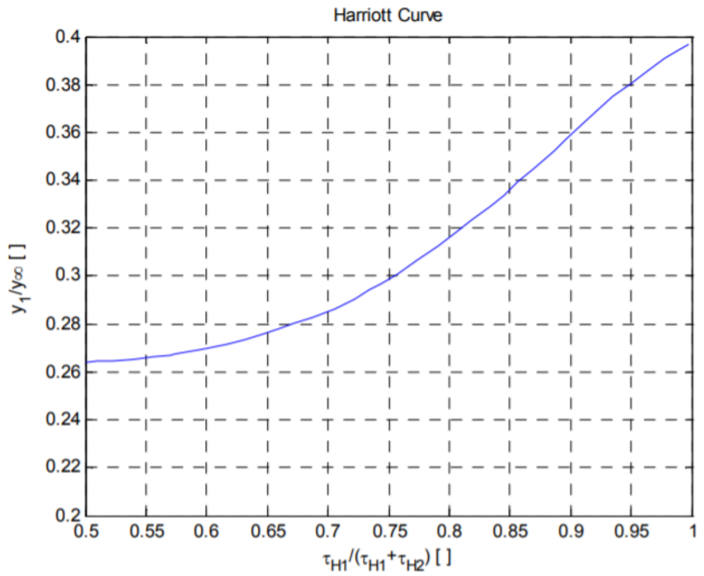
Thus, having the step response, one can find the time value t_{73} where the reaction curve reaches 73% of its final value and, for this instant:

$$\tau_1 + \tau_2 = \frac{t_{73}}{1{,}3}$$

At the same time, Harriot also shows that the greatest dispersion in the curves happens at the point where the normalized time is equal to 0.5. By calling this point $y_{50} = y(t_{50})$ with, $t_{50} = 0{,}5(\tau_1 + \tau_2)$, one can graphically find the value of the time constants using the graph shown in Figure 2.

Figure 2: Harriot curve. In the notation of the source $y_1 = y_{50}y_\infty$ is the permanent regime y_f and τ_{H1} and τ_{H2} value and are the time constants of the second-order model, further assuming that the first constant is greater than or equal to the second.



Source: Jakoubek (2009).

The fact that it depends on the analysis of a graph to find the relationship between the constants allows the use of the method manually but limits its use in an automated way in a computational implementation. Thus, the present work presents an alternative by numerical solution to find the time constants of the model which, in turn, allows the Method to be applied in a computational and automated way.

2 FORMULATION

To formulate a numerical solution to Harriot's method, one starts from the step response of a second-order system with two real and different poles. Without loss of generality, consider that the dead time of the system is zero and that the system has gained unitary. In this case, we have:

$$y(t) = 1 - \frac{\tau_1}{\tau_1 + \tau_2} e^{-\left(\frac{t}{\tau_1}\right)} + \frac{\tau_2}{\tau_1 + \tau_2} e^{-\left(\frac{t}{\tau_2}\right)}$$

Doing:$t = t_{50} = 0{,}5(\tau_1 + \tau_2)$

$$y_{50} = 1 - \frac{\tau_1}{\tau_1 + \tau_2} e^{-\left(\frac{\tau_1+\tau_2}{2\tau_1}\right)} + \frac{\tau_2}{\tau_1 + \tau_2} e^{-\left(\frac{\tau_1+\tau_2}{2\tau_2}\right)}$$

Knowing the value of the sum of the time constants,

$$\tau_1 + \tau_2 = \frac{t_{73}}{1{,}3},$$

One can write τ_2 in terms of τ_1 and solve numerically.

Doing the substitution to rewrite the output value as a $y(t) = y_{50}$ function of the first time constant and defining $f(\tau_1)$ as the resulting function:

$$f(\tau_1) = 1 - \frac{1{,}3\tau_1}{t_{73}} e^{-\left(\frac{t_{73}}{2{,}6\tau_1}\right)} + \frac{1{,}3\left(\frac{t_{73}}{1{,}3} - \tau_1\right)}{t_{73}} e^{-\left(\frac{t_{73}}{2{,}6\left(\frac{t_{73}}{1{,}3} - \tau_1\right)}\right)} - y_{50}$$

one can solve iteratively up to convergence using Newton's method: $f(\tau_1) = 0$

$$\tau_1^{k+1} = \tau_1^k - \frac{f(\tau_1^k)}{f'(\tau_1^k)}$$

Where $f'(\tau_1^k)$ represents the derivative of the function evaluated in the k-th estimate of τ_1. The estimate is updated until it reaches a tolerance criterion or a maximum number of iterations.

3 RESULTS AND DISCUSSIONS

To validate the proposed solution by applying it to the identification of a dynamic model for a physical system, a motor-cogenerator didactic module was used, as seen in Figure 3. The module, described in detail in Pedrisch et al. (2022) is composed of a pair of DC motors where one of them must have its speed controlled and is coupled to a second motor that, acting as a tacokeur, provides speed feedback, allowing control in a closed loop.

Figure 3: Motor-tacoker module used to validate the proposed approach.

Source: Pedrisch et al. (2022).

The motor-tacoker is a stable open-loop system with an underdamped behavior. Doing the phenomenological modeling (Ahmad et al., 2014) it is possible to notice that each DC motor is a second-order system, however, by the difference in speed from the mechanical dynamics to the electric ones (much faster), the system can usually be approximated even by a first-order model (Pedrisch et al., 2022).

For the same system, the techniques of Sundaresan, Hägglund, and Ziegler-Nichols were applied, deterministic identification techniques well established in the literature, and all three also use step response. The Ziegler–Nichols and Hägglund methods identify first-order models with dead time, the former being a method based on tangent lines and the latter on points. They were included in the comparison because the work of Pedrisch et al. (2022) uses a first-order model for the system. Sundaresan's method, in turn, identifies a second-order model over-dampened with dead time, included for fairer competition with Harriot's method, and also by the phenomenological model of a DC engine being, in fact, second-order. The three methods are presented in detail in Jakoubbek (2009) and Coelho (2016). After applying the methods, Table 1 presents the models obtained in each technique:

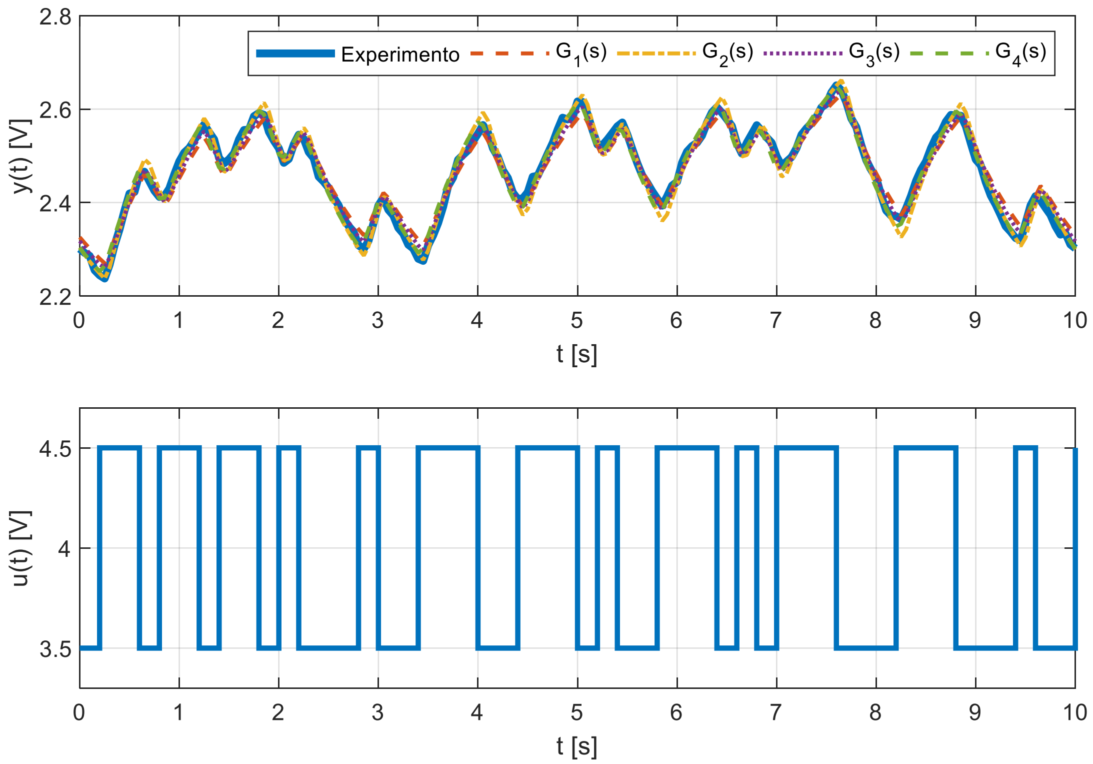
Table 1: Model obtained by deterministic techniques.

Technique	Model
Ziegler-Nichols	$G_1(s) = \frac{0{,}705}{0{,}879s + 1} e^{-0{,}059s}$
Hägglund	$G_2(s) = \frac{0{,}705}{0{,}631s + 1} e^{-0{,}069s}$
Sundaresan	$G_3(s) = \frac{0{,}705}{(0{,}760s + 1)(0{,}032s + 1)} e^{-0{,}032s}$
Harriot Numeric	$G_4(s) = \frac{0{,}705}{(0{,}699s + 1)(0{,}037s + 1)}$

The four models obtain the same static gain since they all do so by the ratio between the variation in the permanent regime of the output and variation in the permanent regime of the input. The time constants of the first-order models and the dominant time constants of the second-order models have distinct but congruent values in terms of magnitude, with a mean of 0.742 seconds and a standard deviation of 0.105. In the three cases where the model includes transport delay, the delay is small when compared to the time constant.

The validation of the models obtained by the four techniques was performed by applying a PRBS signal (pseudo-random binary signal) in the motor-tacoker system and comparing the response of each model with the response of the real system. Figure 4 shows the comparison of the responses of each model with the response of the real system. Visually, it is possible to verify that the Harriot method presents adequate performance, consistent with the other methods, as expected by the models obtained.

Figure 4: Above experimental response of the system compared with the responses of the identified models. The response of the motor-tacoker is presented in blue, the model obtained by Ziegler-Nichols in red, the model by Hägglund in yellow, the model by Sundaresan in purple, and the model by the proposed numerical version of the Harriot method in green. Low: PRBS signal applied to the system.



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To quantify the performance of each model and rank the results obtained, two figures of merit were used: the R² and the Akaike Information Criterion (AIC). R² is a metric that ranges from 0 to 1 and represents how much the model describes the variance of the data. A unitary result represents a perfect description. The AIC measures the loss of information and penalizes models with more parameters. A more negative result describes a better model.

The values of the metrics obtained from the response to the PRBS are presented in Table 2, which shows that the numerically solved Harriot method as proposed has a performance not only equivalent but even slightly superior to other deterministic identification techniques that use step response.

Table 2: Comparison between deterministic techniques through the mé.

Technique	R^2	AIC
Ziegler-Nichols	0,9505	-1543,2
H	0,9655	-1615,5
Sundaresan	0,9730	-1663,1
Harriot Numerical	0,9815	-1741,1

4 CONCLUSIONS

The present work was able to present a numerical solution for the Harriot Method, allowing its use computationally. The solution is based on analytically finding the step answer to an overdamped second-order system and, using the point of coincidence and the point of greatest dispersion in Harriot's normalized graph, it is possible to solve the equation in terms of one of the time constants using Newton's method.

When compared to other established deterministic identification techniques, the method performs well and still has the advantage of being a point-based technique, less sensitive to measurement noise than techniques based on tangent lines, thus being a promising method to meet a specific niche of problems within the identification of systems.

In future work, it is intended to use the technique as part of the automatic tuning scheme of PID controllers.

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