

## Elliptical geometry in undergraduate classes: A possible application

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### ABSTRACT

This work addresses current challenges in mathematics education, particularly the difficulty many students face in engaging deeply with the subject. It focuses on an innovative educational approach aimed at introducing high school students to non-Euclidean geometries, with a specific emphasis on Elliptic Geometry. The initiative stemmed from observations during face-to-face classes where students showed a keen interest in advanced mathematical topics. The study methodically reviewed relevant literature and developed practical teaching strategies, including interactive sessions that explored spherical triangles and their properties. By contextualizing these concepts with historical and practical examples like nautical charting, the approach aimed to enhance students' understanding and appreciation of geometry's real-world applications.

**Keywords:** Elliptic geometry, Exploratory teaching, OBMEP.

### INTRODUCTION

Mathematics education faces a significant challenge today: many students, including those naturally inclined to the subject, have difficulties in connecting and engaging deeply with the content presented (ALMOULOU; SILVA, 2012). This issue is not just restricted to students who are considered problematic but covers a wide range of audiences. In the face of these difficulties, it becomes imperative to explore new approaches that make mathematics more dynamic and captivating for everyone.

In this context, this work aims to report an innovative didactic session experience, focused on intuitively introducing concepts related to non-Euclidean geometries, with emphasis on Elliptic Geometry. The target audience was high school students participating in the Junior Scientific Initiation Program (PIC) in the city of Sobral, in the state of Ceará.

The authors were motivated by direct interaction during face-to-face classes at the PIC, where they noticed the students' interest in advanced mathematics topics. By discussing some of these more complex concepts in an accessible way, they noticed that students not only understood but were also actively involved in the discussions. This underscored the need to develop proposals that offer high school students a solid grounding in mathematical subjects often reserved for higher education.

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## **MATERIALS AND METHODS**

In this article, we explore in detail the research methodology used, which included a thorough literature review and consultation of fundamental works, such as SILVA (2017), COUTINHO (2018), CARMO (1987), and CRUZ and SANTOS (201?). The main objective of the research was to extract and contextualize the main concepts of elliptic geometry, aiming at its practical application in didactic sessions. Based on the solid theoretical foundation acquired, the concepts of triangles on spherical surfaces and the sum of angles in spherical triangles were carefully selected, which were presented in an intuitive way and enriched by practical examples, such as those necessary for the elaboration of nautical charts.

These contents were developed during a class of the Scientific Initiation Program (PIC), aimed at the level 3 class, composed of medalist students of the Brazilian Public School Mathematics Olympiad (OBMEP), who have access to an education especially directed to Olympic training in mathematics. During the classroom interactions, the design of triangles with the sum of angles greater than  $180^\circ$  was explored with the students, a concept that was skillfully refuted based on the students' solid prior knowledge.

In addition, a fascinating historical dive related to the great navigations was carried out, seeking to contextualize the crucial importance of developing accurate maps. This approach allowed students to vividly understand the formation of triangles into spheres and their distinct properties. In addition, intriguing aspects of non-Euclidean geometries were discussed, underscoring their significant influence on modern technologies such as satellites and global positioning systems (GPS).

This study thus emphasizes the crucial importance of contextualizing the teaching of geometry and its various applications, including spherical geometry, in practical and historical situations. Such an approach not only broadens the understanding of mathematical concepts, but also demonstrates their direct and applied relevance in the contemporary world.

## **RESULTS**

At the end of the activity, there was a notable increase in the students' interest in the contributions of modern mathematics. The program, which aims to introduce students to the academic environment, encouraging them to explore this area of knowledge more deeply, potentially enabling future specializations.

Students recognized that mathematics goes far beyond what is taught in traditional schools, motivating them to seek more information about current innovations in the subject. This new interest has significantly increased the time they devote to mathematical studies, reflecting a renewed engagement and a deeper appreciation for the challenges and discoveries of contemporary mathematics.



## **FINAL THOUGHTS**

Among the various responsibilities that a math teacher must take on, motivating their students stands out as one of the most crucial. Therefore, it is essential to present stimulating and challenging situations throughout a didactic sequence to ensure learning success.

Exploring non-Euclidean geometries in a way that is accessible to high school students may represent an innovative approach to capture the attention of these students. This approach aims not only to arouse the curiosity of students, but also to effectively introduce them in the context of scientific initiation, showing that Mathematics, far from being a static discipline, offers vast fields of study in constant evolution, accessible to all who wish to pursue an academic career.

It is important to highlight that basic knowledge about different types of geometries is not something unattainable for Basic Education students. On the contrary, it encourages them to reinterpret the world around them, stimulating curiosity and critical thinking in the face of simple everyday situations. Ultimately, this approach indirectly fosters the formation of well-rounded citizens who are prepared to interact meaningfully in complex contemporary societies.



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