SEVEN publicações acadêmicas
EM PARCERIA COM: IEMS UNIASSELVI

Fractals in High School: A proposal to assist in the teaching of geometry

Aline Picoli Sonza[1].

ABSTRACT
For a long time, Euclidean geometry was responsible for describing, analyzing, and explaining everything around us. However, due to the complexity of the world, which is not limited to points, lines, planes and some regular shapes, over time this geometry has come to be considered insufficient. Other geometries emerged over the centuries and were classified as non-Euclidean, among them, the geometry of fractals. There are countless contributions of this geometry to various areas of knowledge. In biology, for example, we can better understand the growth of plants, the study of irregular surfaces in physics, the detailing of the internal anatomy of the human body, and the analysis of some types of cancer for medicine are among these contributions.

Keywords: Euclidean geometry, Fractals, Mathematics teaching.

INTRODUCTION

For a long time, Euclidean geometry was responsible for describing, analyzing, and explaining everything around us. However, due to the complexity of the world, which is not limited to points, lines, planes and some regular shapes, over time this geometry has come to be considered insufficient. Other geometries emerged over the centuries and were classified as non-Euclidean, among them, the geometry of fractals. There are countless contributions of this geometry to various areas of knowledge. In biology, for example, we can better understand the growth of plants, the study of irregular surfaces in physics, the detailing of the internal anatomy of the human body, and the analysis of some types of cancer for medicine are among these contributions.

In addition to all the applications in the field of science, fractals are considered very important in the teaching of various topics in mathematics and can contribute to the study of functions, sequences, arithmetic and geometric progressions, geometric shapes, perimeter, area, volume, symmetry, and notions of limits, among many others.

Because of the wide possibility of approaching fractal geometry, its diversity of shapes and colors, it can be associated with the use of constructions and exploration of creativity, in addition to enabling the use of the computer, which usually arouses the interest, curiosity and attention of students. Sallum (2005) states that by introducing the study of fractals in high school, the teacher will be satisfying the curiosity of those who have heard of them and also

[1] Southern Riograndense Federal Institute – R/S

> It provides the opportunity to work with iterative processes, write general formulas, create algorithms, calculate areas and perimeters of figures with increasing complexity, introduce an intuitive idea of the concept of limit and is an excellent topic for applying geometric progressions and stimulating the use of tables. (SALLUM, 2005, p. 1)

We can easily see that most man-made creations are based on Euclidean Geometry. It is commonly possible to identify constructions based on prisms, cylinders, cones, or pyramids, for example. Because Euclidean geometry, over many centuries, has served to describe the forms present in nature, it has also served as an inspiration for architects and engineers in their design of monuments, buildings or constructions in general. However, with the realization that Euclidean geometry could not contemplate all the forms present in nature, fractal geometry emerged. In other words, from the need to describe and explain natural phenomena or objects that do not have a defined shape, fractal geometry emerged. In general, it is possible to classify it as a geometry that presents geometrically complex structures, infinitely varied and that studies the behavior of the so-called fractals.

The French mathematician Benoit Mandelbrot (1975) is considered the father of fractals because he gave them their name. From the Latin *fractus*, the term fractal arose referring to 'irregular' or 'broken' and from the corresponding verb *frangere* which means to break or create irregular fragments. There are indications that studies on fractals were carried out before the twentieth century, but they had no scientific value and were called 'mathematical monsters' or 'demons'. Mandelbrot, however, used some of these researches that had already begun but had not been completed in his studies of fractals.

There are several definitions for fractals, although none of them are absolute. Mandelbrot (1998) proposes the discussion by questioning the need to rigorously define a fractal and states that any definition could be early and, therefore, should remain open and intuitive to be continuously improved.

Although the definition of fractals is still open, it is possible to identify them by their characteristics since they maintain their shapes even changing the scale under which they are analyzed. In other words, you can zoom in or out hundreds of times and it will remain with its original characteristics.

Mandelbrot's statement leads us to the analysis of the importance of the emergence of the study of the geometry of fractals for understanding the world around us. Mandelbrot (1998) states that "clouds are not spheres, mountains are not cones, shorelines are not circles, tree bark is not smooth, nor does lightning propagate in a straight line." (MANDELBROT, 1998, p. 468)

The teaching and learning of mathematics can also be something capable of bringing the feeling of satisfaction. Barbosa (2002) states that Mathematics provides the mathematician, the teacher and should also allow the student to have forms of satisfaction related to ways of thinking, seeing or acting.

> Often they emerge from overcoming difficulties; Thus is, for example, the pleasurable state that emerges from the simple successful search for roots in the solution of an equation or a

> numerical or geometric problem-situation whose solution leads to finding only a few numbers or certain points in a plane. (BARBOSA, 2002, p. 13)

From this point of view, it is possible to imagine how much the study of fractals can meet the teacher's desire to provide the student with the study of a more interesting and thought-provoking mathematics. Silva (2011) highlights some activities involving fractals as fascinating because of their applications in various areas of knowledge. In the study of the functioning of dynamic and complex systems such as the formation of clouds, its importance in the diagnosis of cancer because it helps in the measurement of the tortuosity of the edge where the tumor is located. In addition to assisting, also in veterinary medicine, in the financial or stock market, he contributes in the area of arts by creating, with the help of the computer, abstract drawings and music from the colors of the fractals generated.

OBJECTIVE

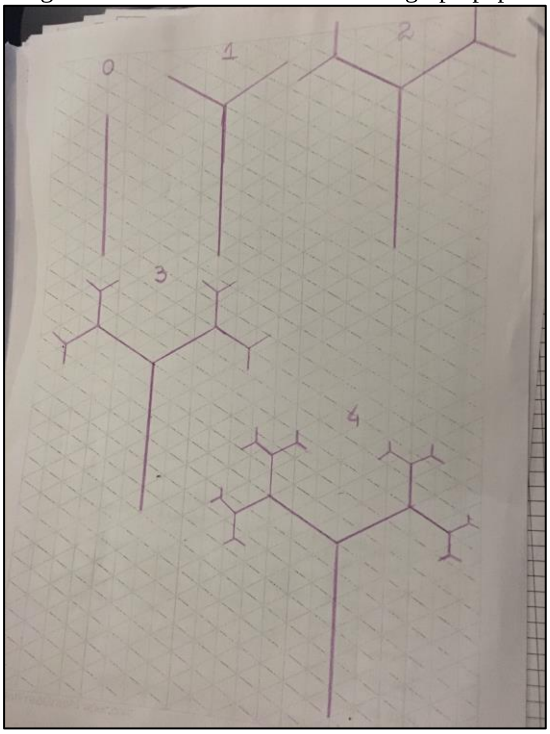
In the face of everything that surrounds us, technological evolutions and scientific advances, we cannot restrict the study of geometry to a few primitive concepts that cannot explain the world as it appears to the eyes of students. For all the contributions that the study of fractal geometry can bring to the teaching of mathematics at the high school level and the possible contextualizations with other areas of knowledge, a didactic sequence was elaborated containing an introductory part, fractal constructions, analysis and application of formulas and, finally, an evaluation. This didactic sequence was implemented in a second-year high school class with the aim of making the study of geometry more attractive and interesting for the student.

The geometry of fractals is not part of the high school curriculum in the institution where the activity was carried out, however the implementation of the didactic sequence was possible using some hours of mathematics in parallel with the study of spatial geometry. Perceiving, over the years, that spatial geometry is a theme that brings with it a certain fear and a lot of difficulties on the part of students, the didactic sequence was built with the objective of introducing notions of fractal geometry so that students can, through the construction of fractals, identify and explore their characteristics and recognize them in their daily lives as well as exploring and applying mathematical content already studied.

METHODOLOGY AND DEVELOPMENT

The implementation of the didactic sequence followed the following organization: in the first class there was a discussion about geometric shapes in nature based on Euclidean geometry and its limits, the presentation of objects and images of fractals, emphasizing the importance of the study of non-Euclidean geometry. In the second period, the students were given graph paper to start, as instructed, the sketch of some fractals. Next, the students began to construct fractals using graph paper, rulers, a protractor and a compass. The constructions were based on guidelines provided and inspired by Silva's (2011) suggestions for activities. At this moment, there was an explanation about the constructions and analysis directing to the application of fractals to topics studied in High School Mathematics. The students took measurements with rulers and made their records for later discussion.

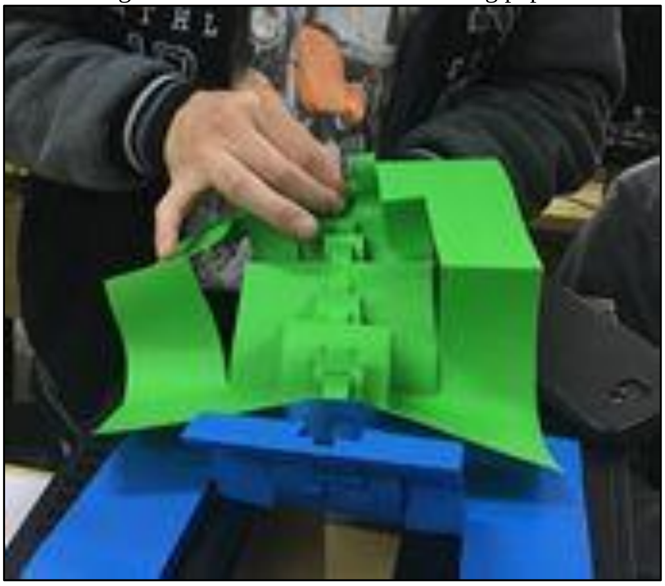
Figure 1: Fractal constructions with graph paper



Source: author herself

The questions proposed to the students were intended to lead them to conclusions about the characteristics of fractals and the relationships with topics already studied in mathematics. In the next class, the students were able to construct the fractals with colored folding paper. Each student chose the fractal they would like to build and received the corresponding instructions. For this moment, two hours of classes were made available. Figure 2 is the image of a construction made by the students in folding paper.

Figure 2 – Constructions with folding paper.



Source: author herself

In the next meeting, the final stage took place in which the students were invited to record their observations, perceptions and everything else they wanted about the realization of the activities. The students were asked about the following aspects: importance of carrying out the activity; what you found most interesting; It was possible to perceive the relationship of fractals with the contents studied and between them the study of fractals could help. Finally, the students shared their impressions of the activity.

Throughout the moments that involved the construction of fractals, both with graph paper and with folding paper, some students showed greater ability while others needed help from the teacher or classmates. The students realized, throughout the constructions, that they could not make cutouts in any way and that everything needed to be measured and/or calculated. In the end, everyone was able to complete their constructions.

Analyzing the students' answers to the questions asked in the last stage, it was possible to see that all of them considered it important to carry out the activity and highlighted many interesting points. The relationships with the topics studied were mentioned based on the comments made during the activities. Some reports from the students about the activity:

> "I never imagined this was fractal. And music then, never. I found it so interesting that mathematics could be just like that, with things that catch our attention but are important to study." (Student A).
>
> "I had never stopped to think that there are many things that the geometry of the school does not explain. I really like geometry, especially spatial geometry, and I found these classes very interesting. But I found some calculations difficult as well." (Student B).
>
> "I thought it was important to study this. I had already seen a question in ENEM and my mother had explained it to me a little, but I hadn't understood it very well. I wish I had other classes like that." (Student C).

Analyzing the performance of the activity as a whole, it is possible to highlight several aspects that demonstrate a very positive return. The fact that it is a topic that provokes curiosity and enchantment reflects the instant interest and continuous involvement of the students.

FINAL THOUGHTS

It is possible to highlight many positive points, related to teaching, learning and the fact that there are many areas of knowledge involved that could be integrated in the development of activities like this, and are capable of justifying the introduction of fractal geometry in high school.

Today's student profile points to the need for teachers to rethink teaching methods and methodologies that are capable of arousing interest, creativity, criticality, autonomy and the ability to put into practice what is worked on in the classroom.

The development and implementation of this didactic sequence demonstrated how rewarding teaching can be when structured with a focus on the student and their learning. As a two-way street in which the student participates in the process and the teacher is motivated to look for other ways to teach mathematics.

REFERENCES

Barbosa, R. M. (2005). Descobrindo a geometria fractal: Para a sala de aula (2nd ed.). Belo Horizonte: Autêntica.

Candau, V. M. (1999). Educação em direitos humanos: Uma proposta de trabalho. In V. M. Candau & M. N. Zenaide (Eds.), Oficinas aprendendo e ensinando direitos humanos (pp. 25–50). João Pessoa: Programa Nacional de Direitos Humanos; Secretaria da Segurança Pública do Estado da Paraíba; Conselho Estadual da Defesa dos Direitos do Homem e do Cidadão.

Carvalho, H. C. de. (2005). Geometria fractal: Perspectivas e possibilidades para o ensino de matemática [Master's thesis, Universidade Federal do Pará]. http://repositorio.ufpa.br/jspui/bitstream/2011/1857/1/Dissertacao_GeometriaFractalPerpectivas.pdf

Gonçalves, A. G. N. (2007). Uma sequência de ensino para o estudo de progressões geométricas via fractais [Master's thesis, Pontifícia Universidade Católica de São Paulo]. http://www.sapientia.pucsp.br//tde_busca/arquivo.php?codArquivo=4953

Lorenzato, S. (Ed.). (2006). Laboratório de ensino de matemática e materiais didáticos manipuláveis. In O laboratório de ensino de matemática na formação de professores (pp. 85–110). Campinas, SP: Autores Associados.

Mandelbrot, B. (1998). Objetos fractais: Forma, acaso e dimensão (2nd ed.). Lisboa: Gradiva. (Original work published 1975, 1984, 1989)

Rêgo, R. M., & Rêgo, R. G. do. (2006). Desenvolvimento e uso de materiais didáticos no ensino de matemática. In S. Lorenzato (Ed.), O laboratório de ensino de matemática na formação de professores (pp. 111–130). Campinas, SP: Autores Associados.

Sallum, E. M. (2005). Fractais no ensino médio. Revista do Professor de Matemática, 57, 33–41. Sociedade Brasileira de Matemática.

Silva, K. B. R. da. (2011). Noções de geometrias não euclidianas: Hiperbólica, da superfície esférica e dos fractais (1st ed.). Curitiba, PR: CRV.

Vargas, A. F., Silva, B. B. da, Lira, C. G., & Lopes, M. M. (2014). Aprendendo matemática através dos fractais: Uma experiência no Ensino Médio. In Anais... XX EREMAT - Encontro Regional de Estudantes de Matemática da Região Sul (pp. 1–12). Fundação Universidade Federal do Pampa (UNIPAMPA), Bagé/RS, Brasil.

Vieira, E., & Volquind, V. (2002). Oficinas de ensino: O quê? Por quê? Como? (4th ed.). Porto Alegre: Edipucrs.