



Study of the affine function and applications in mathematics and physics with an emphasis on contextualized teaching

Estudo da função afim e aplicações nas áreas da matemática e física com ênfase ensino contextualizado

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ABSTRACT

This work deals with a study on the affine function or polynomial function of the first degree, emphasizing some applications in the areas of Mathematics and Physics emphasizing everyday life. First, it reports how some important researchers gave theoretical impulses so that the study of affine function became consistent. Therefore, with the development of that theme, it became



possible to solve a series of everyday problems, such as in the commercial area, in mathematics itself and in the study of kinematics in body movements that are governed by constant and different from zero velocities. Also, the application of this function is verified in tariff applications, such as in company plans in cost per minute run on cell phones, in taxi fares, etc. Therefore, the study of the affine function has a large number of practical applications and the teacher, when teaching the content, must promote a methodology that is linked to the theoretical-practical part so that the student is able to perceive the relevance that the affine function assumes in the everyday context. In this way, the research is concluded by showing the viability that the teacher must articulate in the classroom, considering a study that is connected with the classroom and with the student's daily life where he is able to understand the interaction between the two teaching processes and learning with problems that clarify and give more meaning to the theoretically taught content.

Keywords: Affine function, Mathematics, Physics, Problems, Contextualization.

1 INTRODUCTION

The study of functions in the area of mathematics has been developed in the classroom by some education professionals and should have the character of methodological innovations (RIBEIRO, 2007). As stated by Lima (2013) and Saviani (2008 and 2015), the important thing was "doing"; for the New Pedagogy, it was "learning to learn"; and for the Technician Pedagogy, the important thing is "learning to do".

In a globalized and technological world that today's society offers, the teacher must work together in groups or through research, looking for ways to modify the teaching and learning process in the classroom, seeking to promote a theoretical-practical study in order to cause motivation and curiosity, managing to arouse interest in the content taught. (POWELL, 2004)

There are countless applications in practical terms (ÁVILA, 2005), such as the uniform movement of a body, the plans offered by cell phone companies, in the commercial environment and so on. It is worth noting that the study of this subject is of great importance in various areas of human knowledge, in engineering, physics, chemistry and mathematics itself. (Paul TIPLER, 2000), (D. HALLIDAY, 1996), (D. HALLIDAY, R. RESNICK and J. WALKER, 1996)

However, this subject, when far from contextualization, is considered difficult for many students due to the lack of methodology on the part of some professionals who do not seek to innovate knowledge based on a relationship with everyday life.

However, this work deals with a study of the affine function (GIOVANNI, J. 2005), emphasizing its application in everyday life. However, its general aim is to show that knowledge of the affine function is highly relevant, just like any other subject, as long as it is related to practice.



In order to ensure the consistency of this work, the specific objectives are to provide a study of the affine function and its definition, developing the mathematical formulations, such as root calculation, growth and decrease; to verify that the study of functions can be applied in many everyday situations, such as telephony, the study of kinematics, etc.; to show the historical context that has resulted in the precise mathematical formalism of an affine function.

The methodology is bibliographical, looking at books, websites and articles on the subject in order to build a solid foundation for the study. The justification is to show that once in the classroom, the content between the teacher and the student should not be restricted to the whiteboard and the paintbrush. It must go beyond the classroom so that the student does not build up an erroneous idea of mathematics, taxing it as something purely abstract.

2 HISTORY OF MATHEMATICS IN THE DEVELOPMENT OF THE FUNCTION

2.1 THE HISTORY OF MATHEMATICS

The History of Mathematics is a field of research that has grown a lot both in scientific terms and in the educational field and consequently has made great contributions, especially with regard to the socialization of mathematical knowledge. As such, many topics emphasized by mathematics need to be taught by qualified professionals who can innovate teaching methodologies in search of a better teaching and learning process. By portraying the past, i.e. by looking back in history, in an attempt to uncover procedures or knowledge used by ancient civilizations, scholars from all over the world are trying to reveal facts that have not yet been revealed in the field of the history of mathematics.

In ancient India, reference is made to a very popular pastime for Hindu mathematicians of the time: solving puzzles in public competitions, in which one competitor set problems for another to solve. Mathematics was very difficult at that time. Without any signs, without any variables, only a few wise men were able to solve the problems, using many tricks and laborious geometric constructions. Today, we have the exact language to represent any puzzle or problem. All you have to do is translate them into the language of algebra.

2.2 ORIGIN OF THE FUNCTION

From the time of the Greeks until the Modern Age, the influential theory was Euclidean Geometry, whose basic elements were the point, the line and the plane. From this period onwards, a new theory was born, Infinitesimal Calculus, an expression used to denote objects that are so small that there is no way of seeing or measuring them, but are greater than zero. This theory



contributed to the development of contemporary mathematics, as the notion of functions was one of the foundations of Infinitesimal Calculus.

The appearance of the word "function" as a clearly individualized concept and as an object of study in mathematics only dates back to the end of the 17th century. It has undergone extensive theoretical enrichment over the centuries, especially in the Analytical Method, which revolutionized mathematics.

The introduction of the concept of function, a unifying element for the various branches of mathematics, already represented an attempt to adapt to the latest studies, one of whose fundamental characteristics was to break down the barrier between mathematical fields (MIORIM, 1998, p.106).

Differential and Integral Calculus was spread by Isaac Newton (1642 -1727). He created the "fluxion method", which was based on the categorical discovery that the integration of a function is merely the inverse expression of the differentiation. In doing so, he brought together many different techniques developed previously to solve visibly unrelated problems, analyzing differentiation as a basic operation.

It was Gottfried Wilhelm Von Leibniz (1646 -1716) who was credited with creating the term function in 1694. He used this nomenclature to describe a quantity related to a curve. He also introduced the terms "constant", "variable" and "parameter". Leibniz and Newton are credited with the development of Modern Calculus, in particular the Integral and the Product Rule.

Until 1716, the term "function" was still unknown in the mathematical vocabulary. It was then that Johann Bernoulli (1667 -1748), in 1718, published a wide-ranging article containing his own definition of a function, which was defined as a quantity that is composed of any form of that variable and constant. One of Europe's most admirable mathematicians was Leonhard Euler (1707-1783), who excelled with his discoveries in the varied fields of Calculus and Graph Theory.

He also made many contributions to modern mathematics in the area of terminology and notation, in particular to Mathematical Analysis, such as the notion of a mathematical function. As the study of functions evolved, there were many applications in mathematics and other sciences. As a result, various observations arose that sought an expression to clarify problems encountered in everyday life. In this way, the function became the mathematical example that elucidated the relationship between variables.

The concept of functions is one of the most important concepts in mathematics (LIMA, 2001), especially when studying the concepts of Limits, Derivatives and Integrals. Each of these concepts deals with an analysis and description of what happens to functions. These, "in turn, describe the behaviour of any phenomenon in which some kind of pattern can be seen." Over the



centuries, various mathematical discoveries have been made, so that today we can see the great value of using functions. It has been disseminated in the most varied fields of modern science and can be evaluated as a pillar of countless bases of knowledge.

2.3 HISTORY OF MATHEMATICS AND DIFFICULTIES IN THE TEACHING AND LEARNING PROCESS

Including history in the educational environment as a preview of a particular topic arouses curiosity in students, as they come to realize and know that in the distant past, many men, with their knowledge and research, made important contributions to promoting a much more consistent study of that content. In other words, what guarantees the human species is the transmission of knowledge that helps and expands solutions to many problems, resulting in ease and stability.

The inclusion of history followed by theoretical/practical methodologies contributes to a better teaching and learning process, as the student starts to look at the subject of mathematics from another angle, which they didn't perceive before and builds an interest in learning, being motivated in search of meaningful learning.

The difficulties faced in the classroom by many teachers and students during the teaching-learning process in the study of affine functions or any other subject can be attributed to the lack of resources that could improve the process of teaching mathematics, with a view to a better understanding of the subjects covered. Today's society has an exorbitant amount of computer resources and countless laboratory instruments which, once used, become strong allies for a better teaching and learning process.

It is satisfactory for the teacher to emphasize, before approaching a function study, motivating and involving the student first in the historical context, showing the main mathematicians who contributed to the development and mathematical formalism of the function or any other subject associated with mathematics.

The second step is to develop a theoretical-practical methodology, seeking in the classroom the issue of contextualization that the PCNs clarify as an integral part and improvement for teaching the theme proposed in this work.

Therefore, methodological innovations have crucial value in the teaching and learning process, because if the teacher is able to increasingly qualify and go in search of new experiences with other professionals in the field and build a meaningful teaching process that will motivate, interest and awaken the student to the subject of mathematics (PAIVA, 2008).



3 STUDY OF THE AFFINE FUNCTION

3.1 THE AFFINE FUNCTION.

This study begins with a practical example of an affine function. Consider the following problem: José Gerson takes a regular cab and charges R\$3.00 for the fare and R\$1.50 per kilometer traveled. On this trip, he wants to go to his girlfriend's house, which is 20 km from where he takes the cab. How much will José Gerson spend on a cab?

He will have to pay the 20 X R\$ 1.50 for the distance traveled and another R\$ 3.00 for the flag, i.e. $R\$ 3.00 + R\$ 30.00 = R\$ 33.00$.

If your girlfriend's house was 10 km away, the price of the ride (in reais) would be: $1.50 \times 10 + 3.00 = 15.00 + 3.00 = 18.00$.

Finally, for each distance x covered by the cab, there is a certain price $c(x)$ for the ride. The value $c(x)$ is a function of x . You can easily find the law that expresses $c(x)$ as a function of x . That is,

$$C(x) = 1,50 \cdot x + 3,00$$

Which is a particular case of an affine function.

3.2 DEFINITION OF THE AFFINE FUNCTION

The affine function is expressed as follows:

$$f(x) = ax + b, \quad (1)$$

where a e b are real numbers and a is also non-zero. It is important to understand the meaning of this function. The meaning of function is intrinsic to mathematics and remains the same for any type of function, whether it is a function of the 1st or 2nd degree, or an exponential or logarithmic function.

Therefore, the function is used to relate numerical values of a given algebraic expression according to each value that the variable x assumes. Thus, the function of the 1st degree will relate the numerical values obtained from algebraic expressions of the type $(ax + b)$, thus forming the **function** $f(x) = ax + b$.

You can see that to define a function of the 1st degree, all you need is an algebraic expression of the 1st degree. As previously stated, the purpose of the function is to relate for each value of x a value for $f(x)$.



Note that the numerical values change as the value of x is changed, so we get several ordered pairs, made up as follows: $(x, f(x))$. Note that for each coordinate x we get a coordinate $f(x)$. This helps us to construct function graphs. Therefore, in order to study functions of the 1st degree successfully, you need to understand how to construct a graph and the algebraic manipulation of unknowns and coefficients.

3.3 FUNCTION COEFFICIENTS

Construct a graph of an affine function, as well as this graph being represented by a line. From there, you can check the behavior of the graph of an affine function, based on its coefficients, as we vaguely mentioned in the previous section. In the graphs shown above, the lines form an angle with the axis x . This angle will be called α (alpha).

In the affine function, the coefficient a is mathematically called the angular coefficient or slope, and is associated with the slope of the line representing the graph, i.e. the angle α . The coefficient b is called the linear coefficient, where its value corresponds to the ordinate of the point where the line cuts the y -axis. When the angular coefficient is maintained (a) and the linear coefficient (b) you can see a translation of the lines in the Cartesian plane, where the angle α is maintained and the point where the line cuts the y -axis (linear coefficient = b).

As already mentioned, an affine function is any function of \mathbb{R} on \mathbb{R} given by a law of the form $f(x) = ax + b$ where a e b are real numbers given and $a \neq 0$. The number a is called the angular coefficient and b is called the constant term or linear coefficient.

3.4 DETERMINING THE ZERO OF THE FUNCTION

The value of the real number x for which we have $y = 0$ is called the zero of the polynomial function of the 1st degree" (GIOVANNI JR., 2007). Geometrically, the graph of the function on the Cartesian plane and locating its zero, is the point where the line intersects the axis x . Therefore, to determine the zero or root of a function, simply consider $f(x) = 0$. In other words,

$$\begin{aligned} f(x) = ax + b \rightarrow f(x) = 0 \rightarrow ax + b = 0 \rightarrow ax = -b \rightarrow \\ x = -b/a \end{aligned} \quad (2)$$

Where $x = -b/a$ is the point where the line intersects the abscissa axis.



3.5 REPRESENTATION IN THE CARTESIAN PLANE

An affine function is represented on the Cartesian plane by a line. The function can be increasing or decreasing, which determines the position of the line. The graph of a function, $y = ax + b$ with a is a line oblique to the Ox and Oy axes in the plane.

The graph of the affine function $y = ax + b$ is a straight line. The coefficient of x is called the angular coefficient of the line and, as we'll see below, a is linked to the slope of the line in relation to the axis Ox . The constant term b is called the linear coefficient of the line. For $x = 0$, we have $y = a \cdot 0 + b = b$. Thus, the linear coefficient is the ordinate of the point where the line cuts the axis Oy .

When a line has two known points, to obtain its law of formation, you just need to solve a first degree system. In other words, you need to calculate the values of the angular and linear coefficients of the line. Let the function be

$$f(x) = ax + b$$

Being

$$A(x_1, y_1) \text{ e } B(x_2, y_2)$$

Therefore: For point A: $ax_1 + b = y_1$ and for point B: $ax_2 + b = y_2$

If you subtract B from A, you get:

$$\begin{aligned} ax_2 + b - (ax_1 + b) &= y_2 - y_1 \rightarrow ax_2 + b - ax_1 - b = y_2 - y_1 \\ ax_2 - ax_1 &= y_2 - y_1 \rightarrow \\ a &= \frac{y_2 - y_1}{x_2 - x_1} \end{aligned} \quad (3)$$

Which represents the angular coefficient of the line.

For the linear coefficient b , it follows that:

$$ax_1 + b = y_1 \rightarrow \frac{y_2 - y_1}{x_2 - x_1} x_1 + b = y_1 \rightarrow b = y_1 - \frac{-x_1 y_2 + x_1 y_1 + y_1 x_2 - y_1 x_1}{x_2 - x_1}$$

Then:

$$b = \frac{-x_1 y_2 + y_1 x_2}{x_2 - x_1} \quad (4)$$



Which represents the linear coefficient b of the line. Therefore, the line will have the following equation:

$$f(x) = \frac{(y_2 - y_1)}{x_2 - x_1} x + \frac{-x_1 y_2 + y_1 x_2}{x_2 - x_1} \quad (5)$$

Given the points

$$A(x_1, y_1) \text{ e } B(x_2, y_2),$$

It follows that:

$$f(x_1) = a x_1 + b \text{ e } f(x_2) = a x_2 + b,$$

Hence

$$f(x_2) - f(x_1) = a(x_2 - x_1),$$

So:

$$a = f(x_2) - f(x_1) / x_2 - x_1 \quad (6)$$

a → angular coefficient (determines the slope of the line in relation to the axis ox).

b → linear coefficient (point where the line intersects the axis oy).

The law of the function $f(x) = ax + b$ represents the equation of a straight line.

Example 1:

The table below shows the temperature of the water in the Atlantic Ocean (at the equator) as a function of depth.

DEPTH (M)	TEMPERATURE (°C)
Surface	27
100	21
500	7
1000	4
3000	2,8888

Assuming that the temperature variation is approximately linear between each of the two depth measurements taken, determine the predicted temperature for the 400m depth:

Solution:

The first thing to note about this example is which values from the table we should use to solve it. As we want to determine the temperature for a depth of 400 m, and this value is between 100m and 500m, then we set up a system with these values.



$$21 = 100a + b$$

And

$$500a + b = 7$$

Making the difference between

$$\begin{aligned} 7 - 21 &= 500a + b - (100a + b) \\ -14 &= 500a - 100a \rightarrow 400a = -14 \rightarrow a = \frac{-7}{200} \end{aligned}$$

Substituting the value of a, we find b:

$$\begin{aligned} 21 &= 100a + b \rightarrow 21 = -100 \cdot \left(\frac{-7}{200}\right) + b \rightarrow b = 21 + 3,5 \\ &\rightarrow b = 24,5 \end{aligned}$$

Using the expression in (1), it follows that,

$$f(x) = ax + b$$

Then:

$$f(x) = -\frac{7}{200}x + 24,5$$

For $x = 400m$, it follows that:

$$f(400) = -\frac{7}{200} \cdot 400 + 24,5 \rightarrow f(400) = 10,5^\circ C$$

At a depth of 400m the temperature is $10,5^\circ C$. This example shows the importance of contextualizing the study of functions with everyday problems.

3.6 GROWTH AND DECLINE

The function is said to be decreasing because the angle formed by the line and the abscissa axis is obtuse ($a < 0$) and increasing for $a > 0$. In other words,

- Increasing function: For $a > 0$: if $x_1 < x_2$, then $ax_1 < ax_2$. Hence, $ax_1 + b < ax_2 + b$, from which comes $f(x_1) < f(x_2)$.



- Decreasing function: For $a < 0$: if $x_1 < x_2$, then $ax_1 > ax_2$. Hence, $ax_1 + b > ax_2 + b$, from which comes $f(x_1) > f(x_2)$.

Table 1 illustrates these cases better,

Table 1: Signs of a and b for studying the growth and decay of a function

COEFFICIENTS	FUNCTION
$a > 0$ and $b > 0$	Growing
$a > 0$ and $b < 0$	Growing
$a < 0$ and $b < 0$	Decreasing
$a < 0$ and $b > 0$	Decreasing

Source: Authors' own

Example 2:

The cost of manufacturing x unit of a product is $C = 100 + 2x$. Each unit is sold at the price $p = R\$ 3,00$. To make a profit equal to $R\$ 1.250,00$ must be sold k units must be sold. Determine the value of K .

Let $L = 1.250,00$ e $p = 3,00$, then

$$C = 100 + 2x \rightarrow L(x) = p \cdot x - C(x)$$

Then:

$$L(x) = p \cdot x - 100 - 2x$$

Therefore:

$$F(x) = 3 \cdot x - 100 - 2x$$

Or

$$1.250k = 1.350$$

3.7 FUNCTION SIGN

To study the sign of any $y = f(x)$ is to determine the values of x for which y is positive, the values of x for which y is zero and the values of x for which y is negative. Let's consider an affine function

$$y = f(x) = ax + b$$

Consider the sign for this function cancels out. That is,

$$f(x) = ax + b \rightarrow f(x) = 0 \rightarrow ax + b = 0 \rightarrow ax = -b$$



Then,

$$x = -b/a$$

There are two possible cases: $a > 0$. That is, y is positive for values of x greater than the root; y is negative for values of x less than the root. For $a < 0$, y is positive for values of x less than the root; y is negative for values of x greater than the root.

Example 3:

A health insurance company offers its customers the following monthly payment options:
Plan A: a fixed amount of R\$110.00 plus R\$20.00 per consultation within the period. Plan B: a fixed amount of R\$130.00 plus R\$15.00 per consultation within the period. Analyze the plans in order to demonstrate under which conditions one or the other is more advantageous.

Function of plane A:

$$y = 20x + 110$$

Function of plan B:

$$y = 15x + 130$$

A moment when the plans are exactly the same:

$$A = B$$

$$20x + 110 = 15x + 130 \rightarrow 20x - 15x = 130 - 110$$

$$5x = 20 \rightarrow x = 20/5 \rightarrow x = 4$$

Cost of plan A lower than cost of plan B: $A < B$.

$$20x + 110 < 15x + 130 \rightarrow 20x - 15x < 130 - 110$$

$$5x < 20 \rightarrow x < \frac{20}{5} \rightarrow x < 4$$

Cost of plan B less than cost of plan A: $B < A$.

$$15x + 130 < 20x + 110 \rightarrow 15x - 20x < 110 - 130$$

$$-5x < -20 (-1) \rightarrow x > \frac{20}{5}$$

Then,

$$x > 4$$



If the client has four appointments a month, they can choose any plan. If the number of visits is more than four, plan B has a lower cost. If the number of visits is less than four, plan A costs less. Two cell phone companies offer their customers the following plans:

A: fixed price of 30.00 and 0.25 per minute used; **B:** fixed price of 40.00 and 0.20 per minute used

3.8 SEGMENTING THE LINE

Given the line $y = ax + b$, a much easier way of plotting it in the Cartesian system is to express this line in segmented form. Similar to what is done in analytic geometry in the study of the line. Let the line have equation

$$y = -\frac{x}{2} + 2$$

One way of writing it in segmentary form is to give it the following notation:

$$1 = \frac{x}{p} + \frac{y}{q} \quad (8)$$

According to the above expression, it follows that:

$$y = -\frac{x}{2} + 2 \rightarrow y + \frac{x}{2} = 2 \rightarrow \frac{y}{2} + \frac{x}{4} = 1$$

Therefore, comparing the equation in segmentary form, we have that:

$$p = 2 \text{ e } q = 4$$

The line can be seen in the following figure:

3.9 THE LINE, ANGULAR COEFFICIENT AND A POINT

Another important result that can be observed when studying the determination of a function of the 1st degree is the fact that you can determine the function when you know a point and the angular coefficient of the line. A very convenient way of determining the function is to use the following expression:

$$y - y_0 = tg\theta(x - x_0) \quad (9)$$

Where y_0 e x_0 represents the coordinate of the given point and $tg\theta$ represents the angular coefficient of the line.



Example 4:

A line passes through the point P(2,4) and has an angular coefficient of $m=2$. Determine the equation of the line.

Solution

According to the above expression, the equation of the line can be obtained considering that:

$$y - y_0 = tg\theta(x - x_0)$$

Like: $y_0 = 4$, $x_0 = 2$ e $m = 2$. Since the angular coefficient of the line is a $tg\theta$,

It follows that: $tg\theta = m = 2$.

Then,

$$y - 4 = 2(x - 2) \rightarrow y - 4 = 2x - 4 \rightarrow y = 2x - 4 + 4$$
$$y = 2x$$

The linear coefficient is the number alone at the end of the function, when the function is in general format ($y = ax + b$). And this coefficient is very useful when you want to draw the graph of a function of the first degree, it tells you nothing more and nothing less than the point at which the line *cuts the* Y axis (vertical axis). This is because any point that lies on the Y axis has an X value of zero, and if you put in a function the X being zero, all that remains is to say that y is equal to the linear coefficient!

Another way of writing the graph of a function of the first degree is to use an important result from analytic geometry which states that it is possible to obtain the equation of the line when you know a point and the angular coefficient of the line. This is what we saw in the topic above.

4 APPLICATION OF THE AFFINE FUNCTION IN PHYSICS

Some graphs of movement, studied in Mechanics in Physics with the aim of graphically relating position and velocity to time. The time function of position in uniform motion is given by $s = s_0 + vt$ where S is the final position of a mobile, s_0 is the initial position of the mobile, t is the time spent by this mobile on the journey. This function is of the 1st degree and is represented by an increasing or decreasing line depending on the sign of the velocity (RAYMOND SERWAY, 1996).

The study of the equation of the line developed above has numerous applications in physics, especially in the study of kinematics, which corresponds to the movement of a body performing a uniform movement in a frictionless region, where it is relative to Newton's first law



where we consider that there is no force applied to the body. In this sense, the rate of change of space as a function of time results in a quantity called constant throughout the movement of the body, which can be represented by the following mathematical expression

$$v = \frac{\Delta s}{\Delta t} \quad (10)$$

A function can be continuous throughout its domain or only at a point $x = a$. If there is a limit of $f(x)$ at x , a point in D , tending to "a" and this limit is equal to $f(a)$, this function is said to be continuous at this point "a" and if it has this property at all points in D , then it is said to be continuous throughout its domain. (ÁVILA, 2006)

As will be seen in the expression that relates the position of the body in relation to time, corresponding to a function of the first degree. The Study of Space as a Function of Time - A piece of furniture is moving uniformly when it covers equal spaces in equal times, i.e. the space varies uniformly over time. This only occurs when the speed of the mobile remains constant throughout the journey.

4.1 UNIFORM FORWARD AND BACKWARD MOTION

The direction of the body's movement coincides with the direction set as positive for the trajectory; the speed of the mobile is positive; the spaces increase in relation to the origin. The mover moves against the direction of the trajectory; the speed is negative; the spaces decrease algebraically in relation to the origin.

4.2 TIME FUNCTION OF UNIFORM MOTION

If we represent the initial space by S_0 ($t_0 = 0$) and the final space by S , at any time t , we get the following:

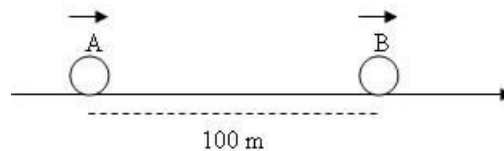
$$x = x_0 + v \cdot t \quad (11)$$

The figure below shows that position in relation to time represents a function of the first degree on the Cartesian plane. In this graph, if the speed is positive, the graph is increasing. For the second graph, if the velocity is negative, the line is decreasing. In this case, we say that the mover is moving backwards.

As there is no acceleration when studying the movement of a piece of furniture in uniform motion, the acceleration graph represents a line that coincides with the abscissa axis. Since velocity is a constant quantity, its graph as a function of time is a line parallel to it.

Example 5:

1-Two cars, A and B, are moving uniformly in the same direction. Their scalar velocities are 5 m/s and 15 m/s respectively. At the instant $t = 0$, the cars are in the positions shown below.



Determine:

a) the instant at which A reaches B;

Solution:

To solve this exercise it is necessary to equate the time functions of the cars. That is, find the time functions of the spaces of each car. Thus,

$$S_A = S_{0A} + v_A \cdot t \text{ e } S_B = S_{0B} + v_B \cdot t$$

$$S_A = 0 + 15 \cdot t \text{ e } S_B = 100 + 5 \cdot t$$

Once you have found the time functions and to determine the instant at which A meets B, simply equate the functions

$$S_A = S_B .$$

$$0 + 15 \cdot t = 100 + 5 \cdot t \rightarrow 15t - 5t = 100 \rightarrow 10t = 100$$

Therefore, $t = 10 \text{ s}$.

In other words, it takes mobile A 10s to reach mobile B. However, the question arises: At what position does mobile A reach mobile B?

b) at what distance from the initial position of A does the encounter occur.

To find the distance from point A that the encounter occurs, simply substitute the value of the time found in the previous item into the time function of the space of car A. Thus:

$$S_A = 0 + 15 \cdot 10 \rightarrow S_A = 150 \text{ m}$$

Therefore, the meeting position is 150 m from the starting position A.

4.3 HEALTH INSURANCE COMPANY

A health insurance company offers its customers the following monthly payment options:



Plan A: a fixed amount of R\$110.00 plus R\$20.00 per consultation within the period.

Plan B: a fixed amount of R\$130.00 plus R\$15.00 per consultation within the period.

Analyze the plans in order to demonstrate under which conditions one or the other is more advantageous.

Function of plan A: $y = 20x + 110$

Function of plan B: $y = 15x + 130$

Moment when the plans are exactly the same: $A = B$

$$\begin{aligned} 20x + 110 &= 15x + 130 \rightarrow 20x - 15x = 130 - 110 \\ 5x &= 20 \rightarrow x = 20/5 \rightarrow x = 4 \end{aligned}$$

Cost of plan A less than cost of plan B: $A < B$.

$$\begin{aligned} 20x + 110 &< 15x + 130 \rightarrow 20x - 15x < 130 - 110 \\ 5x &< 20 \rightarrow x < 20/5 \rightarrow x < 4 \end{aligned}$$

Cost of plan B lower than cost of plan A: $B < A$.

$$\begin{aligned} 15x + 130 &< 20x + 110 \rightarrow 15x - 20x < 110 - 130 \\ -5x &< -20 \quad (-1) \rightarrow x > 20/5 \rightarrow x > 4 \end{aligned}$$

4.4 THE COST OF AN INDUSTRIAL PRODUCT

The cost of an industrial product is given by $C(x) = 250.00 + 10.00x$, where x is the number of units produced and $C(x)$ is the cost in reais. What is the cost of 1000 units of this product?

Solution:

$$C(x) = 250,00 + 10,00x,$$

It follows that

$$C(1000) = 250,00 + 10,00 \cdot 1000 \rightarrow C(1000) = 10.250,00$$

the cost of 1000 units of this product is $C(1000) = 10.250,00$

4.5 THE NUMBER OF UNITS PRODUCED

The number of units produced (y) of a product during a month is a function of the number of employees (x) according to the relationship $y = 60x$. Knowing that 30 employees are



employed, calculate the increase in monthly production in units if 20 more employees are hired.

Solution:

$$y_1 = 60x \rightarrow y = 60.30 \rightarrow y = 1800$$

$$y_2 = 60x \rightarrow y = 60.50 \rightarrow y = 3000$$

Then:

$$\Delta y = 3000 - 1800 \rightarrow \Delta y = 1200$$

4.6 CHOOSING A HEALTH PLAN

A person is going to choose a health insurance plan between two options: A and B. Plan conditions: Plan A: charges a fixed monthly amount of R\$140.00 and R\$20.00 per consultation over a certain period. Plan B: charges a fixed monthly amount of R\$110.00 and R\$25.00 per consultation over a certain period.

Solution:

The total cost of each plan depends on the number of appointments x within the pre-established period.

Let's determine

- a) The function corresponding to each plan.
- b) In which situation plan A is more economical; plan B is more economical; the two are equivalent.

a) Plan A: $f(x) = 20x + 140$

Plan B: $g(x) = 25x + 110$

- b) So that plan A is more economical:

$$g(x) > f(x)$$

$$25x + 110 > 20x + 140 \rightarrow 25x - 20x > 140 - 110$$

$$5x > 30 \rightarrow x > 30/5 \rightarrow x > 6$$

So that Plan B is more economical:

$$g(x) < f(x)$$

$$25x + 110 < 20x + 140 \rightarrow 25x - 20x < 140 - 110$$

$$5x < 30 \rightarrow x < 30/5 \rightarrow x < 6$$



So that they are equivalent:

$$\begin{aligned}g(x) &= f(x) \\25x + 110 &= 20x + 140 \rightarrow 25x - 20x = 140 - 110 \\5x &= 30 \rightarrow x = \frac{30}{5} \rightarrow x = 6\end{aligned}$$

The most economical plan will be:

Plan A = when the number of consultations is greater than 6.

Plan B = when the number of consultations is less than 6.

The two plans will be equivalent when the number of consultations is equal to 6.

4.7 BUYING SHOES

A store manager buys a pair of shoes for R\$45.00 and sells them for R\$75.00. Knowing that the cost of freight is R\$70.00, how many shoes of this model should the store sell to make a profit of R\$9,200.00?

$$\begin{aligned}C(x) &= 45x + 70 \\V(x) &= 75x \\L(x) &= V(x) - C(x) \rightarrow L(x) = 75x - 45x - 70 \\L(x) &= 30x - 70 \rightarrow 9200 = 30x - 70 \rightarrow 30x = 9270 \\x &= \frac{9270}{30} \rightarrow x = 309\end{aligned}$$

4.8 THE PRICE OF A CAB RIDE

The price you pay for a cab ride depends on the distance you travel. The fare y is made up of two parts: a fixed part called the bandeirada and a variable part that depends on the number x of kilometers traveled. Suppose that the fare is R\$6.00 and the kilometer driven is R\$1.20.

a) Express y as a function of x .

Solution:

$$Y = 6,00 + 1,2x$$

b) How much will you pay for a cab ride of 10 km?

$$\begin{aligned}Y &= 6,00 + 1,2x \rightarrow y = 6,00 + 1,2 \cdot 10 \\y &= 6,00 + 12,00 \rightarrow y = 18,00\end{aligned}$$



4.9 DETERMINING A VARIABLE GIVEN A FUNCTION.

Determine the value of p so that the graph of the function $f(x) = 3x + p - 2$ intersects the y -axis at the point of ordinate 4.

Solution:

$$f(x) = 3x + p - 2$$
$$f(0) = 4 \rightarrow f(0) = 3 \cdot 0 + p - 2 \rightarrow p - 2 = 4 \rightarrow p = 2 + 4 \rightarrow p = 6$$

4.10 PRODUCTION OF PARTS IN A FACTORY.

In the production of parts, a factory has a fixed cost of R\$200.00 plus a variable cost of R\$1.20 per part produced. What is the cost of producing 10,000 parts?

How many pieces can be produced with R\$20,000.00?

Law of formation of the function

Note that we have a fixed value of R\$200.00 and a value that varies according to the number of pieces produced, R\$1.20.

$$y = 1,2x + 200$$

Cost of producing 10,000

$$y = 1,2 * 10.000 + 200 \rightarrow y = 12.000 + 200 \rightarrow y = 12.200$$

The cost of producing 10,000 pieces is R\$12,200.00. Number of pieces that can be produced with R\$20,000.00

$$1,2x + 200 = 20.000 \rightarrow 1,2x = 20.000 - 200$$
$$1,2x = 19.800 \rightarrow x = 19.800 / 1,2 \rightarrow x = 16.500$$

16,500 pieces will be produced.

4.11 CHARGING FOR CAB RIDES

A cab driver charges R\$4.50 plus R\$0.90 per kilometer driven. Knowing that the price to be paid is a function of the number of kilometers traveled, calculate the price to be paid for a ride in which 22 kilometers are covered?

$$f(x) = 0,9x + 4,5 \rightarrow f(22) = 0,9 \cdot 22 + 4,5$$
$$f(22) = 19,8 + 4,5 \rightarrow f(22) = 24,3$$



The price to be paid for a race that covered 22 kilometers is R\$24.30.

5 FINAL CONSIDERATIONS

The teaching practice gave the students the opportunity to construct the concept of a polynomial function of the 1st degree, understanding the relationship between the content studied and life outside of school and also within it, through a set of situations that give meaning to this study. Observing the argument above, it is clear that it is only possible to lead students to a greater understanding of the study of functions when the teacher has a methodological practice that aims to build a link between theory and the student's daily life. In this sense, the class opens up a range of opportunities where they can understand and conceptualize what the teacher has proposed. This is the main role of a qualified professional who is responsible for education.

By working on the properties, symbolic representations and exercises relating to the study of the function of the 1st degree, it was possible to facilitate the subject from a connection with everyday life. It is clear that the study of the 1st degree function is important for students when it is applied in a contextualized way. The absence of abstraction is useful if the teaching and learning process is to be effective. As such, teachers who teach functions at secondary school level need to review their concepts and practices to avoid the problems that exist in most state schools. This problem is reflected in the dropout and failure rates in this transition between primary and secondary school.

However, it's obvious to point out that it doesn't just happen at this stage, as it's been found to cover the whole of primary and secondary education. The reason for this is the lack of an innovative methodology that could be acquired through continuing training for teachers, which is not the case in practice. However, although this continuing training is not a real fact, there is the possibility of circumventing the abstract educational work of some professionals. This is abstract because they use an archaic methodology with no pedagogical or methodological basis and no resources that could improve the teaching and learning process.

As such, the aforementioned topic sought to interweave contextualization into this teaching process, involving, throughout the approach, problems related to the student's daily life where they could understand the importance of knowing the presence of the application of the 1st degree function in countless situations. Therefore, it is only fair to point out that the work presented was consistent with the practice experienced by each student who makes up the educational environment and who can, when possible, have a much more comprehensive knowledge of this and other subjects, as long as the teacher "escapes" abstractions and interpolates contextualization into each subject taught as an inherent part of the entire teaching and learning process. Acting in



this way will prevent many of the problems such as evasion and failure from becoming part of educational life.



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