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Angles and areas in plane geometry with emphasis on history and everyday applications

Ângulos e áreas em geometria plana com ênfase na história e aplicações cotidianas

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Luis Augusto Valente da Cunha
Federal University of Pará, Abaetetuba Campus
Manoel de Abreu, S/N, Mutirão

José Francisco da Silva Costa
Federal University of Pará, Abaetetuba Campus
Manoel de Abreu, S/N, Mutirão

José Antônio Farias Nonato
Maranhão State University
Municipal Secretariat of Education in Moju-Pa

Marinaldo Carvalho Lobato
Pará State University-UEPA
Laboratory of Preparation and Computation of Nanomaterials-LPCN, Belém, Pará

José Eduardo Pastana Silva
Federal University of Pará, Abaetetuba Campus
Manoel de Abreu, S/N, Mutirão

Raimundo das Graças Carvalho Almeida
Federal University of Pará
Center for Exact and Natural Sciences-ICEN, Belém, Pará

ABSTRACT
This work has as general objective to understand the historical aspect of plane geometry, calculation of plane areas and applications. For a better approach to the theme, it develops a study on the primitive ideas between straight and plane points, opening a gap to introduce essential elements for a better understanding of the content. Methodology is developed in a bibliographical study based on some authors such as Gelson Iessi, Osvaldo Dulce, Geovane, etc. which contributes to a better solid construction of the exposed content. In terms of specific objectives, it brings as main goals to show that the study of plane geometry is widely used in practice, mainly in house constructions where its relevance is observed in the inherent geometric figures of regular polygons, perimeters and areas that, routinely, are observed and that requires knowledge of plane geometry to consolidate a solid approach to content; Check that plane geometry has the advantage of helping professionals with problems encountered in everyday life, since they need this knowledge to carry out certain measurements that are essential for correct calculations of areas of polygons, such as areas of plane figures.; Analyze the relevance of mathematical formalism for a better understanding of the theory and subsequent practical application. The research concludes considering that plane geometry is commonly found in practice and it is essential to know how to

apply mathematical knowledge in everyday situations. Therefore, it is justifiable that the essence of this content is of fundamental practical and theoretical importance.

Keywords: Plane Geometry, Historical process, Flat areas, application.

1 INTRODUCTION

This paper discusses a study associated with plane geometry with emphasis on everyday applications. It is important to note in this part when referring to everyday life that many educators address this issue in the classroom, giving little importance in a large number of applications that could be linked in the process of teaching and learning as a methodological way that has as its object, the incentive, motivation and interest in this subject. Theory alone is not enough for the teacher to alleviate the difficulties that most students face in the educational environment.

It is necessary that the teacher provides a teaching method that comes together, theory and practice to awaken the student's interest in mathematics. Therefore, plane geometry has a vast field of practical situations that the teacher can use as a resource to show the student how to apply plane geometry in everyday life.

It is also worth considering that the historical part of the plane geometry shows how it became consolidated over time due to the contribution of some philosophers and mathematicians who with their work managed to bring such knowledge to this day. As justification is to consider that plane geometry is linked to everyday life, nature and all objects created by man himself, and, above all, the relationship between theory and practice of the subject studied in the classroom. As for the methodology, it consists of a bibliographical research, supported by the theory of plane geometry linked to some everyday problems, clarifying deep down that science only walks on two footprints: Theory and practice.

1.1 GENERAL OBJECTIVE

Understand the relevance of plane geometry and its usefulness in everyday applications.

1.2 SPECIFIC OBJECTIVES

To approach that the study of the areas of plane figures can be used in different everyday situations;

Verify that the triangle theory as well as Thales' theorem are useful for calculating unknown values of height and length of objects using the similars between triangles or using Thales' theorem;

Show that the study of plane geometry has a great advantage in everyday applications, and that the theorems and properties are of essence for a better understanding of the problems.

2 HISTORY OF GEOMETRY AND THE NATIONAL CURRICULAR PARAMETERS

2.1 THE CREATION OF GEOMETRY

Historians have attributed the creation of geometry to the Egyptians and the Chaldeans. The Chaldeans were people of Semitic origin (Babylonian, Assyrian and Phoenician peoples) who inhabited Mesopotamia, a region of Western Asia, between the Tigris and Euphrates rivers, where Iraq is located today. The word geometry is derived from the Greek geometrein, meaning earth measurement (geo= earth and metrein = measure). Geometry is the branch of mathematics concerned with the properties of space, more in terms of plane (two-dimensional) and solid (three-dimensional) figures. Some mathematicians took decisive steps to make geometry solid through time.

2.2 MATHEMATICIANS AND PHILOSOPHERS WHO CONTRIBUTED TO THE DEVELOPMENT OF GEOMETRY

2.2.1 Euclid

He lived in Byzantium[1] between the years 485 and 410 B.C. At that time, the wise Ptolemy I succeeded Alexander the Great on the throne of Egypt. Under his care, an institution emerged in Alexandria, called the "Museum", which brought together most of the wise men of the time. The Museum was erected next to the royal palace, and had residential quarters, classrooms, lecture halls, and most importantly, the largest library of the time.

Euclid was the first director of the Museum, and, thanks to this, he could organize the results obtained by previous mathematicians (Thales, Pythagoras, Eudoxo and others). This organization is found in his immortal work, modestly titled "The Elements". "The Elements" is a set of 13 books devoted to the logical and systematic foundation and development of geometry. The first book deals with the questions that are fundamental to geometry, and its style, its ordering, served as the guiding standards for all other later works of mathematics.

[1] https://www.infoescola.com/biografias/euclides/

2.2.2 Thales of Miletus

He was born around 624 B.C. in Miletus[2] , Asia Minor (now Turkey), and died around 547 B.C. also in Miletus. He is described in some legends as a businessman, a salt merchant, an advocate of celibacy, or a statesman of vision. But the truth is that little is known about his life. Thales' works have not survived to our days, but based on traditions some ideas can be reconstructed.

Traveling extensively in the ancient centers of knowledge he must have obtained information about astronomy and mathematics, learning geometry in Egypt. In Babylon, under Nebuchadnezzar's rule, he came into contact with the first astronomical tables and instruments and it is said that in 585 B.C. he was able to predict the solar eclipse that would occur this year, astonishing his contemporaries.

Thales is considered the first philosopher and the first of the seven wise men, a disciple of the Egyptians and Chaldeans, and is commonly given the title "first true Mathematician", trying to organize Geometry in a deductive way. It is believed that during his trip to Babylon he studied the result that comes down to us as "Theorem of Thales: an angle inscribed in a semicircle is a right angle". We owe to his studies the knowledge of four more fundamental theorems of Geometry. He was the master of a group of followers of his ideas, called the "Jániá School" and was the first man in history to whom specific mathematical discoveries are attributed. As Aristotle (another Greek mathematician) said, "for Thales the primary question is not what we know, but how we know it.

2.2.3 Pythagoras

Pythagoras[3] lived 2500 years ago and did not leave his written works. What is known about his biography and his ideas is a mixture of legend and real history. The legend begins even before Pythagoras was born: around 580 BC, the priestess of the god Apollo said to a couple who lived on the island of Samos, in the Aegean Sea: "You will have a son of great beauty and extraordinary intelligence, will be one of the wisest men of all time. That same year the couple had a son, naming him Pythagoras.

Legend or no legend, the intelligence of the young Pythagoras amazed the doctors of the best schools of Samos: they could not answer the questions of the boy at 16 years. Under these conditions, there was only one thing to do: they sent Pythagoras to Miletus to study with Thales (the greatest sage of the time). As an adult, Pythagoras decided to broaden their interests. He began

[2] https://brasilescola.uol.com.br/biografia/tales-de-mileto.htm
[3] https://pt-static.z-dn.net/files/d41/7664240ec825d14d1f70e42812999929.pdf

to add, in addition to numbers, ideas about science and religion of other peoples. Believing that we must see to believe, packed his bags and said "see you later" to their patrons: he went to Syria, then Arabia, Chaldea, Persia, India and finally Egypt, where he spent more than 20 years and became a priest to better understand the mysteries of Egyptian religion.

A long time had passed and Pythagoras was already over 50 years old. His desire was to return to Samos and open a school. But Samos had changed and the dictator Polycrates, who ruled the island, did not care either schools or temples. So Pythagoras went to Italy, where he founded his school and could teach arithmetic, geometry, music and astronomy, as well as religion and morals.

More than a school, Pythagoras had managed to create a religious, philosophical and political community. The students who trained left to occupy high positions in the local government, aware of their wisdom twisted the nose before the ignorant masses and supported the aristocratic party. The result: the masses responded with violence, and some say they burned down the school, arrested the teacher, and killed him. Others are more optimistic: they say that Pythagoras was only exiled to Metaponto, further north, in Lucania, where he died forgotten, but in peace, over 80 years old.

Based on the above it is considered that geometric knowledge as it is conceived today was not always so. Geometry emerged intuitively, and like any branch of knowledge, was born of human need and observation. It began naturally through man's observation of nature. To designate this type of event, Subconscious Geometry appeared. Geometric knowledge was also needed by priests. As they were the tax collectors of the time, they were in charge of demarcating the lands that were devastated by the floods of the Nile River in Egypt. The sharing of the land was directly proportional to the taxes paid. Rooted in this purely human need, the calculation of area was born.

Many events took place, still in the field of Subconscious Geometry, until the human mind *was* able to absorb properties of shapes previously seen intuitively. With this achievement, Scientific or Western Geometry was born. This geometry, seen in educational institutions, incorporates a series of rules and logical sequence responsible for its definitions and solutions to geometric problems.

With this assumption in mind, this paper seeks to address a study associated with plane geometry creating a relevant link between theory and practice. To achieve this goal, it develops a study of the primitive ideas between point, line and plane, opening a gap to introduce essential elements for a better understanding of the content. However, to ensure this theoretical development the work reports as methodology a study of bibliographic character based on some authors such as

Gelson Iessi, Osvaldo Dulce, Geovane, etc. where they build a solid base of the exposed content. As for the specific objective is to show that the study of plane geometry is widely used in practice, especially in construction of houses where some regular polygons are observed; Verify that the plane geometry has the advantage of helping the professional in the construction of houses, since it needs this knowledge for the realization of certain measures that are essential for the correct calculation of polygon areas, such as triangle, square and so on; Analyze the relevance of mathematical formalism for a better understanding of the theory and subsequent practical application.

For justification, it considers that plane geometry is inherently found in practice and that it is essential to know how to apply mathematical knowledge in everyday situations. Thus, a bricklayer, a carpenter or any other professional needs to have a full grasp of the contents related to this topic since they need to understand and interpret the mathematical formulations implicit in such practical situations. Therefore, plane geometry has the advantage of helping these professionals to have greater confidence in the measurements of a door, a window or any other area where geometric and analytical knowledge of plane geometry is present. Thus, it becomes justifiable that the essence of this content is of fundamental practical and theoretical importance.

2.3 MATHEMATICS AND THE PROCESS OF TEACHING FOR EVERYDAY LIFE

To see that mathematics is present in everyday life, just walk around the city or even in the neighborhood and observe the shapes of buildings, street signs, streets, and nature. Math should not be seen only as a prerequisite for further studies. It is necessary that teaching is focused on the education of the citizen, who uses more and more mathematical concepts in his routine (PCN - Special Edition, p.51). In this sense, it is necessary that the teacher is able to relate the contents taught in the classroom to the reality in which he lives, becoming a transforming agent of this reality.

> [...] in this scenario, planning a course becomes dependent on the citizen one wants to train. And since nobody has drawers of knowledge in their heads, where the contents rest isolated, the only way out is to plan in a collective way. Links must be sought with other areas and among the contents of the discipline itself (FALZETTA, 2001, p. 54-55).

Math can no longer be thought of as a linear sequence of information, but as a web of relationships. We can no longer cross our arms and be satisfied with just what the textbooks offer, being limited to a poor and meaningless teaching; we need to show what mathematics takes place behind the door,

> [...] within four walls, in the classroom, each teacher is free to do whatever he or she

> wants with his or her students. To "open the door" of the classroom means, therefore, to take the risk of showing what really happens in a classroom. And that means showing not only the successes and certainties, but also the failures, anxieties and uncertainties experienced in a process of innovation (FIORENTINI & MIORIM, 2001, p. 44).

2.4 THE TEACHER'S ROLE IN MATHEMATICS TEACHING AND THE NCP'S IN THE STUDY OF GEOMETRY

In view of the above, the reality shows that this teaching only happens between four walls because some teachers are still unable to dare in their classes, being mediators, facilitators, evaluators and organizers of this wonderful and challenging knowledge that is mathematics. They still have the idea that they should be the holders and transmitters of knowledge. They have not yet discovered that

> [...] to get the students' attention, you have to use words, and lots of words. Forget the traditional class, in which a certain point of the subject is presented on the blackboard, explained and then practiced through exercises. Because it is mechanical, this type of learning does not assess whether or not the student has understood the knowledge. Instead, try to surprise the class. Show the content by doing a lot of talking and making room for the students (PCN, Special Edition, p. 49-50)

It is in this perspective of showing that mathematics does not only happen inside the classroom Geometry, being an area of mathematics, cannot be taught separately, it needs to go outside the four walls of the classroom.

> Geometry and Mathematics have never been disassociated. Except in textbooks of the past and in old curricula, which provided separate classes. Moreover, the notions of point, line and plane are abstract concepts, with no direct relation to life. Finding teaching material for this is not difficult. Just look around. Doors, windows, wheels, balls, scissors... Everything has shape and volume: the world is pure geometry (FALZETA, 2002, p. 22-23).

The National Curricular Parameters say that geometric concepts are an important part of the mathematics curriculum (BRAZIL, 1998, p. 51). Thus, reflecting the attitude of some professionals who still treat the teaching of mathematics, especially the teaching of geometry, as something without relevance to the student's daily life, the proposal to work geometry in the community garden emerged.

> The teaching of geometry, as presented in most textbooks, still seems to follow the Euclidean model. It starts from premises and definitions such as point, line and plane, from which geometric knowledge is structured. Geometry presented and structured only as a set of well determined laws has always bothered me, because it scares and makes students have the false idea that they have never related to absolutely nothing about what they are learning (CRISTOVÃO, 2001, p. 51).

As we know, the study of Geometry is not well explored; almost always the subject is addressed at the end of the year, in the last bimester. This may be because some teachers have difficulty with the content, either because they don't master it, or because they favor other contents that they think are prerequisites for the next grade, not to mention that the students are alienated from the activities proposed to them, thinking that they don't fit the reality they experience. Thus, educators insist on turning Geometry class into a mere repetition of what is in the textbook, making it fall into neglect, and literally left aside. Teaching Geometry requires the teacher to consider it as an inexhaustible source of ideas, motivating, stimulating, reasoning instigator and, why not challenging, with regard to its conceptualization as well as working on the skills and competencies that the subject requires. For it is known that,

> Too often Geometry is considered by elementary school teachers simply as the study of rectangles, line segments, angles, congruences, and the like. Even in the intermediate grades, Geometry is often neglected until the end of the year, when a few figures and terms are hastily introduced and some exercises are done. (DANA, 1994, p.141).

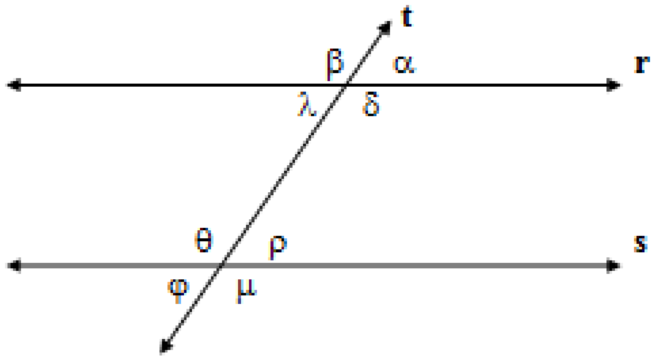
According to the National Curriculum Parameters (1998, p.51), it is essential that the studies of space and shape are explored from objects in the physical world, works of art, paintings, drawings, sculptures and crafts, so that allows the student to establish connections between mathematics and other areas of knowledge. Thus, the motivation to explore geometry further arises, trying to show in practice what the textbook brings.

3 ANGLES, POLYGONS AND AREAS OF PLANE FIGURES

3.1 ANGLES FORMED BY TWO PARALLEL LINES INTERSECTED BY A TRANSVERSAL LINE

Two parallel lines **r** and **s**, intercepted by a transversal line, determine eight angles named thus.

Figure 1: Parallel and competing lines and their eight angles



Fonte: https://br.pinterest.com/pin/angles--438678819926609111/

Corresponding angles:α andρ ,β and, andθλφδ and,; alternate internal angles: andλρ ,δ ; alternate external angles:α and , andφβμ ; collateral internal angles: andλθ ,δ andρ AND collateral external angles:α andμ , and βφ

3.2 PROPERTIES

Internal alternate angles are congruent; external alternate angles are congruent. Corresponding angles are congruent and the sum of the side angles (internal and external) is equal to 180 .º

λ +θ = 180 ºδ +ρ = 180 ºα + = 180μºβ +φ = 180 º

Example 1

The sum of two adjacent angles is 120º . Calculate the measure of each angle, knowing that the measure of one of them is three times the measure of the other minus 40 .º

Solution

$$x + y = 120$$

From the statement, it follows that

$$y = 3x - 40.$$

Taking in the first relation, one has that:

$$x + 3x - 40 = 120 \quad \rightarrow \quad 4x = 160 \quad \rightarrow x = 40$$

Soon,

$$y = 3.40 - 40 \rightarrow y = 2.40 \rightarrow y = 80$$

Example 2

Determine the measure of the supplement of angle 129º 40' 35"

Solution

$$180 - 129° - 40' - 35''$$
$$180° - 129° = 51°$$
$$51º = 50º + 60'$$
$$51º = 50º + 60'$$
$$180 - 129° - 40' = 50º + 60' - 40' = 50° + 20'$$
$$20' = 19' + 60"$$

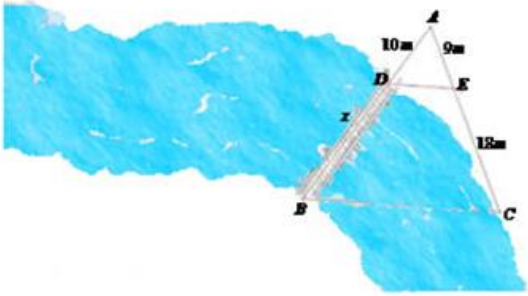
Soon:

$$180 - 129° - 40' - 35'' = 50° + 19' + 25"$$

Example 3: Practical use of Thales' Theorem

Thales' Theorem has several applications in everyday life, being an important tool in Geometry to calculate inaccessible distances and relationships involving similarity between triangles. The best way to visualize the applicability of the Theorem proposed by Thales of Miletus is through some examples**.**

Calculate the length of the bridge that is to be built across the river according to the following diagram



Source: http://mundoeducacao.bol.uol.com.br/matematica/aplicacoes-teorema-tales.htm

According to the figure we have a triangle ABC and the segment DE dividing the triangle, and the triangle ADE is formed. The information we have are the measurements of the following segments: AD = 10m, AE = 9m, EC = 18m and DB = x. The value of DB will be determined through the Theorem of Thales which says: "parallel lines cut by transversals form proportional segments." In this way, we can establish the following relationship:

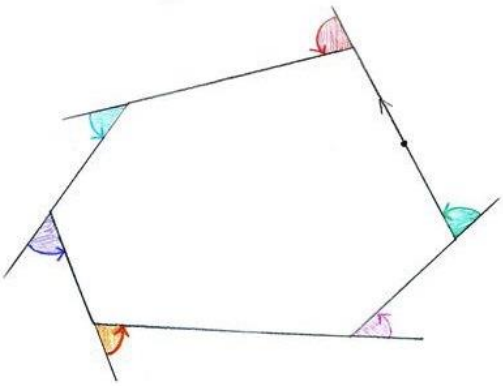
Solution

$$\frac{AD}{DB} = \frac{AE}{EC} \quad \rightarrow \quad \frac{10}{X} = \frac{9}{18} \quad \rightarrow \quad 9x = 180 \quad \rightarrow \quad x = 20m$$

3.3 POLYGONS AND THE SUM OF THE MEASURES OF INTERNAL AND EXTERNAL ANGLES

When a figure is formed by consecutive, non-collinear segments, two by two, these figures are said to be polygons. A, B, C, and D are the vertices of the polygon. AB , BC , CD and DA are the sides of the polygon. Every polygon that has congruent sides and angles is called a regular polygon Consider the n-sided polygon as shown in Figure 2,

Figure 2: Polygons and the sum of their exterior and interior angles



Source: https://gaspacho.matmor.unam.mx/clubmate/secundaria3/191-redondeando-el-hexagono

$$S_i = i_1 + i_2 + i_3 + \ldots + i_n \Rightarrow \quad S_i = (n - 2).180^o \; n \geq 3$$

$$S_e = e_1 + e_2 + e_3 + \ldots + e_n \Rightarrow \quad S_e = 360^o$$

S_i is the sum of the interior angles and S_e is the sum of the exterior angles

The interior and exterior angles of a regular polygon are congruent.

$$a_i = (n - 2).\frac{180}{n}$$

For polygon interior angles and

$$a_e = \frac{360}{n}$$

For exterior angles

Where

$$a_i + a_e = 180$$

a_i is the internal angle of the polygon. a_e is the external angle of the polygon. The sum of the interior angle and the exterior angle of the same vertex is 180° .

Example 1

Find two regular polygons whose ratio of internal angles is 3/5 and the ratio of the number of sides is 1/3.

Solution:

$$a_1 = \frac{(n_1 - 2).180}{n_1} \rightarrow \quad a_2 = \frac{(n_2 - 2).180}{n_2} \quad \rightarrow \quad \frac{a_1}{a_2} = \frac{(n_1 - 2).180}{n_1} . \frac{n_2}{(n_2 - 2)180}$$

$$\frac{a_1}{a_2} = \frac{(n_1 - 2).n_2}{(n_2 - 2).n_1} \quad \rightarrow \quad \frac{a_1}{a_2} = \frac{3}{5} \quad \rightarrow \quad \frac{3}{5} = \frac{(n_1 - 2).h_2}{(n_2 - 2).n_1} \rightarrow \frac{n_1}{n_2} = \frac{1}{3} \quad \rightarrow \quad n_2 = 3n_1$$

$$\frac{3}{5} = \frac{(n_1 - 2).3n_1}{(3n_1 - 2).n_1} \rightarrow \frac{3}{5}(3n_1 - 2) = 3.(n_1 - 2) \rightarrow \ 3n_1 - 2 = 5(n_1 - 2) \rightarrow$$

$$3n_1 - 2 = 5n_1 - 10 \rightarrow \ 3n_1 - 5n_1 = 2 - 10 \rightarrow \ -2n_1 = -8$$

$$n_1 = 4$$

$$n_2 = 3.n_1 \ \rightarrow \ n_2 = 3.4 \rightarrow \ \ n_2 = 12$$

3.4 CALCULATION OF AREAS AND PLANE FIGURES

3.4.1 Calculating the Area of the Triangle

A three-sided polygon is called a triangle. Look at the picture beside. The letter **h** represents the measure of the height of the triangle, as well as letter **b** represents the measure of its base. The area of the triangle will be half of the product of the value of the measure of the base and the value of the measure of the height, as in the formula below:

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$$S = \frac{b.h}{2}$$

In terms of the semi-perimeter, one can use the expression:

$$S = \sqrt{p(p-a).(p-b).(p-c)}$$

The letter **S** represents the area or surface of the triangle. In the case of the equilateral triangle, which has all three internal angles equal, as well as its three sides, the following formula can be used:

$$S = \frac{l^2}{4}\sqrt{3}$$

To calculate the height of this triangle, the Pythagorean theorem is used: That is:

$$L^2 = (\frac{L}{2})^2 + h^2 \rightarrow L^2 - (\frac{L}{2})^2 = +h^2 \rightarrow h^2 = L^2 - \frac{L^2}{4} \rightarrow h^2 = \frac{4L^2 - L^2}{4} = \frac{3L^2}{4} \rightarrow h = \frac{L\sqrt{3}}{2}$$

As the area of the triangle is given by

$$S = \frac{b.h}{2} \rightarrow S = \frac{L..}{2}\frac{L\sqrt{3}}{2} \rightarrow S = \frac{L^2\sqrt{3}}{4}$$

3.4.2 Calculating the Area of a Parallelogram

A quadrilateral whose opposite sides are equal and parallel is called a parallelogram. With h representing the measure of its height and with b representing the measure of its base, the area of the parallelogram can be obtained by multiplying b by h, as in the formula below:

$$S = b\,.\,h$$

3.4.3 Losango Area Calculation

The rhombus is a particular type of parallelogram. In this case not only are the opposite sides parallel, but all four sides are equal. If you have the value of the measures h and b, you can use the parallelogram formula to obtain the area of the rhombus. Another characteristic of the rhombus is that its diagonals are perpendicular.

Notice in the figure on the right, that from the diagonals we can divide the rhombus into four equal triangles. Consider the base b as half of the diagonal d_1 and the height h as half of the diagonal d_2 , to calculate the area of one of these four triangles. We then simply multiply this by 4 to get the area of the rhombus.

$$S = \frac{\frac{d_1}{2} \cdot \frac{d_2}{2}}{2} . 4$$

Making the appropriate simplifications we will arrive at the formula:

$$S = \frac{d_1 . d_2}{2}$$

3.4.4 Square Area Calculation

Every square is also a rhombus, but not every rhombus becomes a square, just as every square is a rectangle, but not every rectangle is a square. The square is a rhombus, which besides having four equal sides, with perpendicular diagonals, also has all its internal angles equal to 90°. Note that the diagonals are not only perpendicular, but also equal. Because the square is a rhombus and because the rhombus is a parallelogram, we can use the same formulas to calculate the area of the square as those used to calculate the area of both the rhombus and the parallelogram.
When you have the measure of the side of a square, you can use the parallelogram formula:

$$S = b \ . \ h$$

Since h and b have the same measure, we can replace them by **l,** and the formula then reads:
Everyday example

A teacher has given her students a rectangular sheet of paper, 1m wide and 80cm high, to be cut into equal squares so that there is no paper left over and the squares are as big as possible. What will be the area of each of these squares?
Solution

Since the teacher wants several equal squares to be formed from the 100cm x 80cm sheet of paper, one realizes that the measure of the sides of these squares must be divisive of both 100 and 80, so that there are no leftovers, and since one wants each of these squares to have the largest possible area, then the measure of the sides of these squares must be the greatest common divisor of 100 and 80, that is, one needs to calculate the MDC(100, 80) to find the measure of the sides of each square.

In the study of Maximum Common Divisor - MDC the calculation of MDC is explained in detail. Calculating the MDC (100,80), the answer is 20, which represents the value of the sides of the squares that must be cut out by the students. Calculating their area, the answer is
the calculation of the area or surface of a square is performed using the formula:

$$S = b\ .\ h$$

Where $L = h = b = 2m \rightarrow \quad S = 20.20 = 400$

Therefore, the area of each of these squares will be 400cm2.

3.4.5 Calculating the Area of the Rectangle

By definition, a rectangle is an equilateral quadrilateral (all internal angles are equal), whose opposite sides are equal. If all four sides are equal, we have a special kind of rectangle, called a square. Because the rectangle is a parallelogram, the calculation of its area is done in the same way. If you call the measures of the sides of a rectangle as in the picture to the right, we will have the following formula:

$$S = b\,.h$$

3.4.6 Calculating the Area of a Circle

Dividing the perimeter of a circle by its diameter will always result in the same value, whatever the circle is. This constant irrational value is represented by the lower case Greek letter pi, spelled as: Because pi is an irrational number, it has infinite decimal places. For everyday calculations, we can use the value 3.14159265. For less precise calculations, we can use 3.1416, or even 3.14. The perimeter of a circle is obtained using the formula:

$$P = 2\pi r$$

The calculation of the area of the circle is performed according to the formula below:

$$S = \pi \,.\, r^2$$

Where **r** represents the radius of the circle.

3.4.7 Calculating the Area of Circular Sectors

The calculation of the area of a circular sector can be done by calculating the total area of the circle and then setting up a rule of three, where the total area of the circle will be for 360°, just as the area of the sector will be for the number of degrees of the sector. With **S** being the total area of the circle, **S_α** being the area of the circular sector, and **α being** its number of degrees, we have:

$$\frac{S}{360} = \frac{S_a}{\alpha}$$

In radians, you have

$$\frac{S}{2\pi} = \frac{S_a}{\alpha}$$

From these sentences we can arrive at this formula in degrees:

$$S = \frac{\pi \,.\, r^2 \,.\, \alpha}{360}$$

And this one in radians:

$$S = \frac{r^2 \,.\, \alpha}{2}$$

Where **r** represents the radius of the circle referring to the sector and **α** is the angle also referring to the sector.

Everyday example

A rectangular table measures 1.2 m x 0.8 m. If you attach a string with a nail to one corner of this table, how long should the string be so that you can span a circular section one-third the area of the table?

The area of the table is equal to:

$$S = b\,.\,h$$

How to

$$b = 1{,}2cm$$
E
$$h = 0{,}8cm$$

Since we have a rectangular table, the angles formed at its edges are 90°. With this data you can calculate the length of string that would have to cover 1/3 of the area of the table, if you use the string as a compass to trace a circular sector on the table:

$$S = b\,.\,h \quad \rightarrow S = 1{,}2.0{,}8 = 0{,}96cm^2$$

Soon:

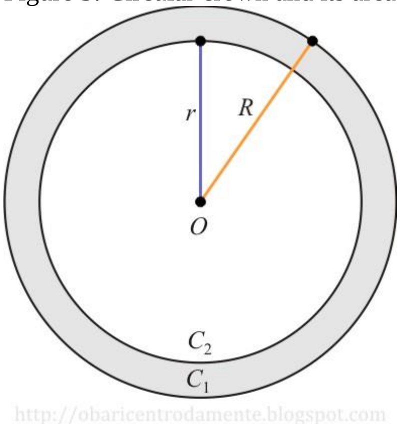
$$S = \frac{\pi\,.\,r^2\,.\,\alpha}{360} \qquad S = \frac{\pi\,.\,r^2\,.\,90}{360} = \frac{\pi\,.\,r^2}{4} \rightarrow S = \frac{\pi\,.\,r^2}{4} \rightarrow r = \sqrt{\frac{4s}{\pi}}$$

Putting the given values together, the length of the string should be approximately 0.6383 m.

3.4.8 Calculating the Area of Circular Crowns

Circular corona is the area between two concentric circles. This area is formed by two circles of radii R and r, where R > r. The circular ring area is widely used in mechanical engineering, in the production of machine parts and accessories.

Figure 3: Circular crown and its area



http://obaricentrodamente.blogspot.com.br/2011/05/area-da-coroa-circular-annulus.html

The calculation of the area of a circular ring can be done by calculating the total area of the circle and subtracting from this, the area of the inscribed circle. We can also use the following formula:

$$S = \pi(R^2 - r^2)$$

Where **R** represents the radius of the circle and **r** represents the radius of the inscribed circle.

3.5 APPLICATION

1-Determine the polygon whose number of diagonals is four times the number of sides.

Solution

$$d = 4.n \to d = \frac{n\ /n - 3}{2} \to 4.n = \frac{n(n-3)}{2} \to$$
$$4.2 = n - 3 \to 8 + 3 = n \to n = 11$$

2-What is the polygon, whose sum of internal angles is worth 1800?º

Solution

$$Si = (n-2).180° \to Si = 1.800° \to 1800° = (n-2).180°$$
$$n - 2 = \frac{1800°}{180°} \to n - 2 = 10 \to n = 10 + 2 \to n = 12$$

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3-Which polygon has the number of sides equal to the number of diagonals?

Solution

$$n = ? \rightarrow d = n \rightarrow d = \frac{n.(n-3)}{2} \rightarrow n = \frac{n.(n-3)}{2}$$
$$2n = n(n-3) \rightarrow n - 3 = 2 \rightarrow n = 3 + 2 \rightarrow n = 5$$

4-The ratio between the internal angle and the external angle of a regular polygon is 9. Determine the number of sides of the polygon?

$$\frac{a_i}{a_e} = 9 \rightarrow a_i = \frac{(n-2).180°}{n} \ \text{ e } \ a_e = \frac{360°}{n} \rightarrow$$
$$\frac{\frac{(n-2).180}{n}}{\frac{360°}{n}} = 9 \rightarrow \frac{(n-2).180°}{360°} = 9$$
$$\rightarrow \frac{(n-2).180°}{2.180°} = 9 \rightarrow n - 2 = 9.2$$
$$n - 2 = 18 \rightarrow n = 20$$

5-Determine the regular polygon whose measure of the internal angle is equal to 4/3 of the measure of a right angle.

$$a_i = \frac{4}{3}.90° \rightarrow a_i = 120° \rightarrow \frac{(n-2).180°}{n} = 120°$$
$$\frac{n-2}{n} = \frac{12}{18} = \frac{2}{3} \rightarrow 3n - 6 = 2n \rightarrow 3n - 2n = 6$$
$$n = 6$$

6- In the figure below ABCD is a square with side equal to cm10 and the arcs indicated have centers, respectively, in points A and C, with radii equal to 10 cm and in EFG and H, with radii equal to 5 cm. Determine the measure of the area of the hatched region is equal to:

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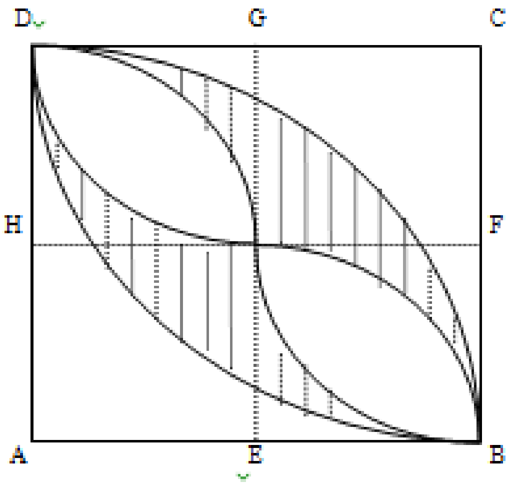


For the large area of the circle one has that:

$$A_t = \frac{l}{2} = \frac{10^2}{2} \rightarrow A_t = 50 \text{ cm}^2$$

$$A_c = \frac{\pi}{4} R^2 \rightarrow A_c = \frac{\pi}{4} . 10^2 = \frac{100}{4} \pi$$

$$A_c = 25\pi \text{ cm}^2$$

$$2. (A_c - A_t) \rightarrow 2. (25\pi - 50) \rightarrow 50(\pi - 2)$$

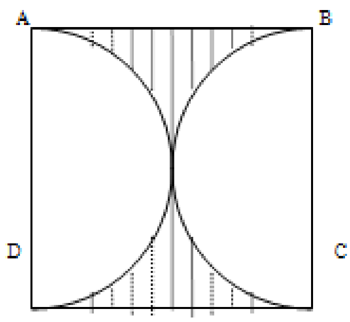
$$A_t = \frac{l^2}{2} = \frac{25}{2} \text{ cm}^2 \rightarrow A_c = \frac{\pi}{4} R^2 = \frac{\pi}{4} . 25$$

$$4. \left(\frac{\pi}{4} . 25 - \frac{25}{2}\right) \rightarrow 25\pi - 50$$

$$\rightarrow 50(\pi - 2) - 25\pi + 50 \rightarrow 50\pi - 100 - 25\pi + 50$$

$$25\pi - 50 \rightarrow 25(\pi - 2)$$

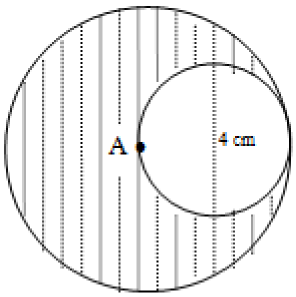
7-A square ABCD has sides of 20 cm. With center at the midpoints of sides AD and BC , draw the arcs AD and BC. What is the area of the hatched region**?**



$$l = 20 \text{ cm} \quad R = 10 \text{ cm}$$
$$A_q = l^2 \rightarrow A_q = 20^2 \rightarrow A_q = 400 \text{ cm}^2$$
$$A_c = \frac{1}{2}\pi r^2 = \frac{\pi}{2}.10^2 = \frac{100}{2}\pi \rightarrow A_c = 50\pi$$
$$\rightarrow A_{TC} = 2.50\pi \rightarrow A_{TC} = 100\pi$$
$$A = A_q - A_{TC} \rightarrow A = 400 - 100\pi$$
$$A = 100(4 - \pi)$$

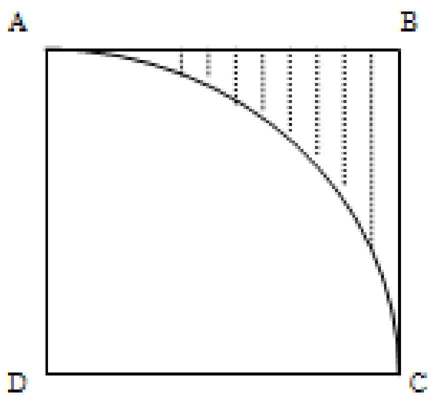
8- Calculate the area of the hatched regions in the figures below:

a) A is the center of the largest circle.



$$r = 2 \text{ cm} \quad R = 2.r \rightarrow R = 2.2 \rightarrow R = 4 \text{ cm}$$
$$A = A_R - A_r \rightarrow A = \pi R^2 - \pi r^2$$
$$A = \pi.4^2 - \pi.2^2 \rightarrow A = 16\pi - 4\pi$$
$$A = 12\pi \text{ cm}^2$$

b) The square ABCD has sides 4 cm. With center at D, we draw the arc AC.



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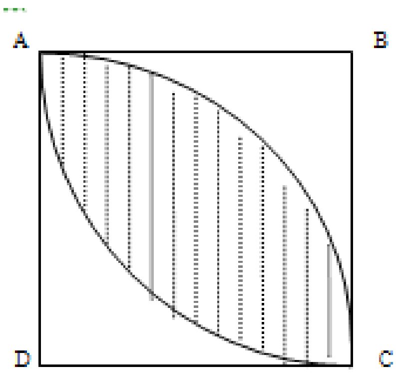
$$l = 4 \text{ cm} \rightarrow A_q = A_2 + A_1 \rightarrow l = \frac{1}{4}A_c + A_1$$

$$4^2 = \frac{1}{4}.\pi R^2 + A_1 \rightarrow 16 = \frac{1}{4}.\pi.l^2 + A_1$$

$$16 = \frac{16}{4}\pi + A_1 \rightarrow A_1 = 16 - 4\pi$$

$$A_1 = 4(4 - \pi)$$

c) The square ABCD has sides of 8 cm,



$$l = 4 \text{ cm}$$

$$R = 4 \text{ cm}$$

$$A_t = \frac{b.h}{2} = \frac{A_q}{2} \rightarrow A_t = \frac{l^2}{2} = \frac{4^2}{2} = \frac{16}{2}$$

$$A_t = 8 \text{ cm}^2$$

$$A_c = \frac{1}{4}\pi R^2 = \frac{1}{4}\pi l^2 \rightarrow A_c = \frac{1}{4}\pi.4^2 = \frac{16}{4}\pi \rightarrow$$

$$A_c = 4\pi$$

$$\rightarrow A = A_c - A_t \rightarrow A = 4\pi - 8 \rightarrow A = 4(\pi - 2)$$

$$\rightarrow A_t = 2.A \rightarrow A_t = 2.4(\pi - 2)$$

$$A_t = 8(\pi - 2)$$

d) The square ABCD has sides of 8cm. With centers at B and D we draw the centers at EFGH , we draw the arcs AB, BC, CD arcs AC. AND AD**.**

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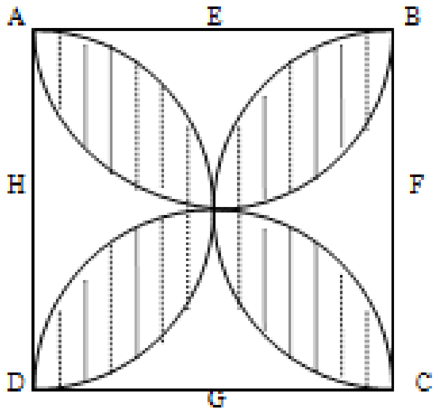


$$A_q = \frac{l^2}{4}, \quad \text{como} \quad l = 8 \text{ cm} \quad \rightarrow Ag = \frac{64}{4} \rightarrow$$

$$A_q = 16 \text{ cm}^2$$

$$A_c = \frac{\pi r^2}{4} = \frac{\pi}{4}.\left(\frac{l}{2}\right)^2 = \frac{\pi}{4}.4^2 = \frac{16\pi}{4} \rightarrow$$

$$A_c = 4\pi$$

$$2A_1 + A_3 = A_q \rightarrow 2\left(A_q - A_c\right) + A_3 = A_q$$

$$2(16 - 4\pi) + A_3 = 16 \rightarrow 32 - 8\pi + A_3 = 16$$

$$A_3 = 8\pi - 32 + 16 \rightarrow A_3 = 8\pi - 16$$

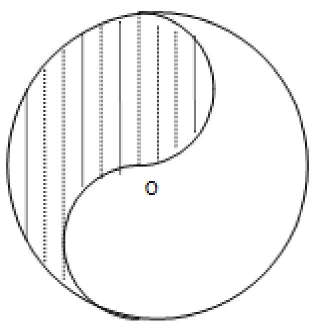
$$A_3 = 8(\pi - 2)$$

As there are four areas A_3we have that:

$$A_t = 4.A_3 \rightarrow A_t = 4.8(\pi - 2)$$

$$A_t = 32(\pi - 2)$$

e) The larger circle has radius R. The two smaller ones tangent at O.



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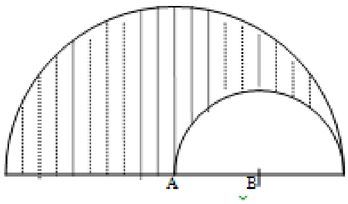
By filling the top part into the bottom part, you get an area of a larger semicircle. Then:

$$A_c = \frac{\pi r^2}{2}$$

f) In the figure we have two semicircuncles one has center A and diameter 8 cm, and the other has center B.



$$A_1 = \frac{1}{2}\pi . R^2$$

How to $R = 2r \rightarrow D = 2R \rightarrow 8 = 2R \rightarrow$

$$R = 4 \text{ cm}$$
$$R = 2r \rightarrow r = \frac{R}{2} = \frac{4}{2} \rightarrow r = 2\text{cm}$$
$$A_1 = \frac{1}{2}\pi . R^2 = \frac{1}{2}\pi . 4^2 = \frac{16}{2}\pi$$
$$A_1 = 8\pi$$
$$A_2 = \frac{1}{2}\pi r^2 = \frac{1}{2}\pi . 2^2 = \frac{4}{2}\pi \rightarrow$$
$$A_2 = 2\pi$$
$$A = A_1 - A_2 \rightarrow A = 8\pi - 2\pi$$
$$A = 6\pi$$

9-What is the size of the angle that is equal to twice the size of its complement?

Let x be the angle and $90 - x$ be the complement.

Then according to the statement, it follows that:

$$x = 2.(90 - x) \ \rightarrow \ x = 180 - 2x \rightarrow \ 3x = 180 \ \rightarrow \ x = 60$$

10-Find the measure of the angle that is equal to 1/5 of the measure of your supplement.

From the statement, it follows that:

$$x = \frac{1}{5}(180 - x) \rightarrow 5x = 180 - x \rightarrow 6x = 180 \rightarrow x = 30°$$

What is the angle that exceeds its complement by 76 ?º

$$x = 90 - x + 76 \rightarrow 2x = 166 \rightarrow x = 83°$$

4 FINAL CONSIDERATIONS

Observing what was addressed throughout the text, it appears that plane geometry occupies a privileged place in the field of mathematics, in view of the theory addressed coupled with the practice can contribute to a better understanding in the classroom. According to what was observed, the NCP warns about the issue of content practices taught by education professionals. This point, which refers to contextualization, has been much discussed and the teacher, as the main actor in the teaching and learning process, should be aware of this fact, not allowing his work in the classroom to be a reason for lack of interest and motivation for the students. As a professional, he or she must use resources that are capable of providing the student with a better approach to the subject of mathematics, building classroom problems that are within their reality.

Many times school failure is not only due to the student, because the teacher must constantly rethink his work in the classroom. Innovate their methodologies and discuss them so that students learn in the best possible way what is being discussed. Therefore, teaching is not an easy task when the teacher does not seek methodological strategies that are crucial to the best method of teaching and learning. Therefore, it is not enough to be just a teacher, but an educator in constant evolution, varying and innovating the teaching methodologies so that it is possible to build a consolidated base of contents that are not only approached in the classroom from an analytical and obscure point of view, being far from the students' daily problems.

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